CMSC 471: Games
MCTS

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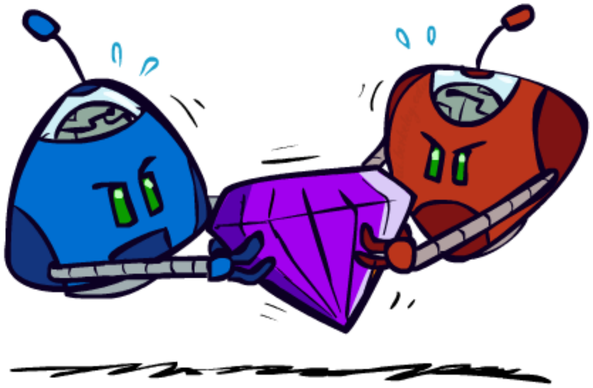
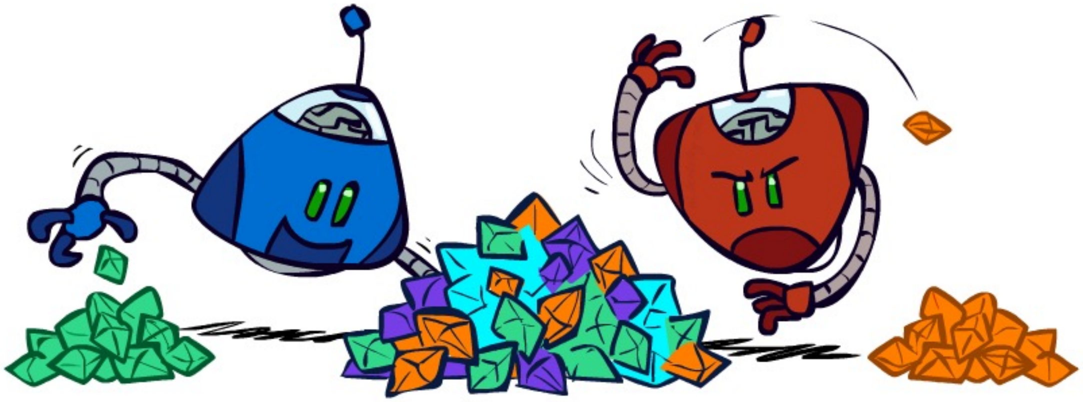
Some materials adopted from slides by Dan Klein and Pieter Abbeel, Stuart Russell and Dawn Song at UC Berkeley
[http://ai.berkeley.edu]

Zero-Sum Games

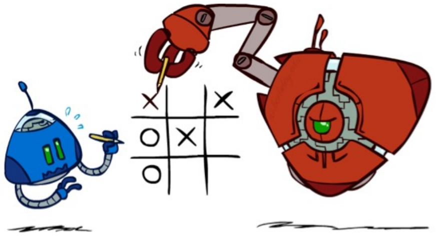


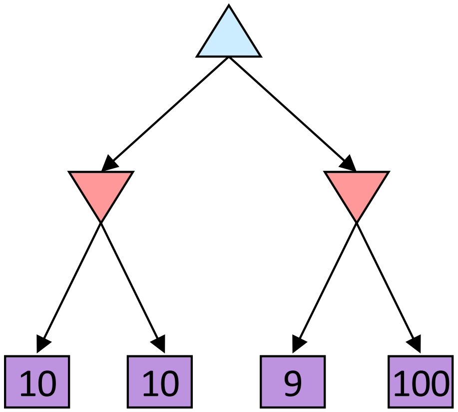


- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - **Lets us think of a single value that one maximizes and the other minimizes**
  - Adversarial, pure competition

- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, and more are all possible
  - More later on non-zero-sum games

Chance outcomes in trees

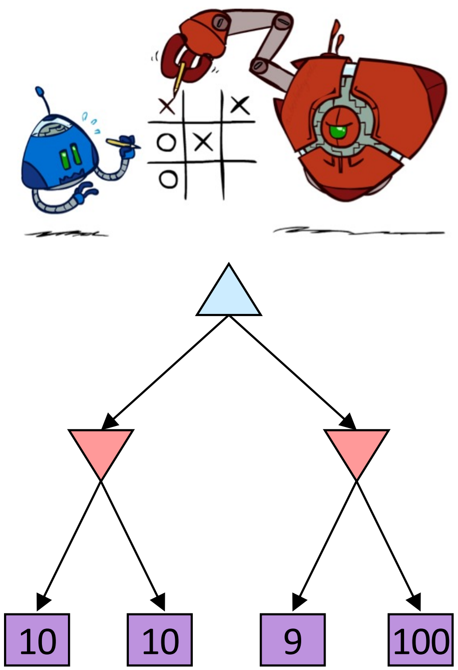




Tictactoe, chess

Minimax

Chance outcomes in trees



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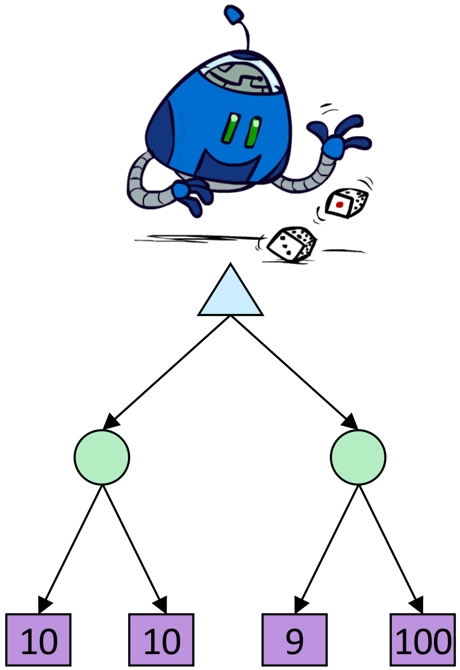
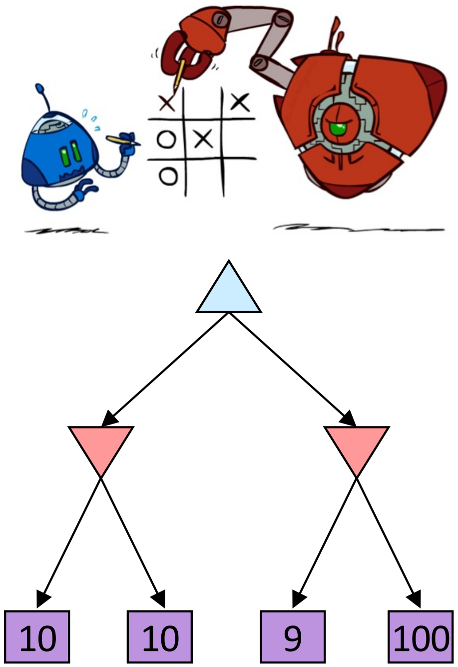


Tetris, investing
Expectimax

Chance outcomes in trees



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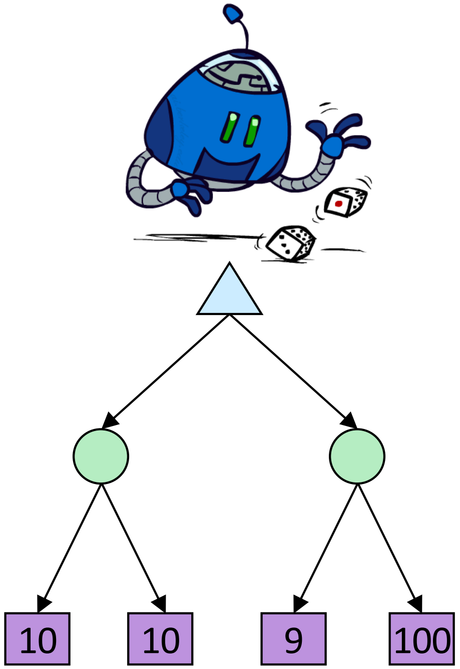

Tetris, investing

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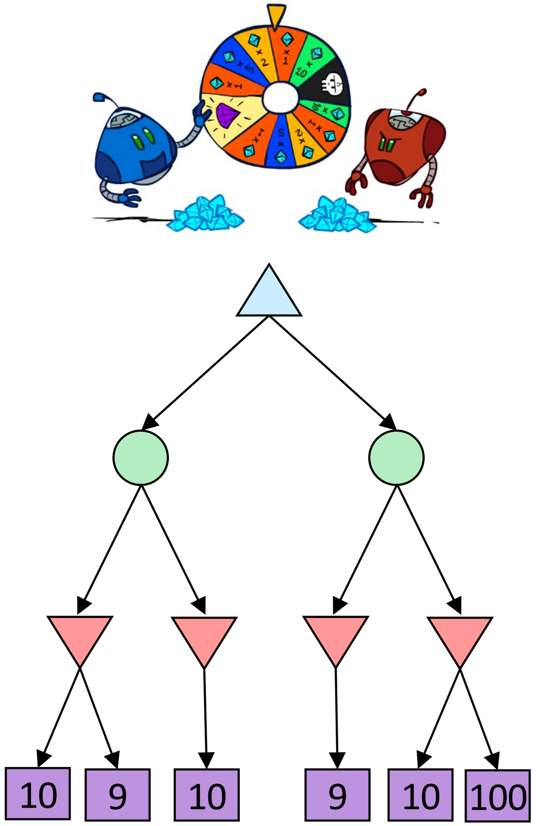
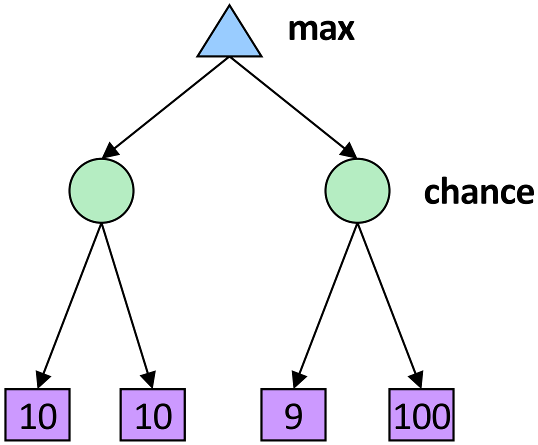


Backgammon, Monopoly

Expectiminimax

Expectimax Search

- Why wouldn't we know what the result of an action will be?
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the ghosts respond randomly
  - Actions can fail: when moving a robot, wheels might slip
- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes



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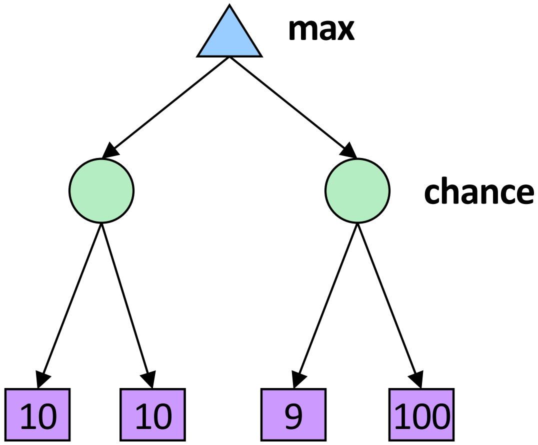
- Expectimax search: compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their expected utilities
  - I.e. take weighted average (expectation) of children

- Later, we'll learn how to formalize the underlying uncertain-result problems as Markov Decision Processes



Expectimax Pseudocode

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def value(state):
        if the state is a terminal state: return the state's utility
        if the next agent is MAX: return max-value(state)
        if the next agent is EXP: return exp-value(state)
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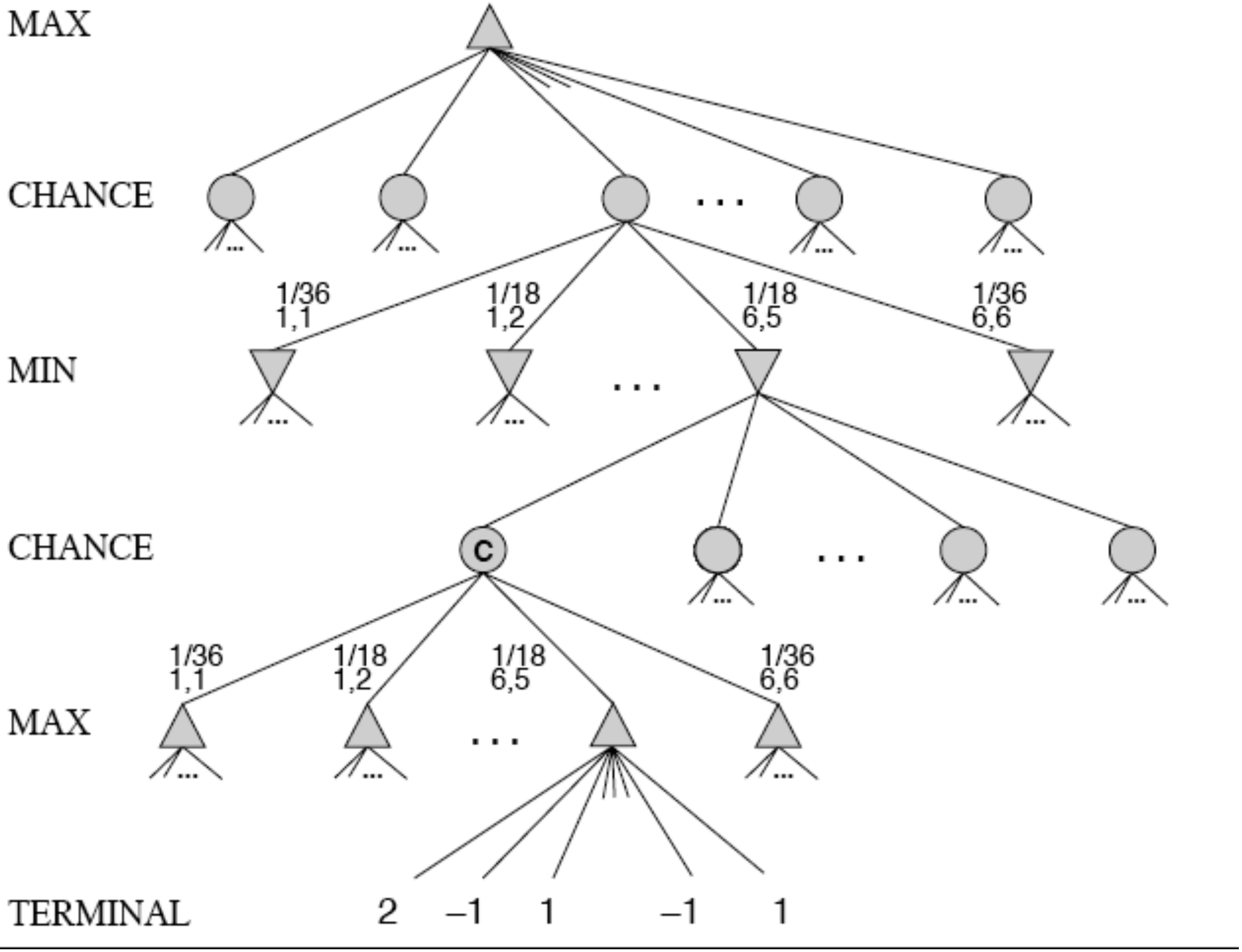
```
def max-value(state):
      initialize v = -∞
      for each successor of state:
            v = max(v, value(successor))
      return v
```

```
def exp-value(state):
      initialize v = 0
      for each successor of state:
                p = probability(successor)
            v += p * value(successor)
      return v
```

