# CMSC 471
# Artificial Intelligence

# Search

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Slide adoption courtesy: Tim Finin, Frank Ferraro (UMBC),
Charles R. Dyer (UW-Madison), Dan Klein and Pieter Abbeel (CS188, UC Berkeley)

# A General Searching Algorithm

Core ideas:

1. Maintain a list of frontier (fringe) nodes
   1. Nodes coming *into* the frontier have been explored
   2. Nodes going out of the frontier have not been explored
2. Iteratively select nodes from the frontier and explore unexplored nodes from the frontier
3. Stop when you reach your **goal**

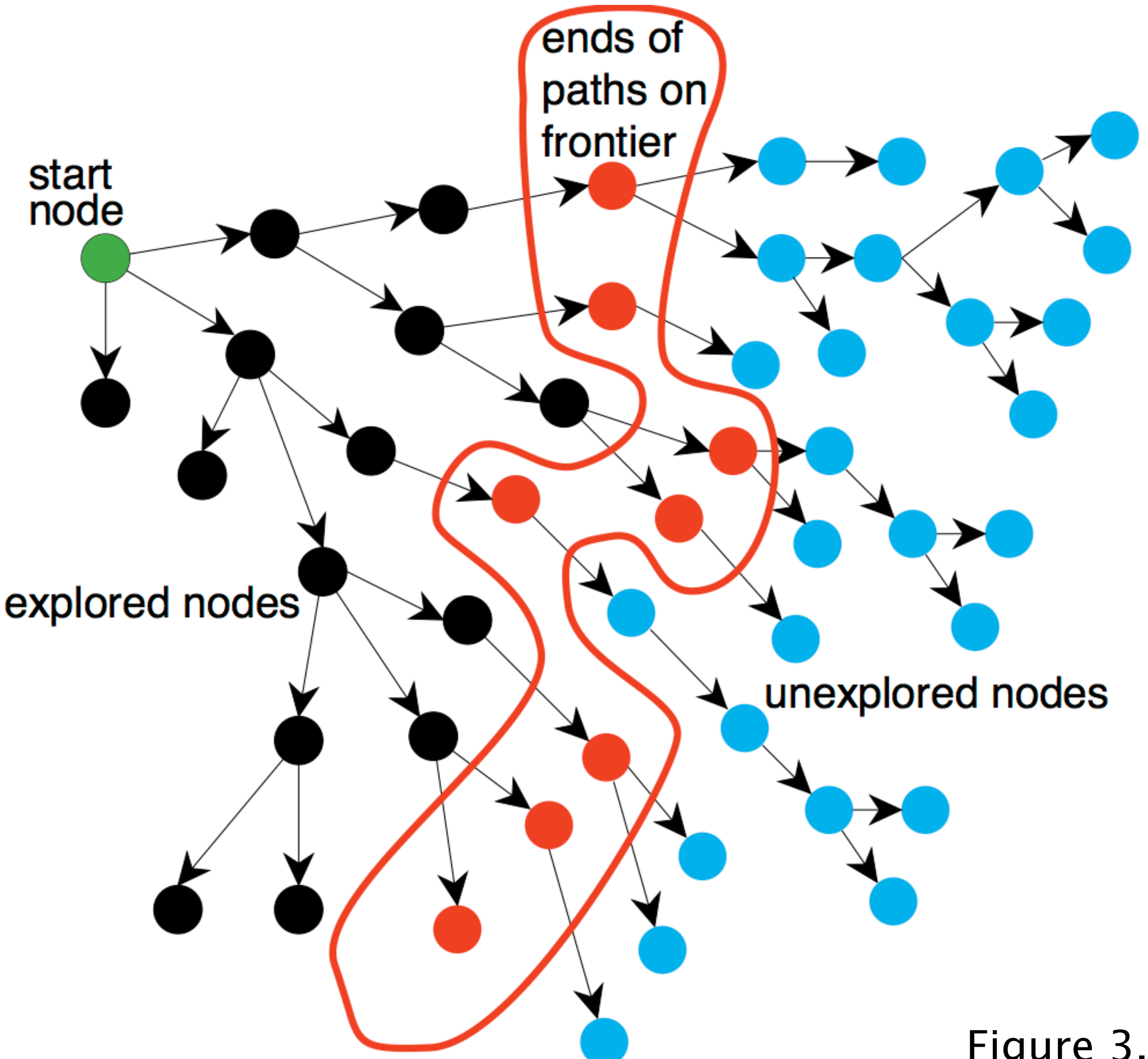



Figure 3.3

# State-space search algorithm

*;; problem describes the start state, operators, goal test, and operator costs*

*;; queueing-function is a comparator function that ranks two states*