MiniMax trees with Chance Nodes



High-Performance Game Programs

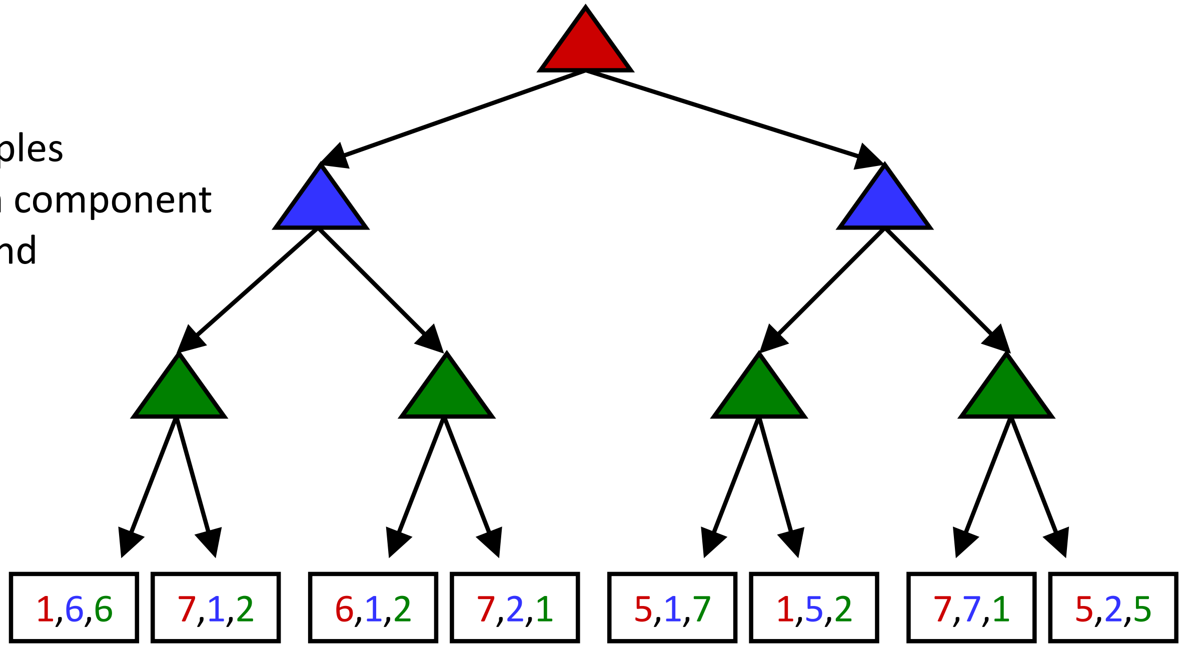
- Many programs based on alpha-beta + iterative deepening + extended/singular search + transposition tables + huge databases + ...
- Chinook searched all checkers configurations with ≤ 8 pieces to create endgame database of 444 billion board configurations
- Methods general, but implementations improved via many specifically tuned-up enhancements (e.g., the evaluation functions)

Other Issues

- Multi-player games, no alliances
  - E.g., many card games, like Hearts
- Multi-player games with alliances
  - E.g., Risk
  - More on this when we discuss game theory
  - Good model for a social animal like humans, where we must balance cooperation and competition

Multi-Agent Utilities

- What if the game is not zero-sum, or has multiple players?
- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically…



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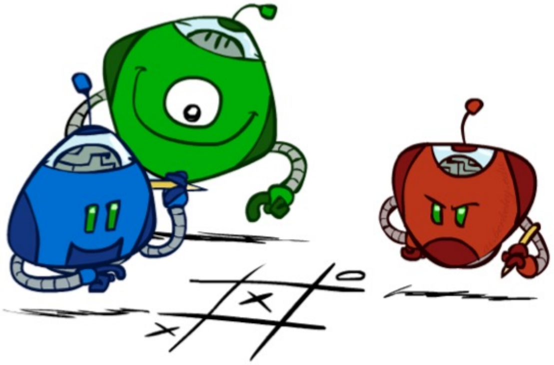

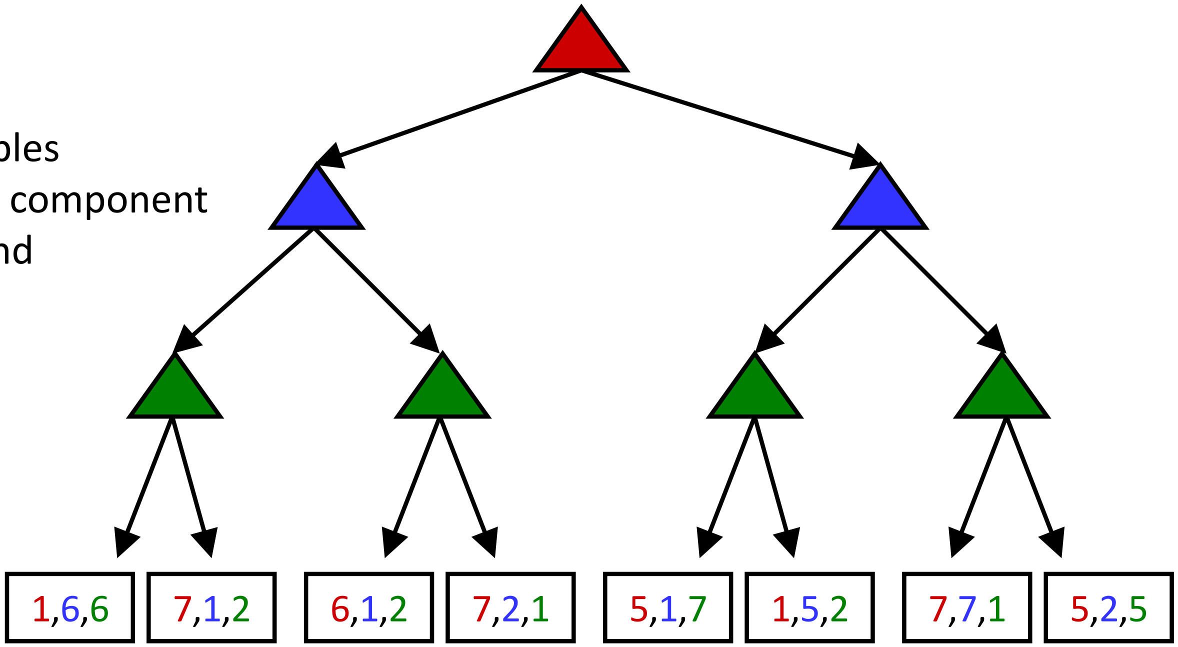

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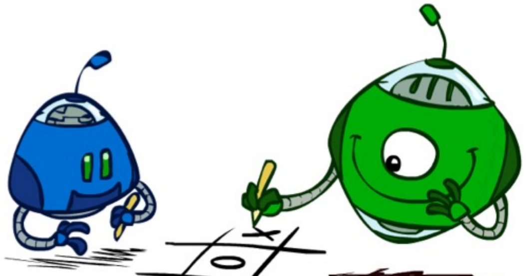

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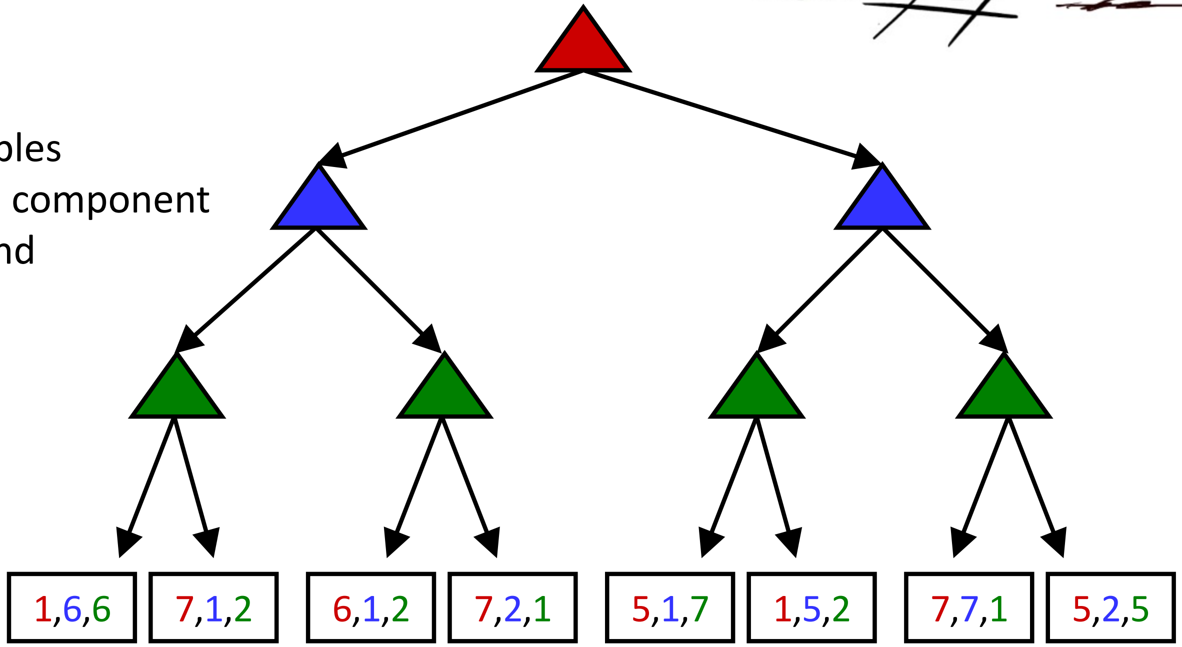
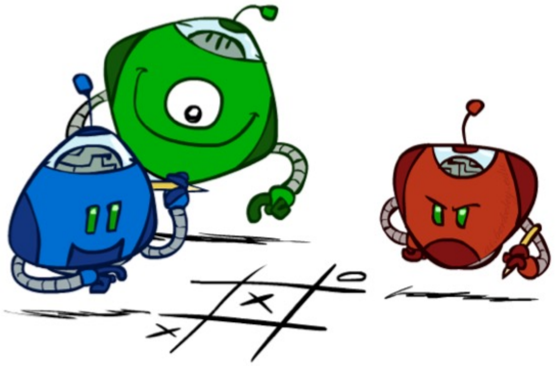
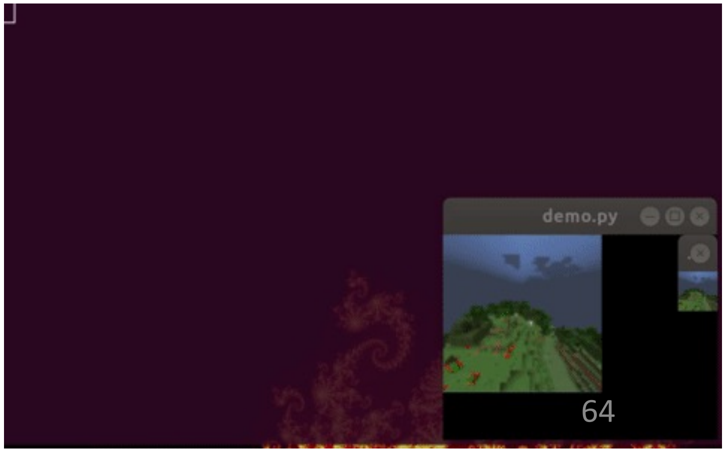




AI and video Games

- Many games include agents run by the game program as
  - Adversaries, in first person shooter games
  - Collaborators, in a virtual reality game
  - E.g.: AI bots in Fortnite Chapter 2
- Some games used as AI/ML challenges or learning environments
  - MineRL: train bots to play Minecraft
  - MarioAI: train bots for Super Mario Bros



AlphaGO

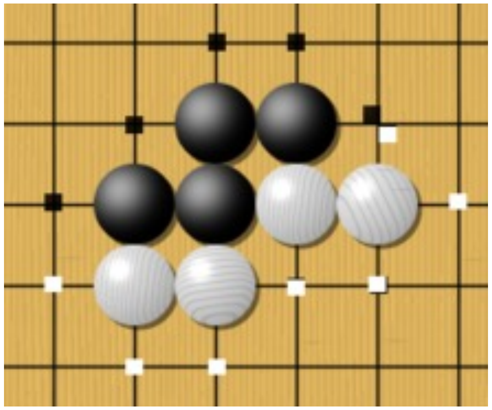
- Developed by Google’s DeepMind
- Beat top-ranked human grandmasters in 2016
- Used Monte Carlo tree search over game tree

  expands search tree via random sampling of search space
- *Science* Breakthrough of the year runner-up

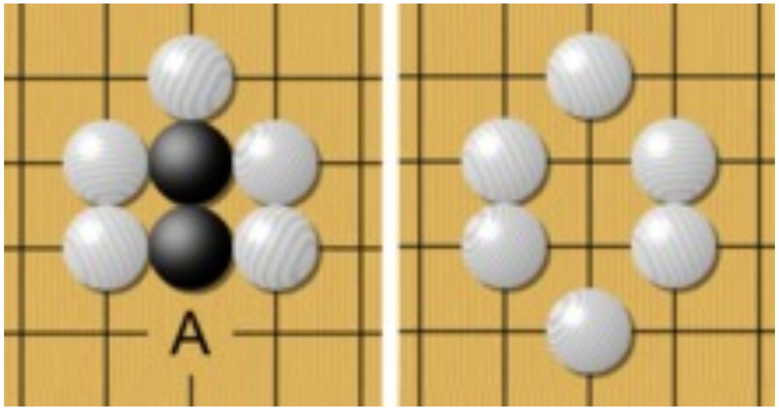
  Mastering the game of Go with deep neural networks and tree search, Silver et al., *Nature*, 529:484–489, 2016
- Match with grandmaster Lee Sedol in 2016 was subject of award-winning 2017 AlphaGo

Go - the game

- Played on 19x19 board; black vs. white stones
- Huge state space $O(b^d)$: chess:~35^{80}, go: ~250^{150}
- Rule: Stones on board must have an adjacent open point ("liberty") or be part of connected group with a liberty. Groups of stones losing their last liberty are removed from the board.



liberties



capture

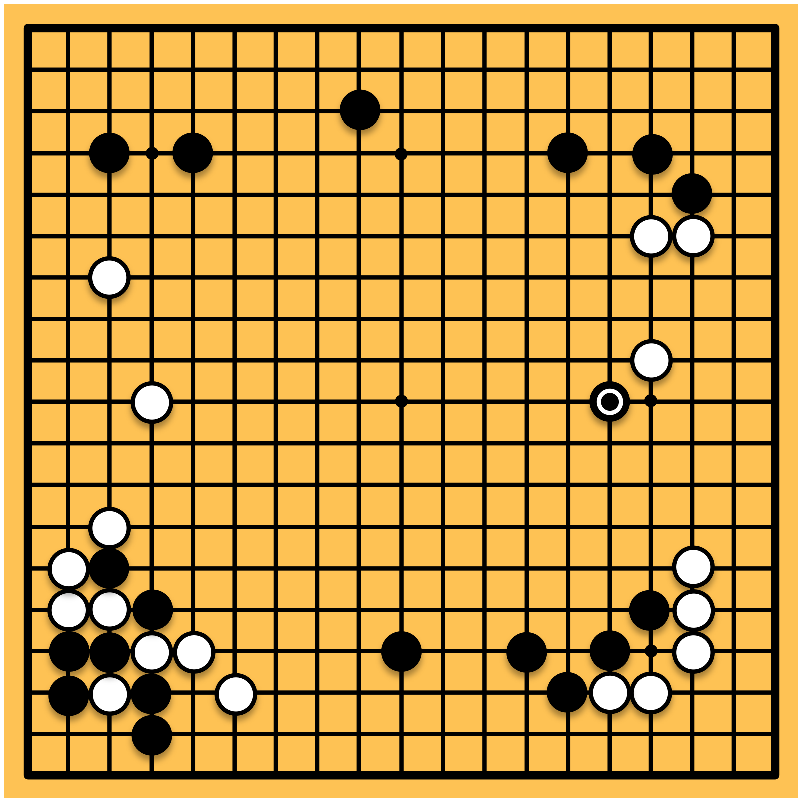
Monte Carlo Tree Search

- Methods based on alpha-beta search assume a fixed horizon
  - Pretty hopeless for Go, with $b > 300$
- MCTS combines two important ideas:
  - ***Evaluation by rollouts*** – play multiple games to termination from a state s (using a simple, fast rollout policy) and count wins and losses
  - ***Selective search*** – explore parts of the tree that will help improve the decision at the root, regardless of depth

Rollouts

- For each rollout:
  - Repeat until terminal:
    - Play a move according to a fixed, fast rollout policy
  - Record the result

“Move 37”



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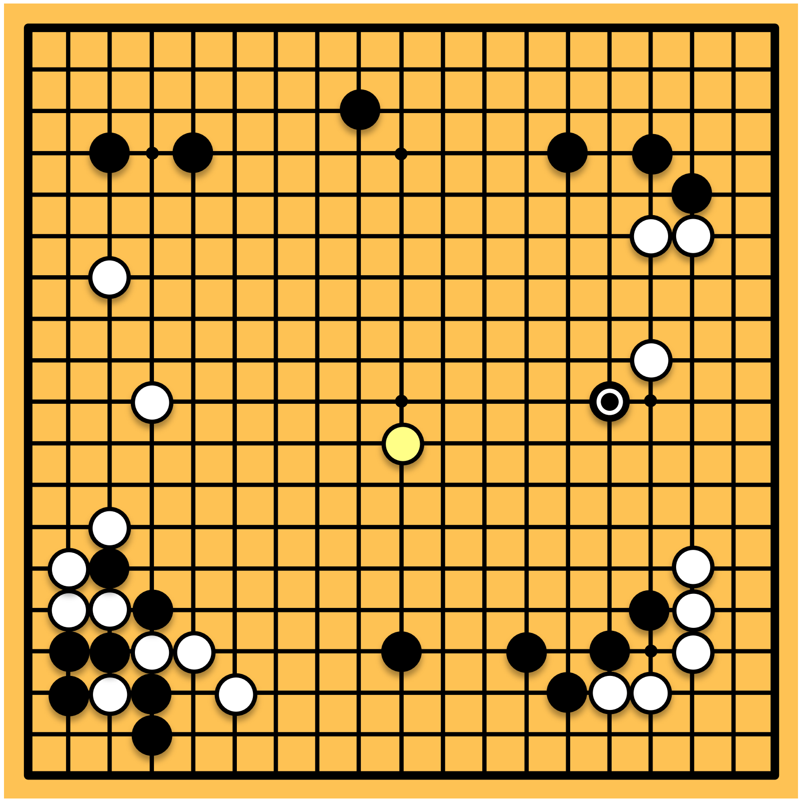

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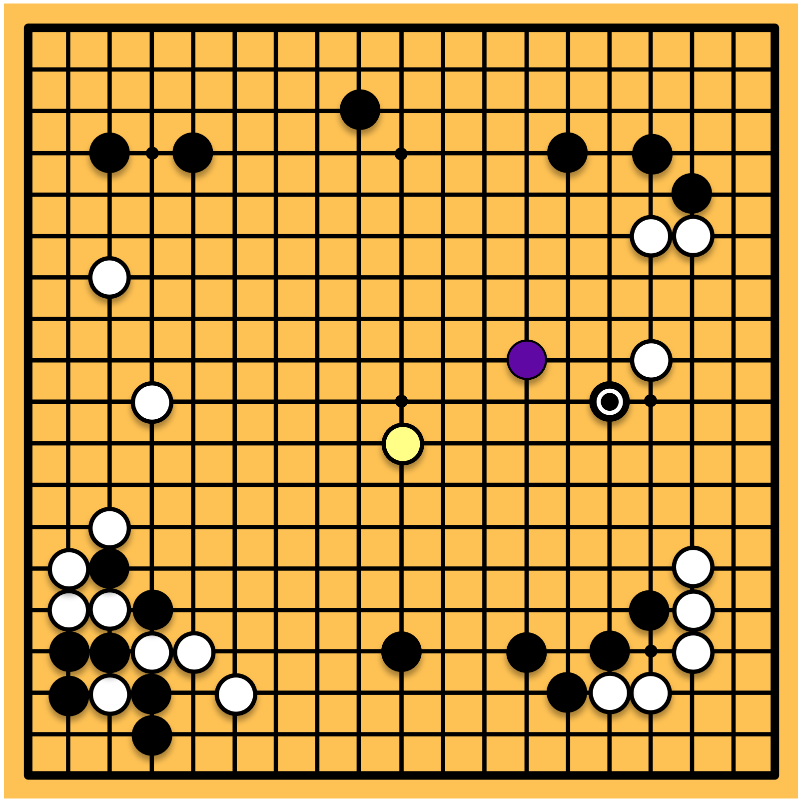

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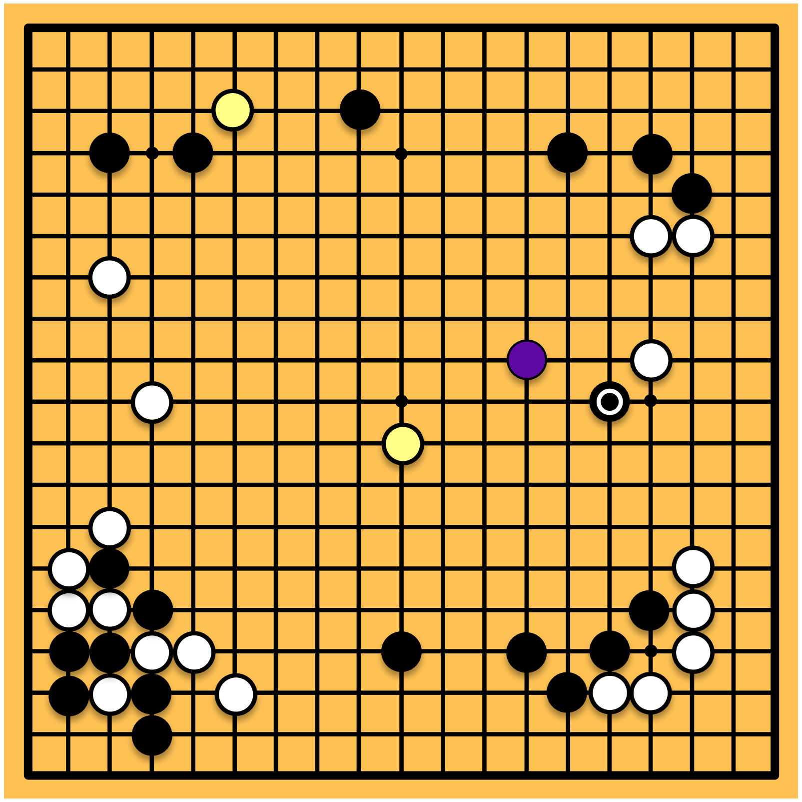

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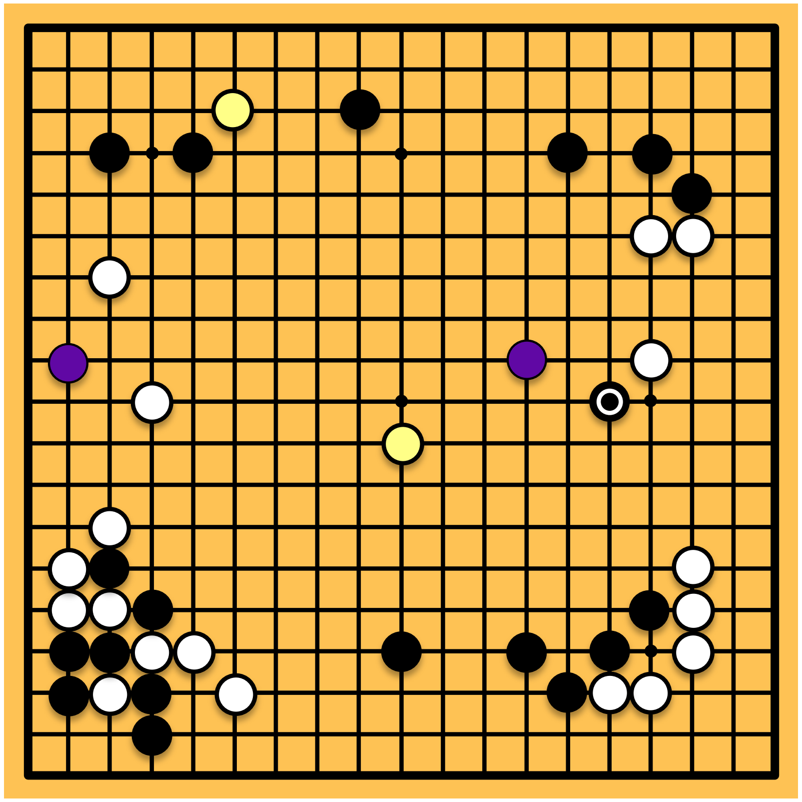
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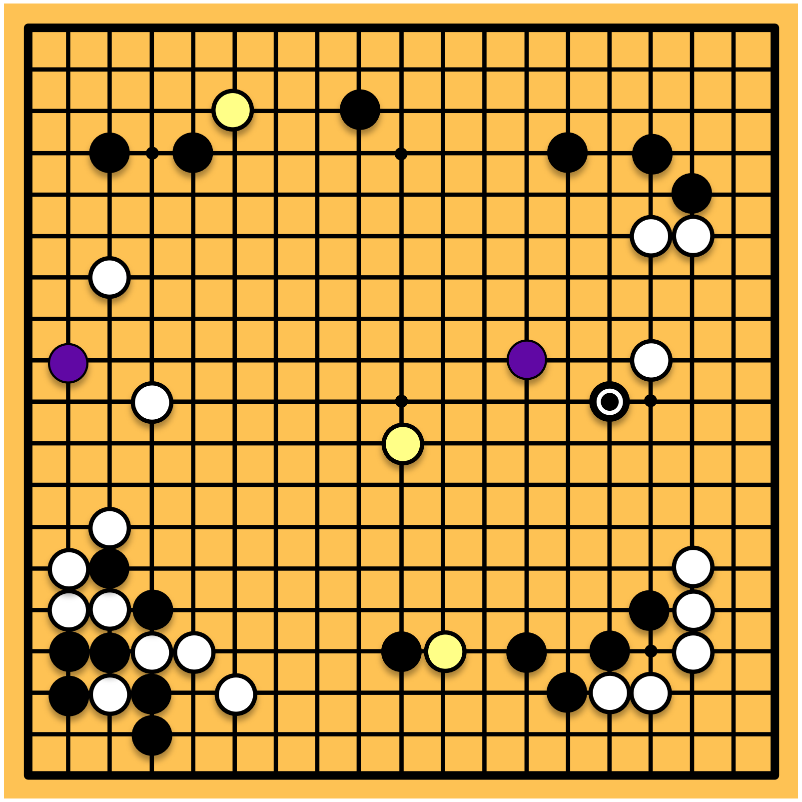

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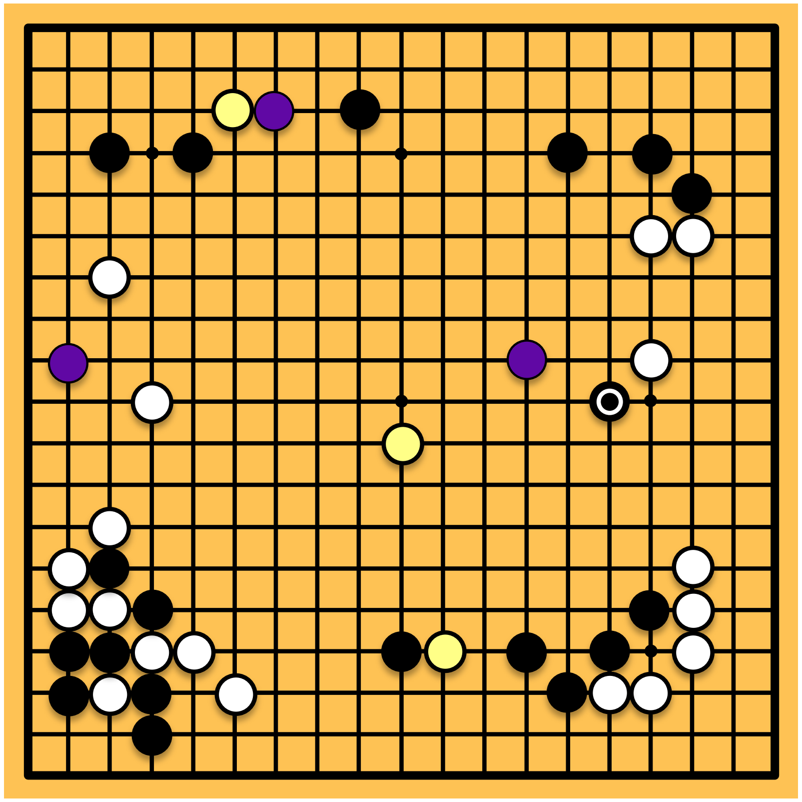

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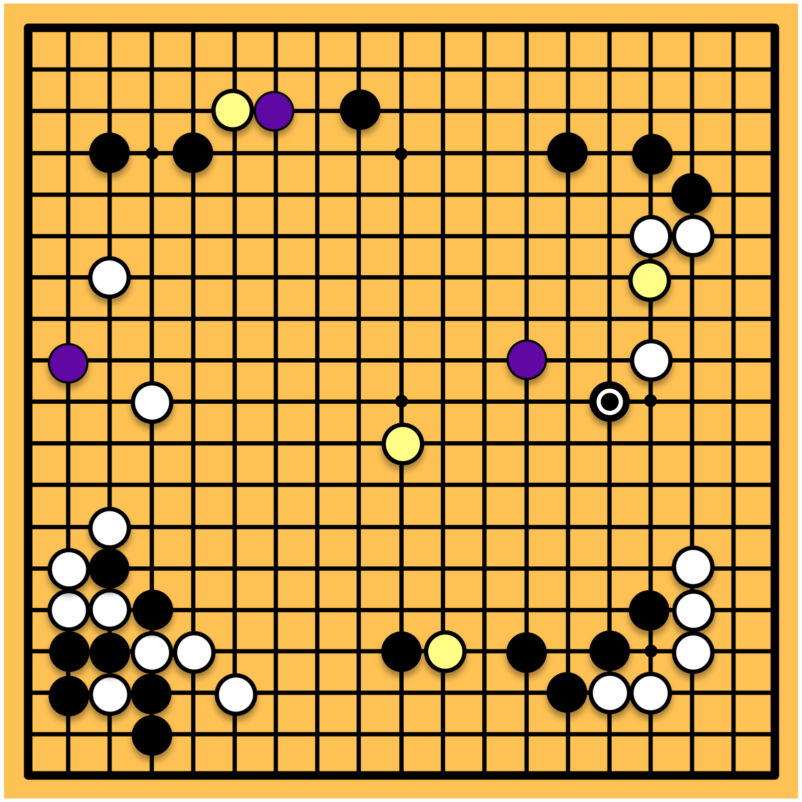

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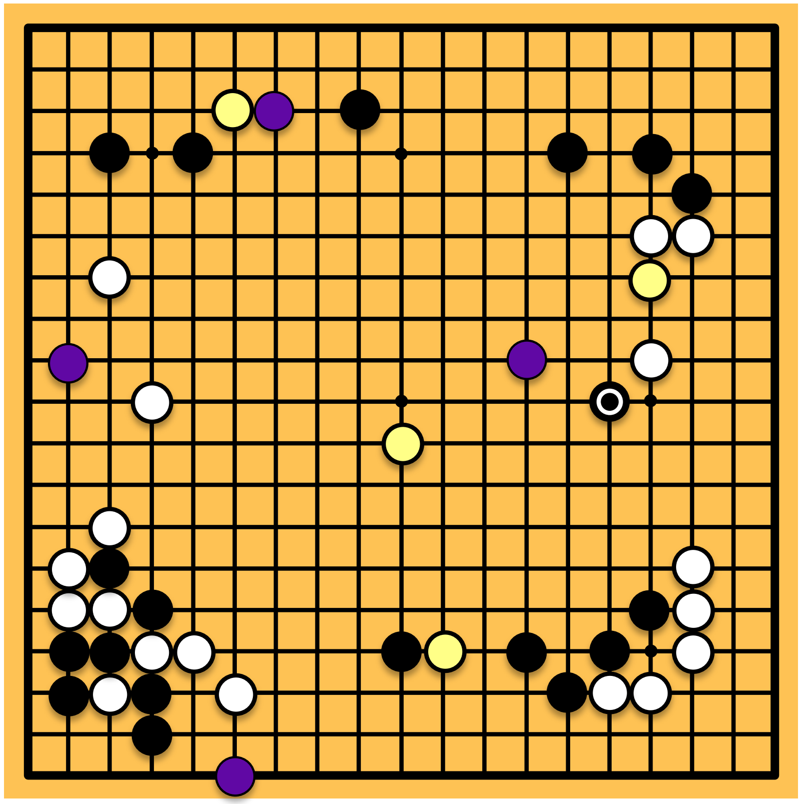

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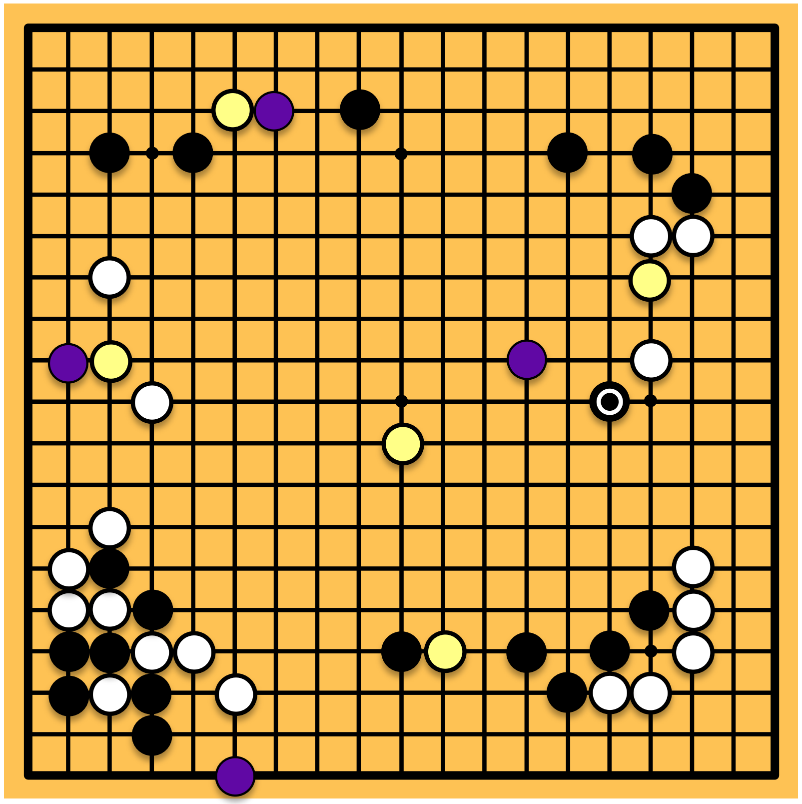

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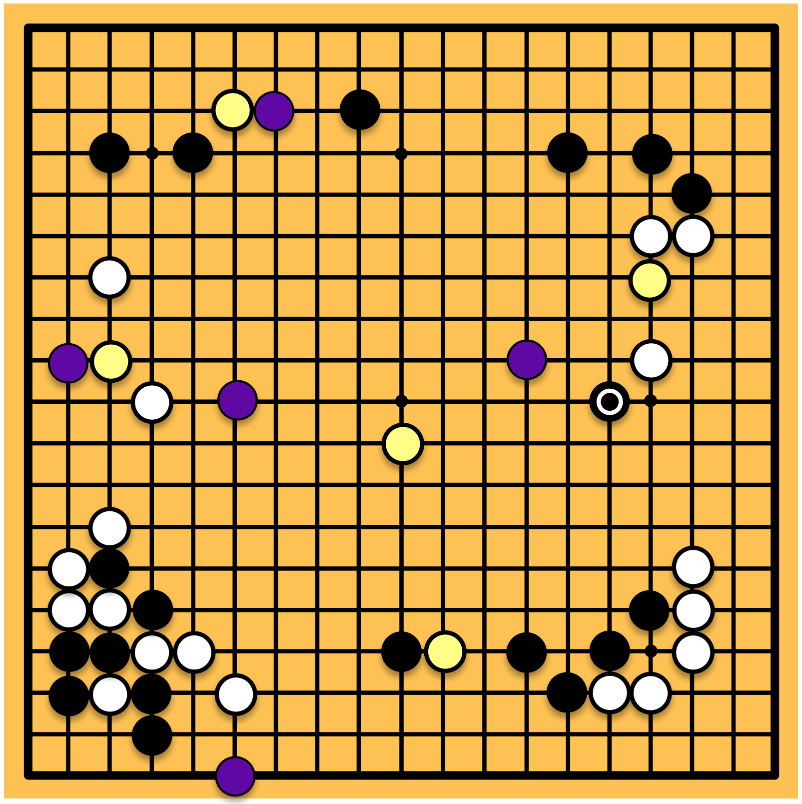

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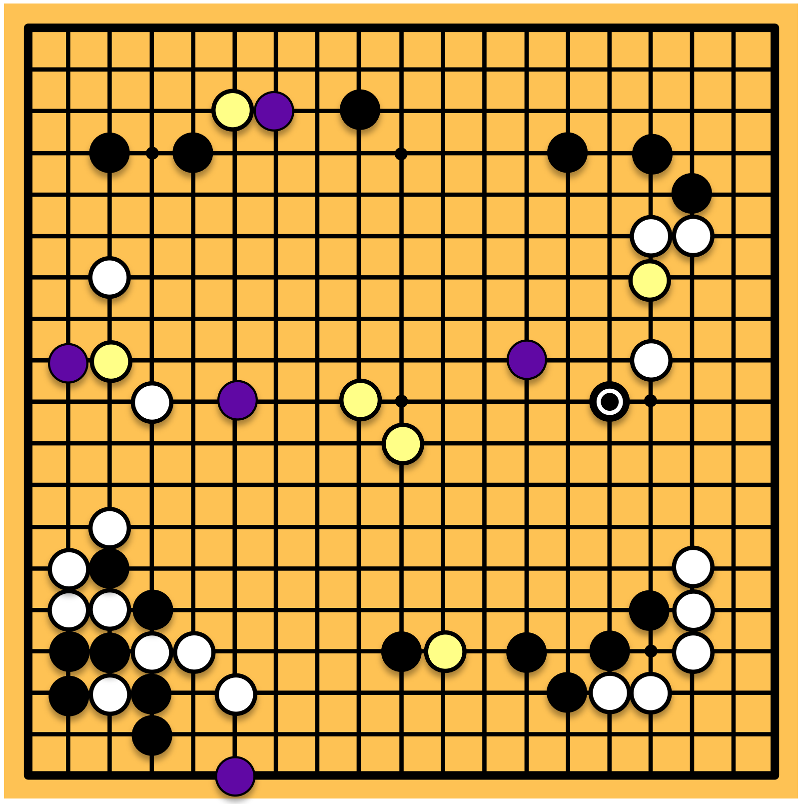

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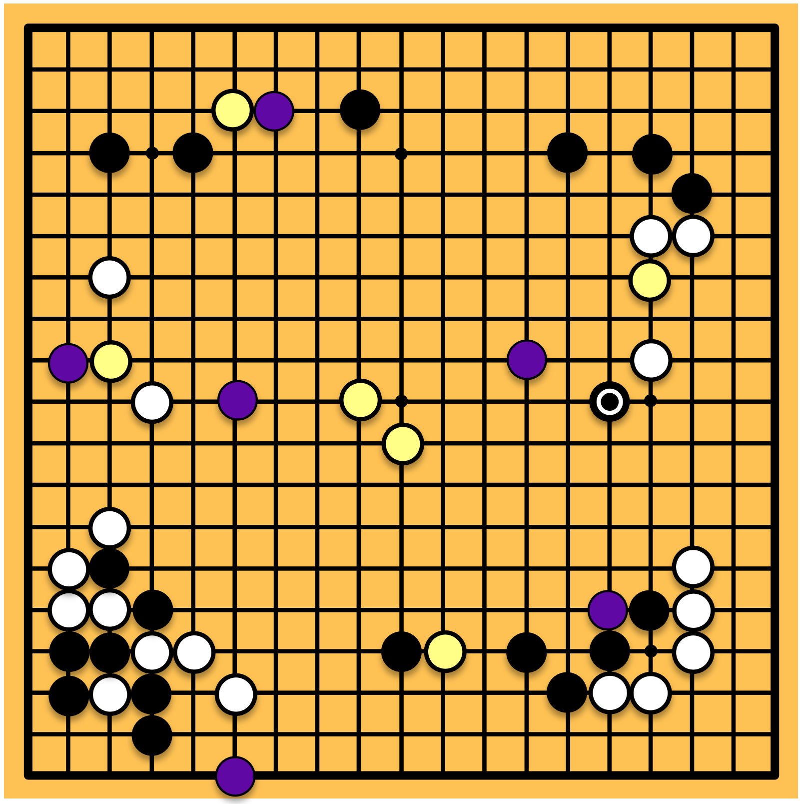

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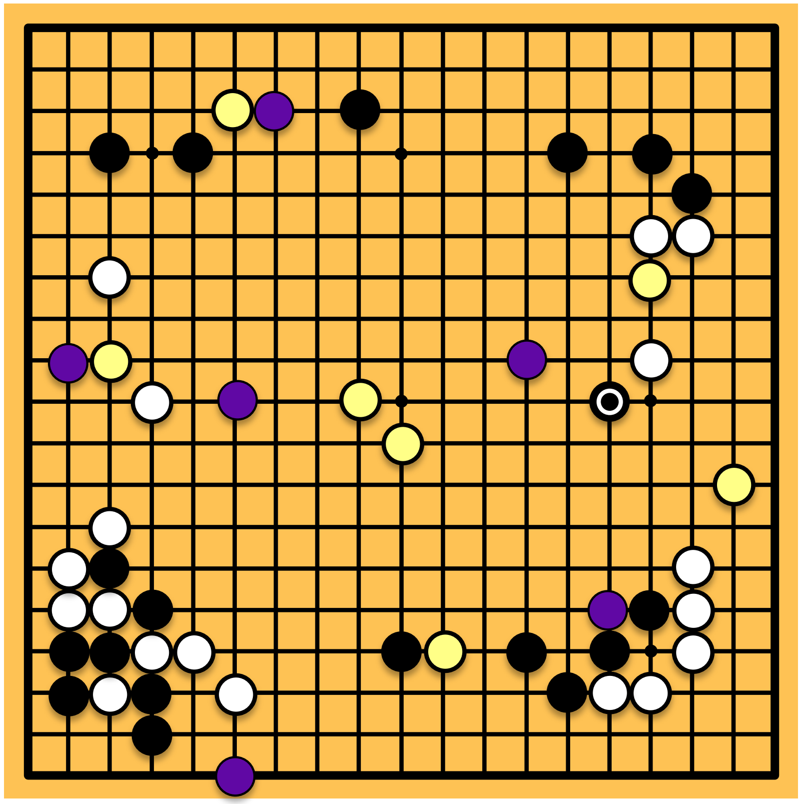

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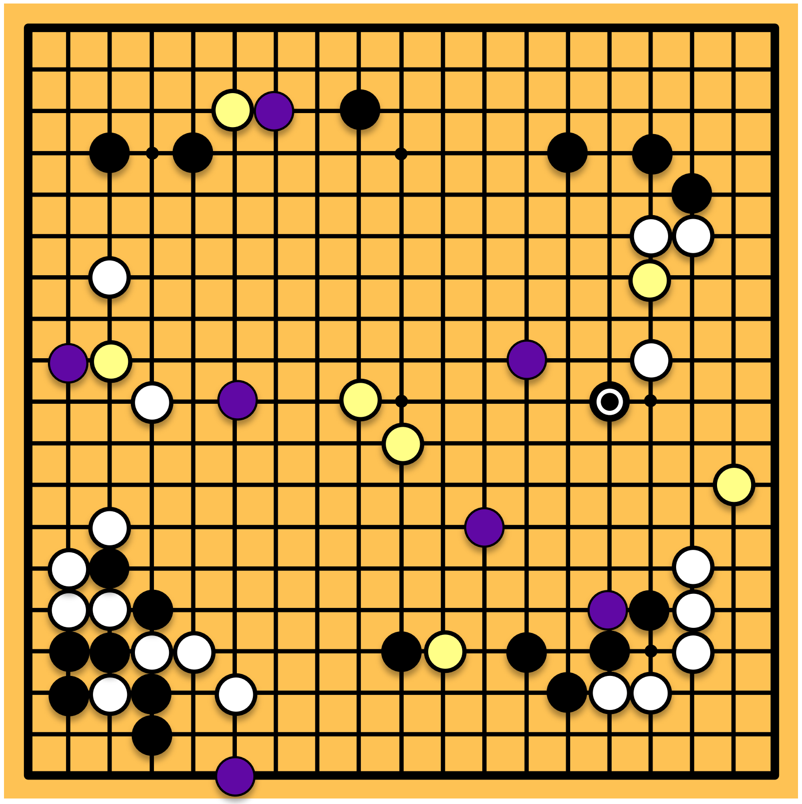

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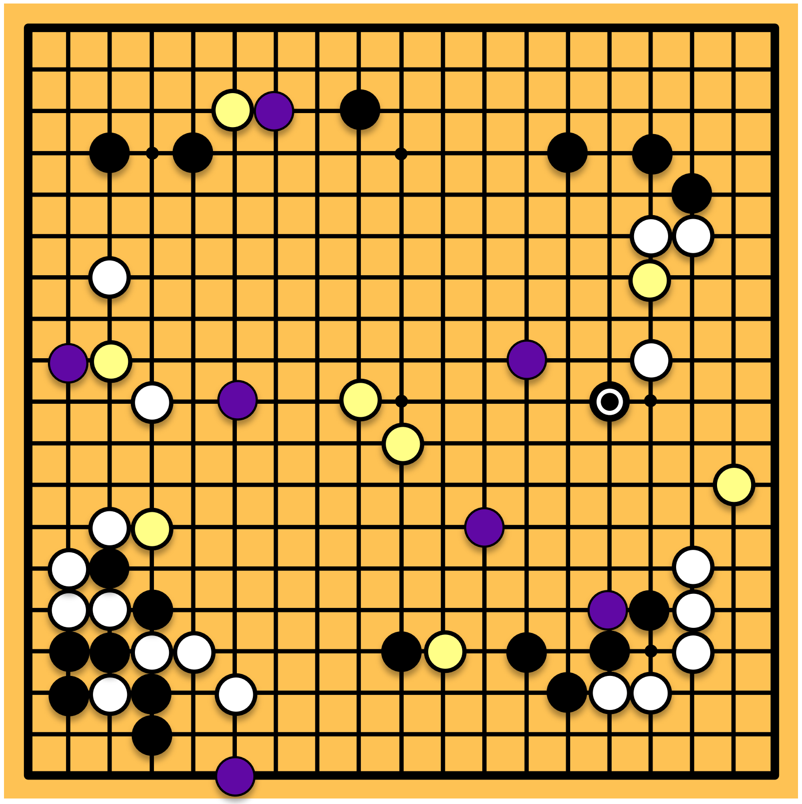

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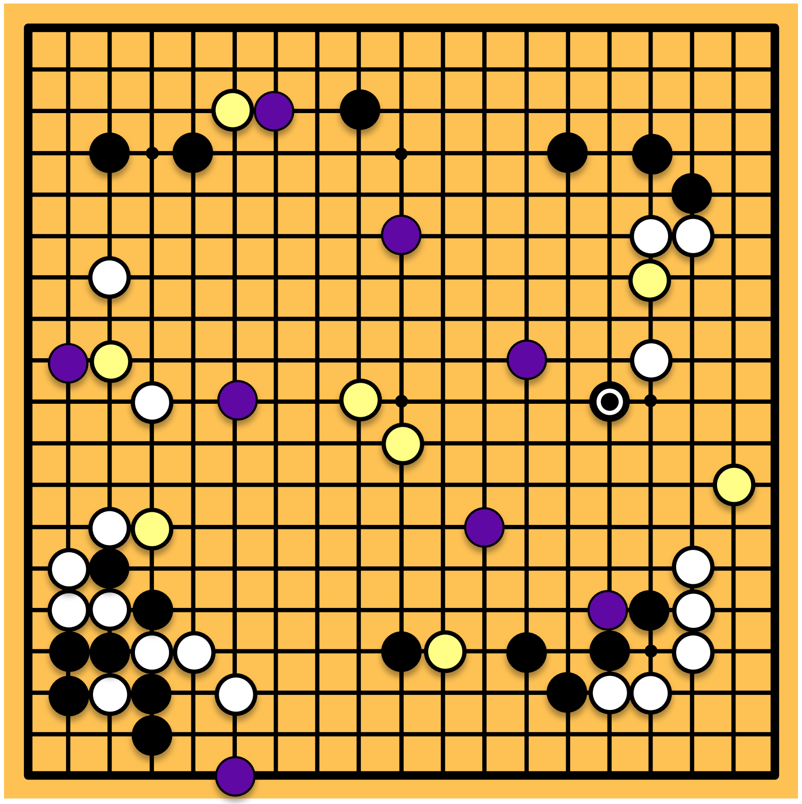

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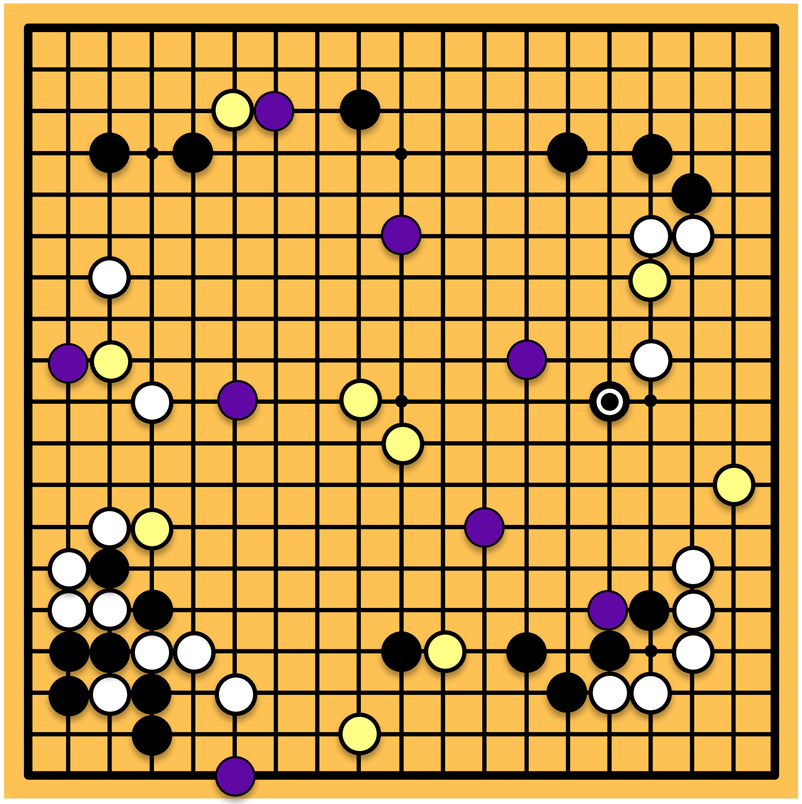

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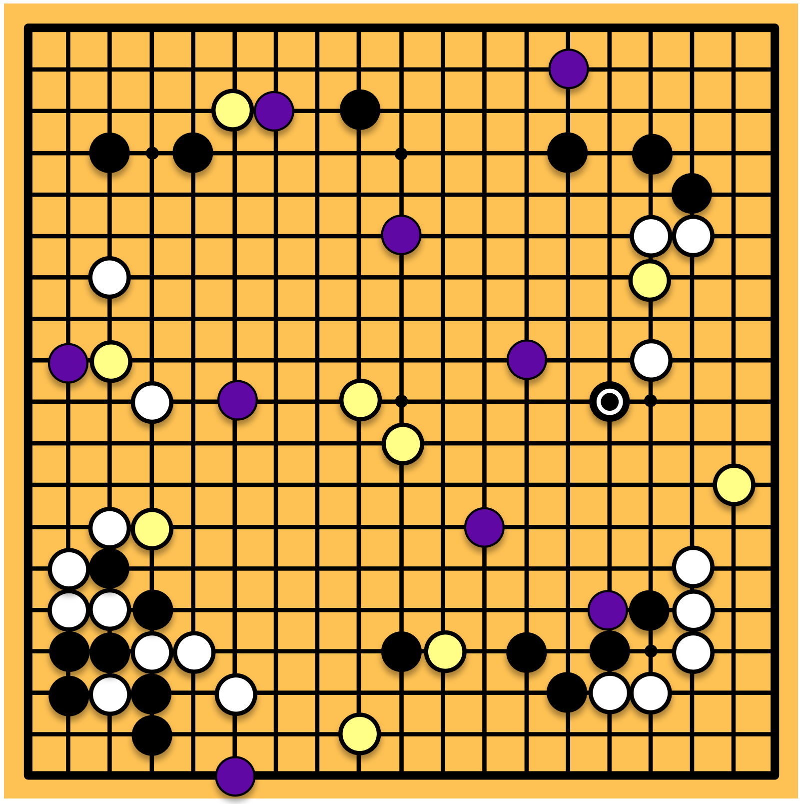

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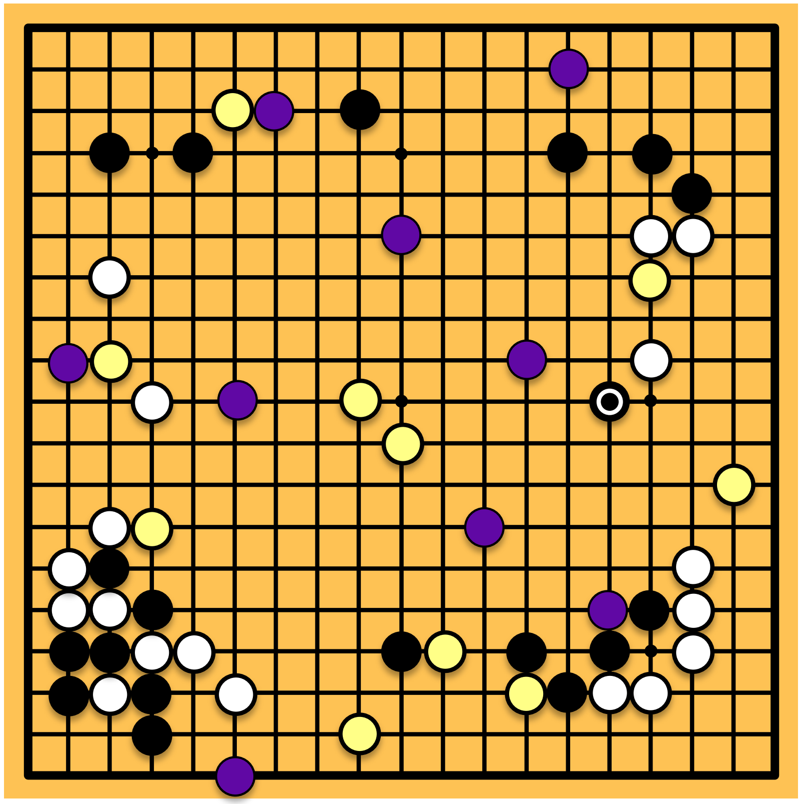

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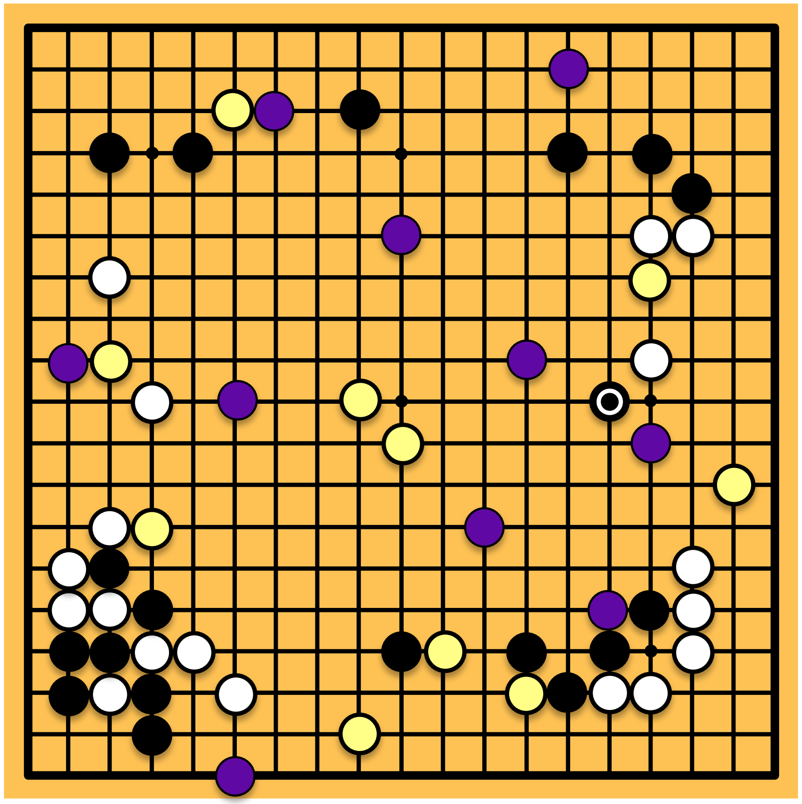

Rollouts

- For each rollout:
  - – Repeat until terminal:
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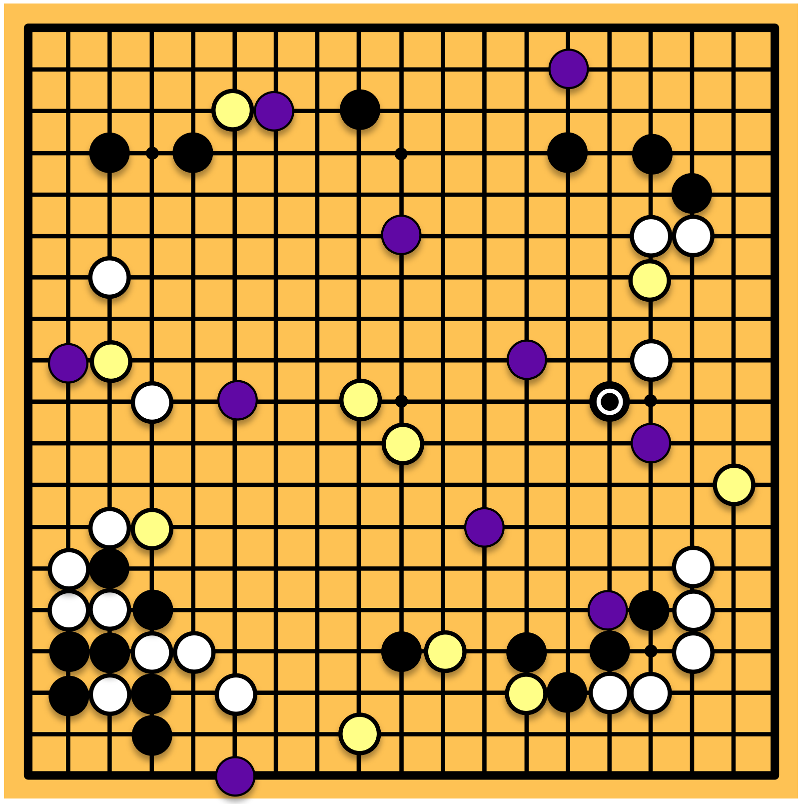
  - – Record the result
- Fraction of wins correlates with the true value of the position!

“Move 37”



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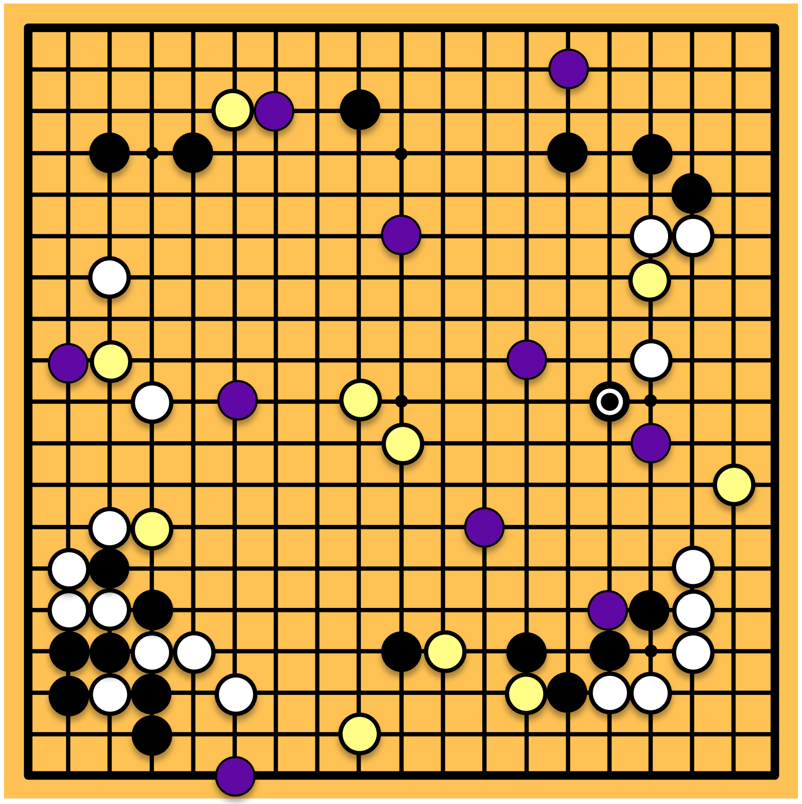
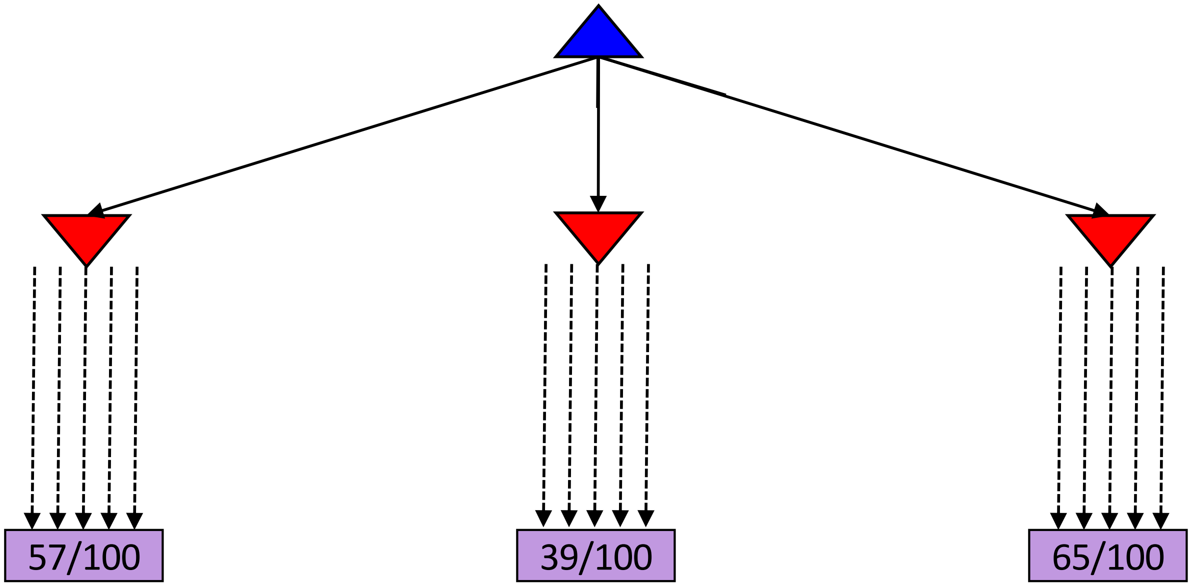
- Fraction of wins correlates with the true value of the position!
- Having a “better” rollout policy helps

“Move 37”



MCTS Version 0

- Do *N* rollouts from each child of the root, record fraction of wins
- Pick the move that gives the best outcome by this metric



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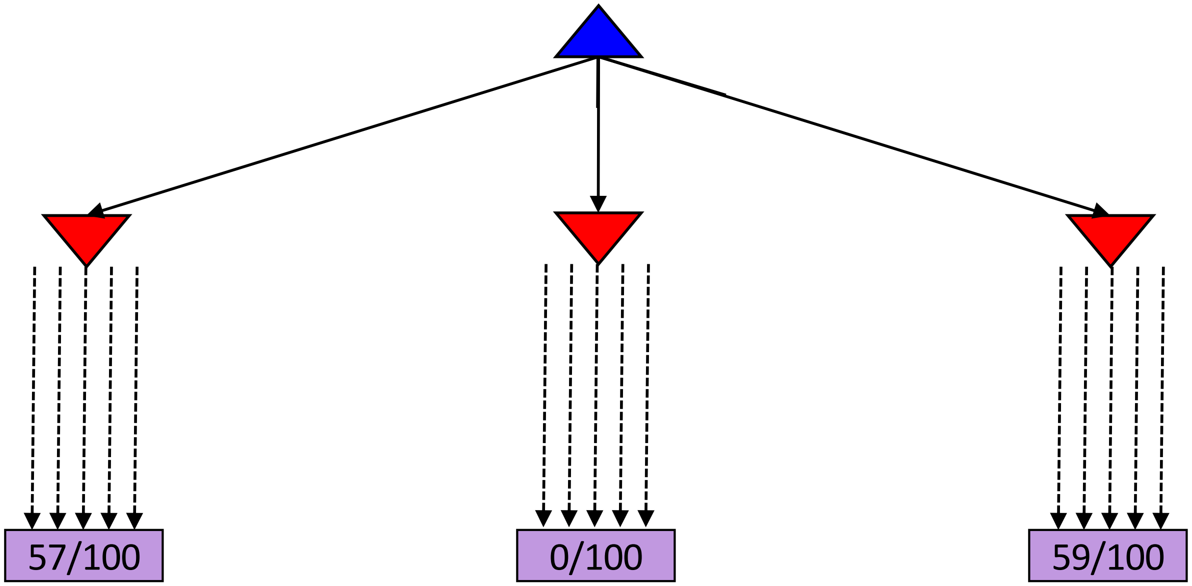

MCTS Version 0.9

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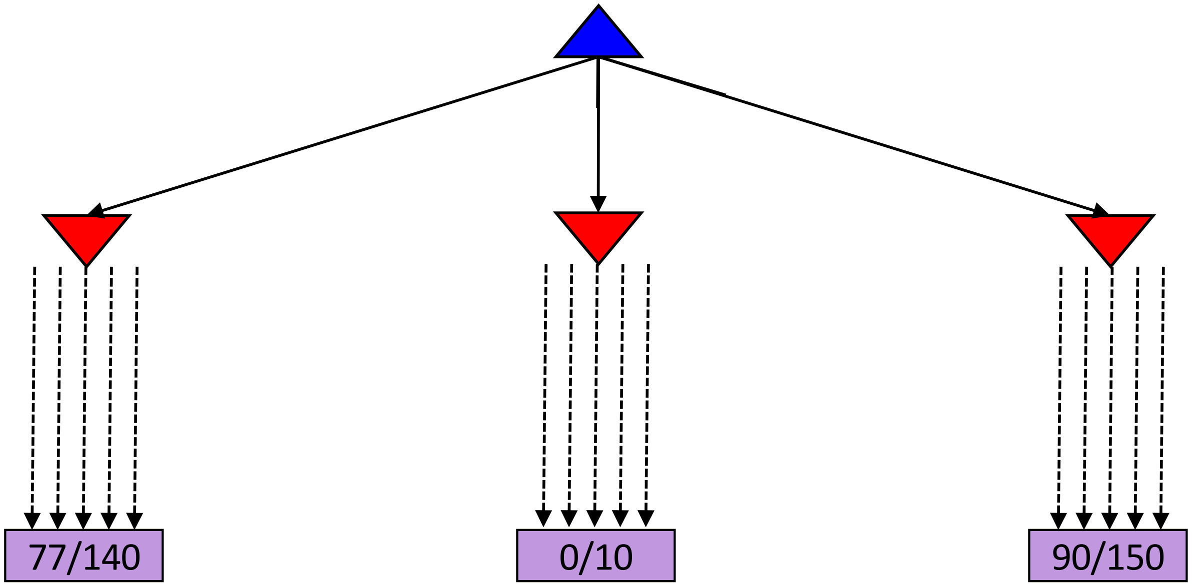

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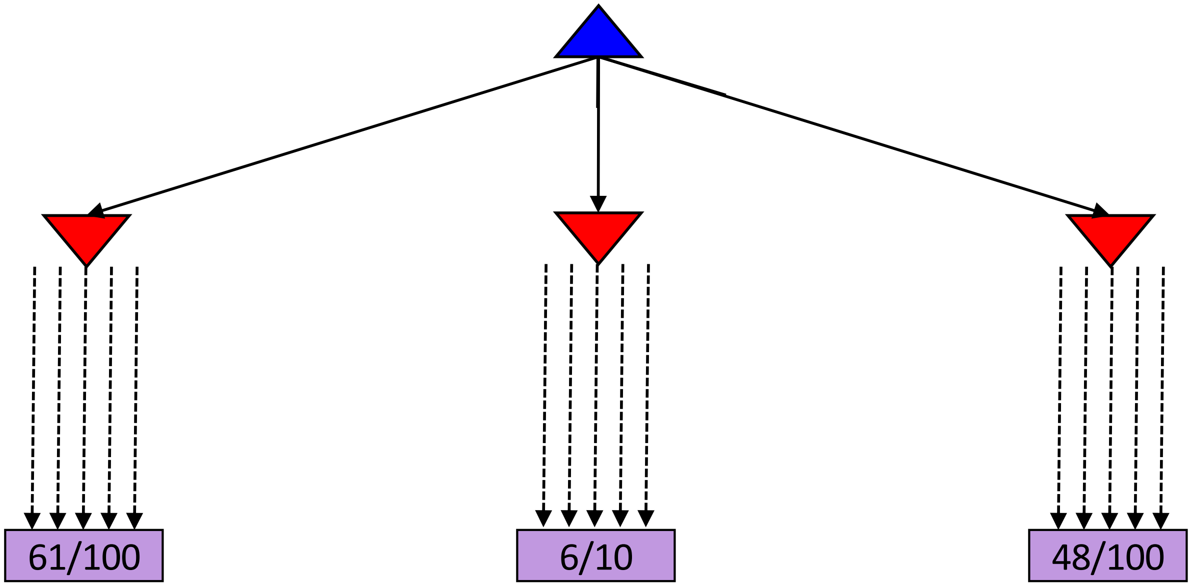
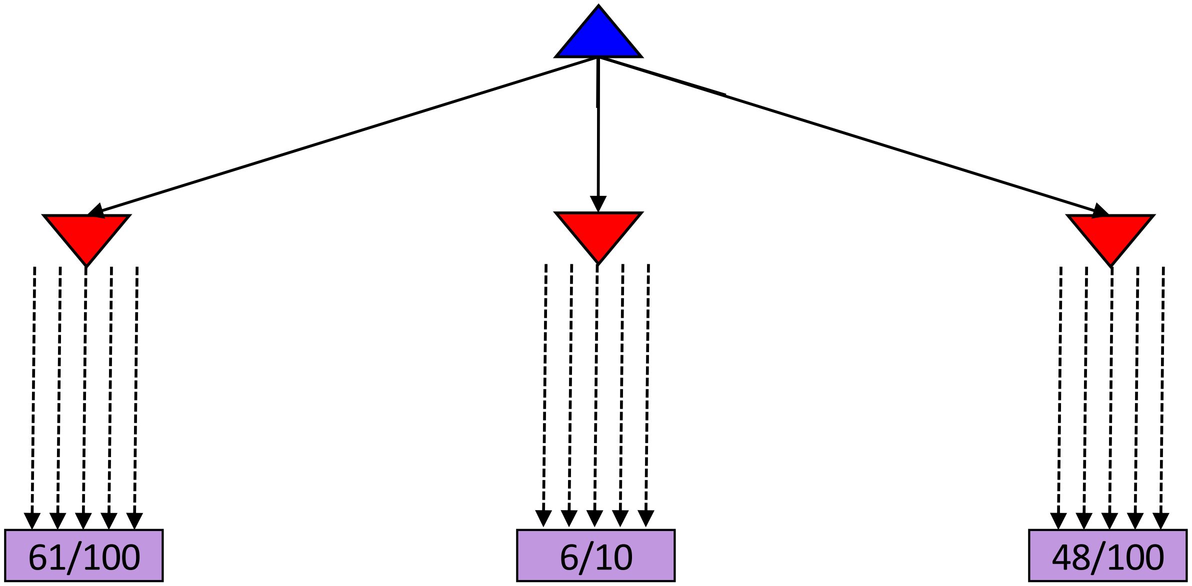


MCTS Version 1.0

- Allocate rollouts to more promising nodes
- Allocate rollouts to more uncertain nodes



Upper Confidence Bounds (UCB) heuristics

- UCB1 formula combines "promising" and "uncertain":
  - C is a parameter we choose to trade off between two terms

$$UCB1(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(\text{Parent}(n))}{N(n)}}$$

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- $U(n)$ = total utility of rollouts (# wins) for player of Parent(*n*)
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- High for small N
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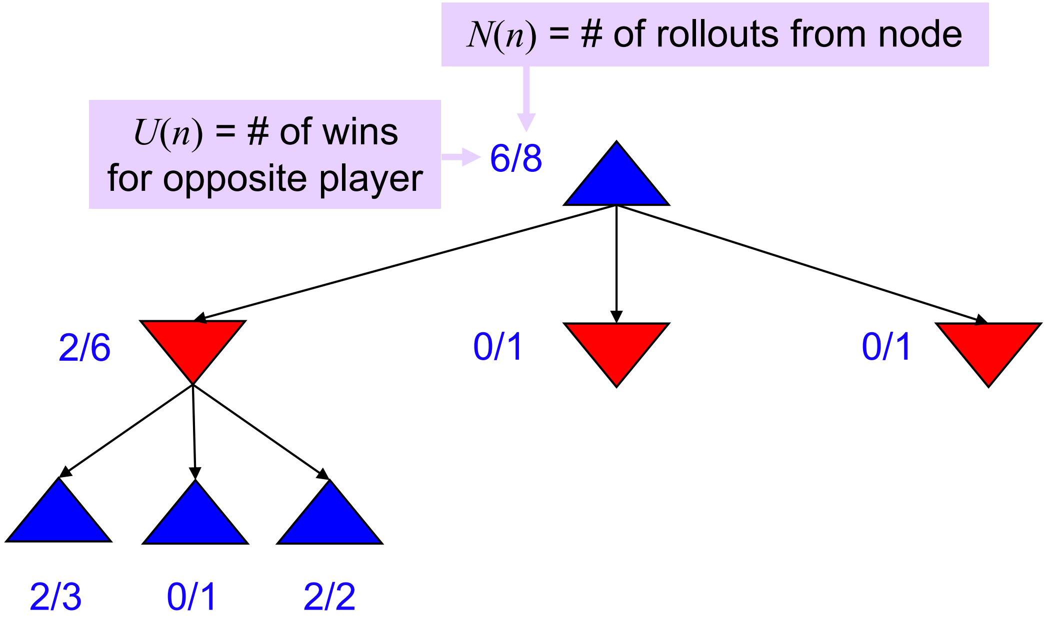
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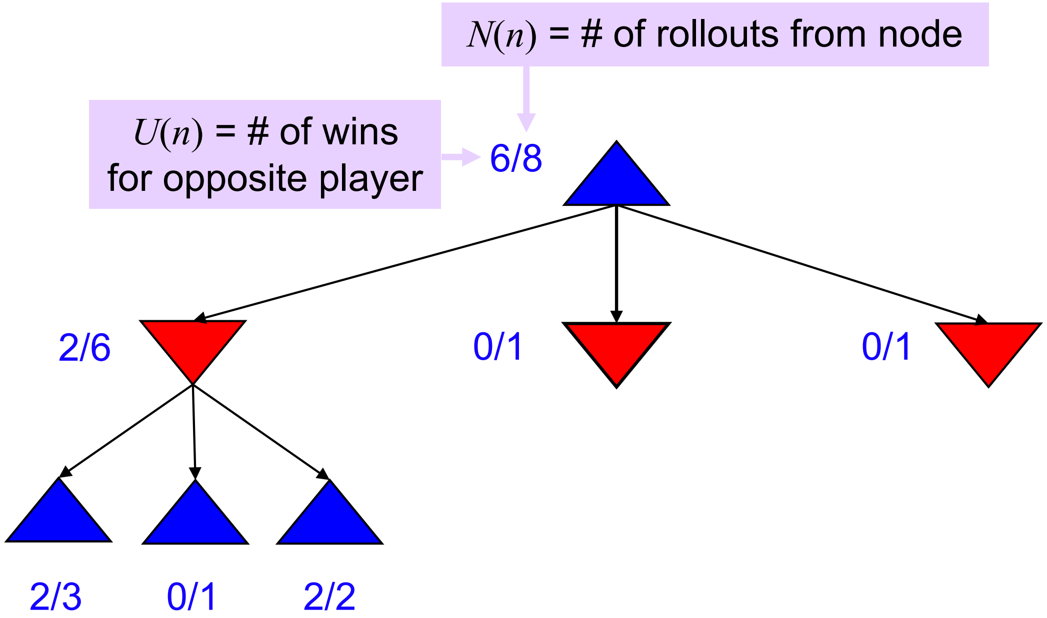

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For 3 red nodes above the UCB values (with C=1) are:

$$\frac{2}{6} + \sqrt{\frac{\log 8}{6}} \quad 1.06 \qquad \frac{0}{1} + \sqrt{\frac{\log 8}{1}} \quad 1.75 \qquad \frac{0}{1} + \sqrt{\frac{\log 8}{1}}$$

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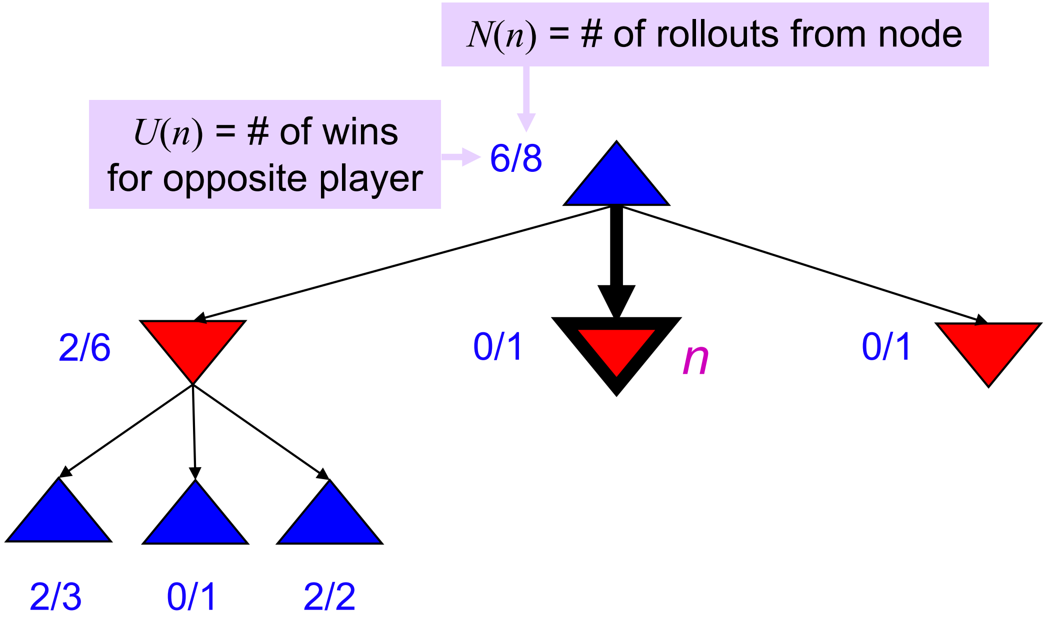

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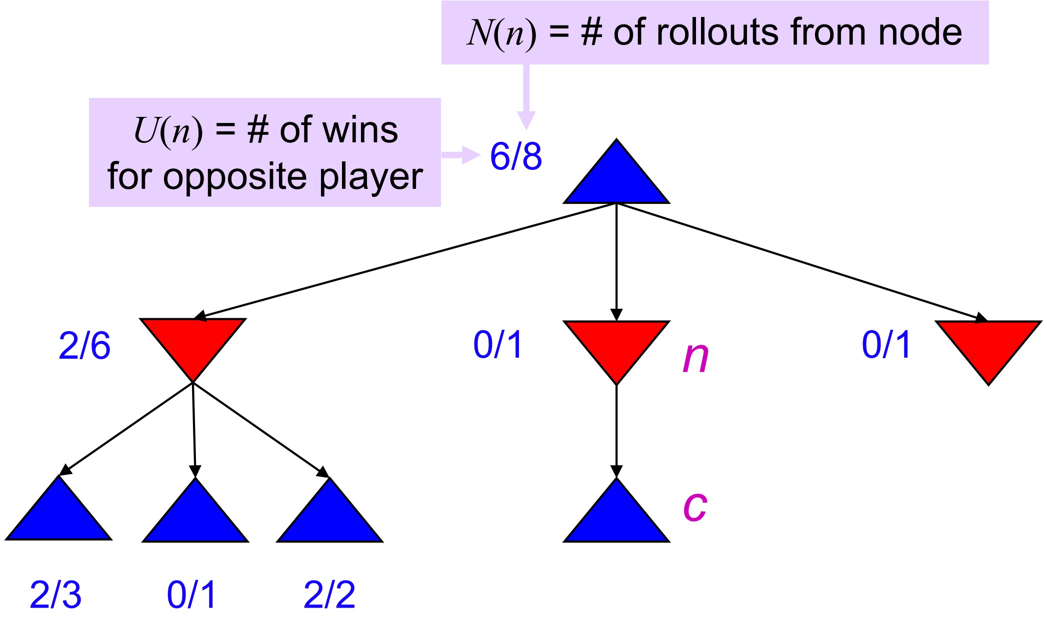

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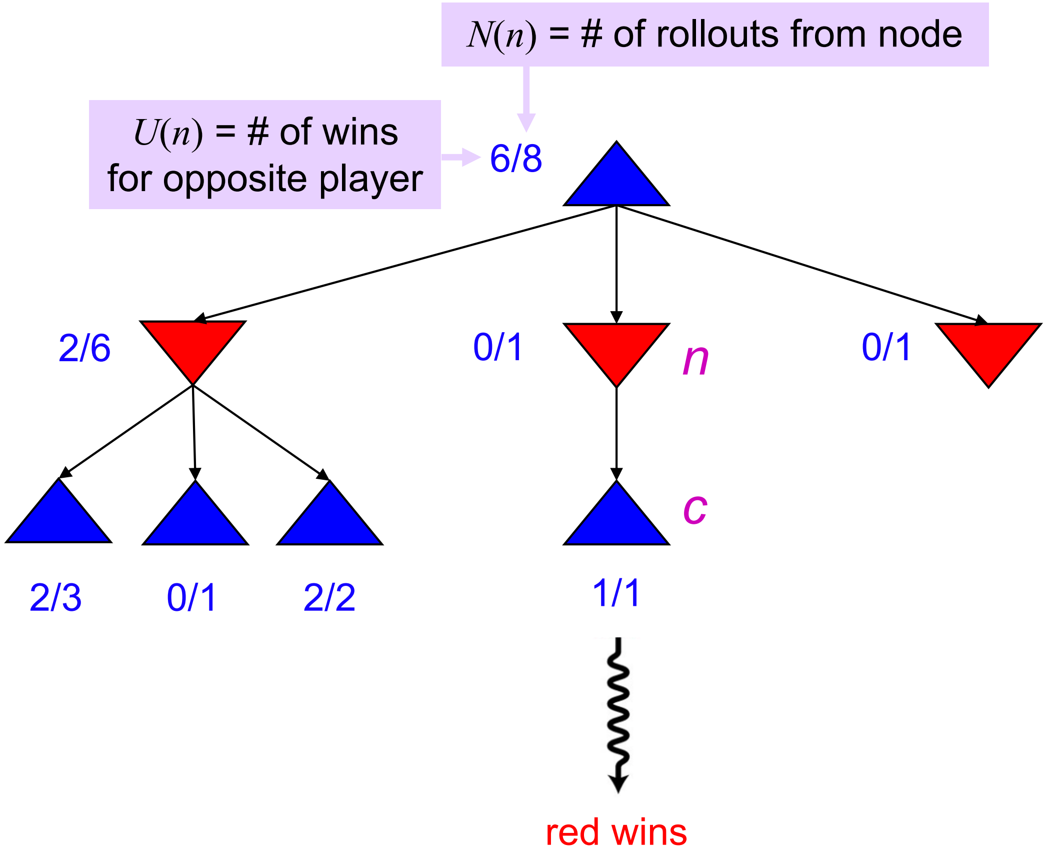

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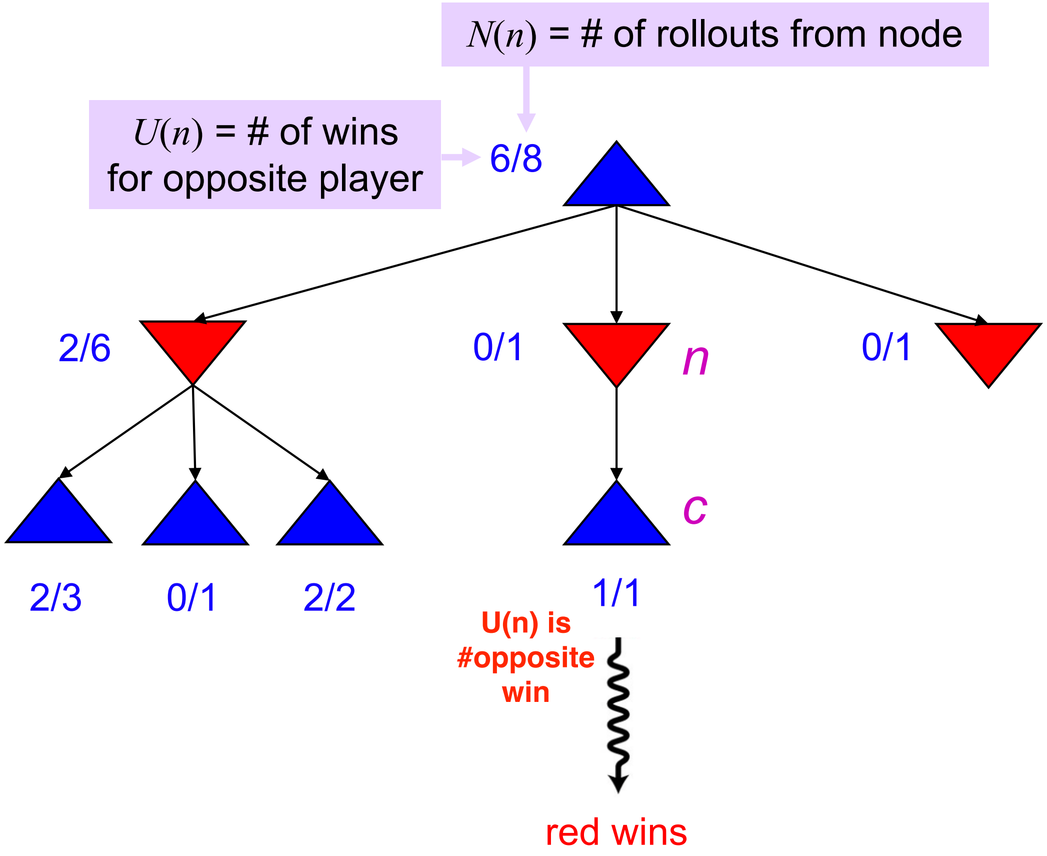

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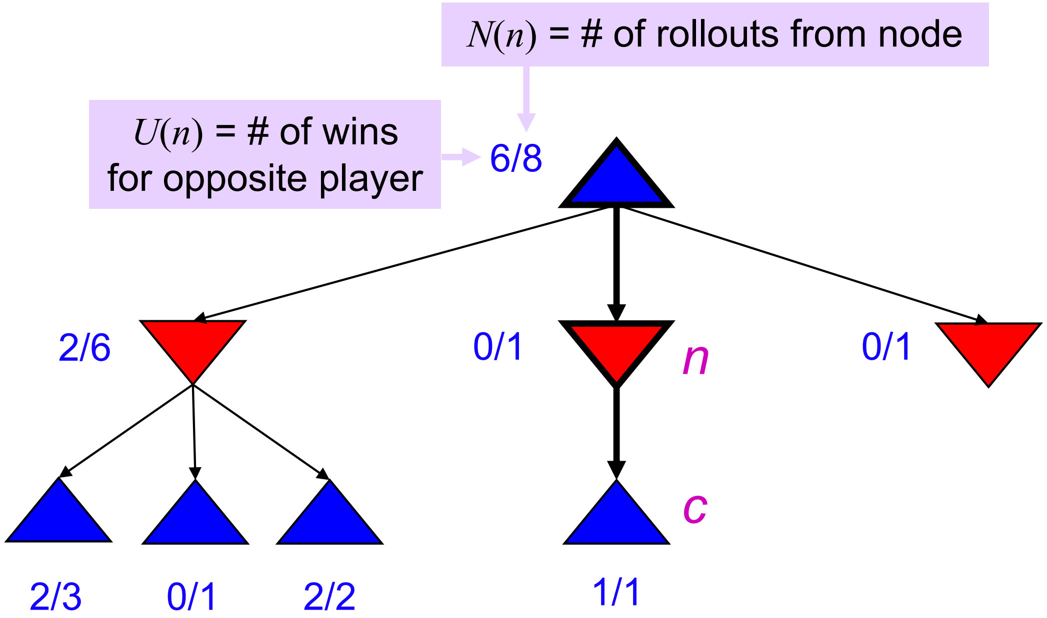
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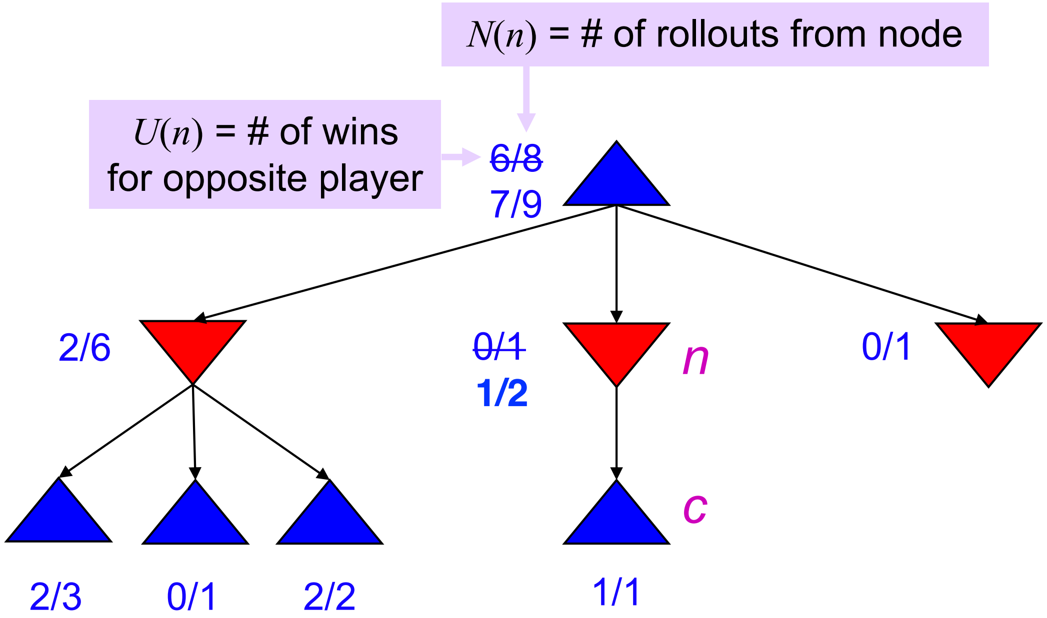

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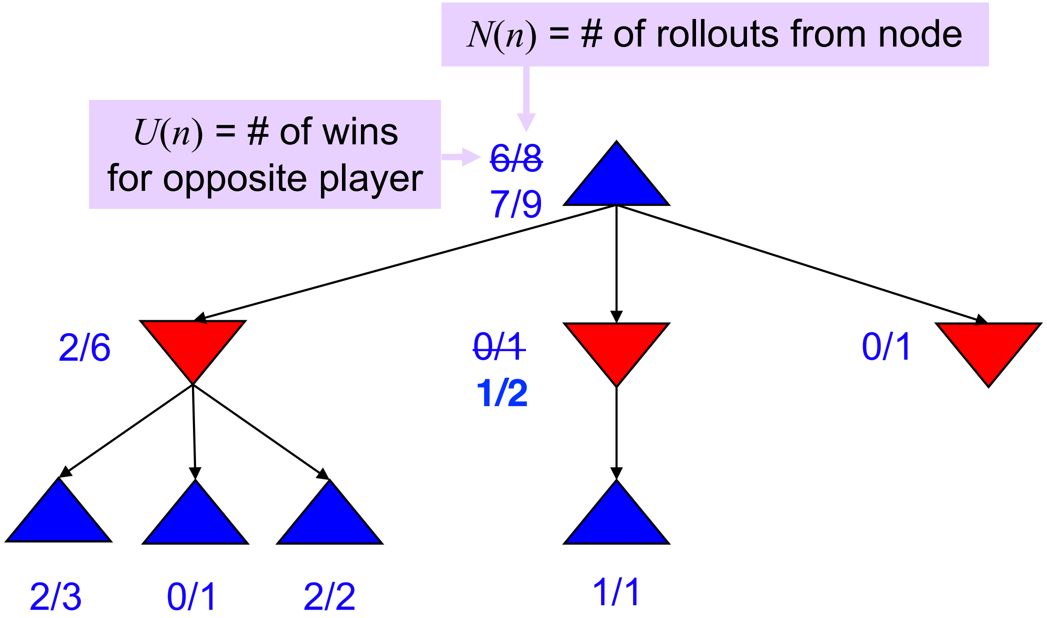

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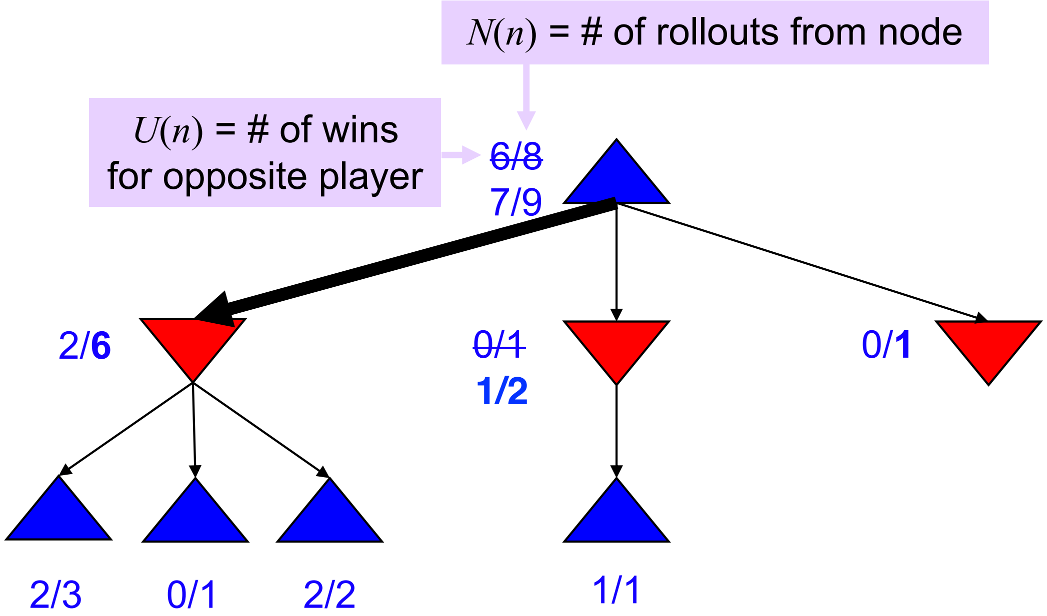

MCTS Summary

- MCTS is currently the most common tool for solving hard search problems
- Why?
  - Time complexity independent of b and m
  - No need to design evaluation functions (general-purpose & easy to use)
- Solution quality depends on number of rollouts N
  - Theorem: as $N \rightarrow \infty$ UCT selects the minimax move
- Example of using random sampling in an algorithm
  - Broadly called *Monte Carlo* methods
- MCTS can be improved further with machine learning

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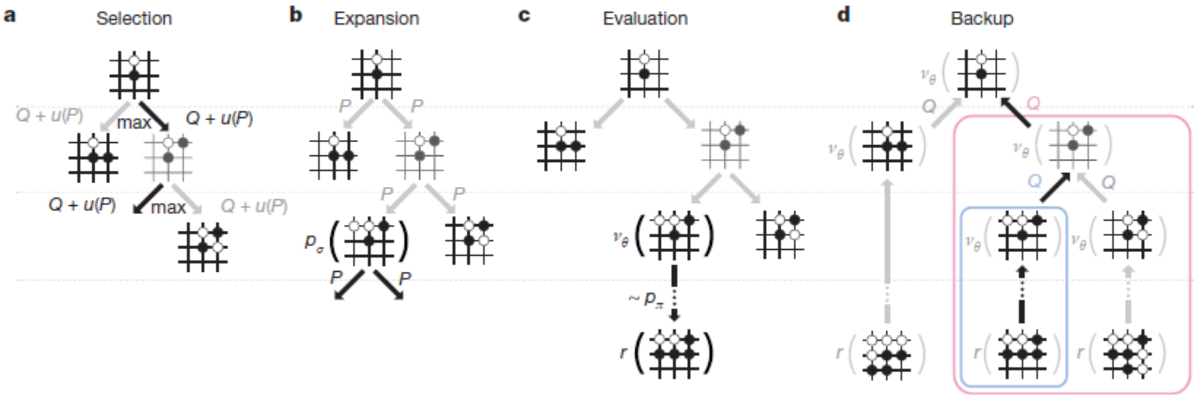
  - (but N never approaches infinity!)

AlphaGo implementation

- Trained deep neural networks (13 layers) to learn **value function** and **policy function**
- Performs Monte Carlo game search
  - –explore state space like minimax
  - –random "rollouts"
  - –simulate probable plays by opponent according to policy function

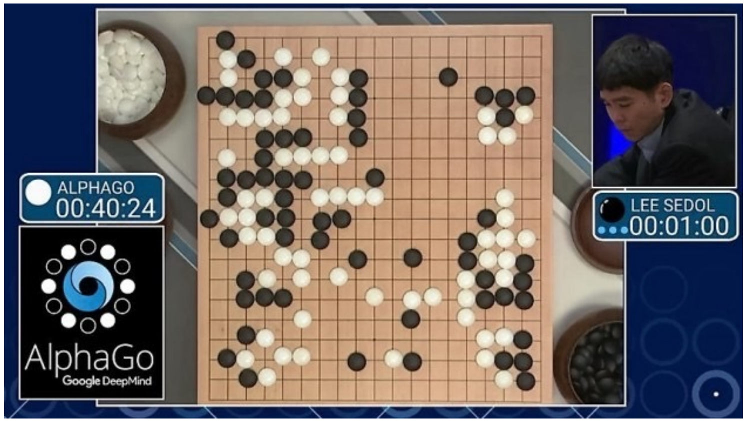


AlphaGo implementation

- Hardware: 1920 CPUs, 28O GPUs
- Neural networks trained in two phases over 4-6 weeks
- **Phase 1:** supervised learning from database of 30 million moves in games between two good human players
- **Phase 2:** play against versions of self using reinforcement learning to improve performance

MCTS + Machine Learning: AlphaGo

- Monte Carlo Tree Search with additions including:
  - Rollout policy is a neural network trained with reinforcement learning and expert human moves
  - In combination with rollout outcomes, use a trained value function to better predict node’s utility



[Mastering the game of Go with deep neural networks and tree search. Silver et al. Nature. 2016]

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  - Solution methods for partial-information games in economics (poker)
- Video games present much greater challenges – lots to do!
  - $b = 10^{500}$, $|S| = 10^{4000}$, $m = 10{,}000$, partially observable, often > 2 players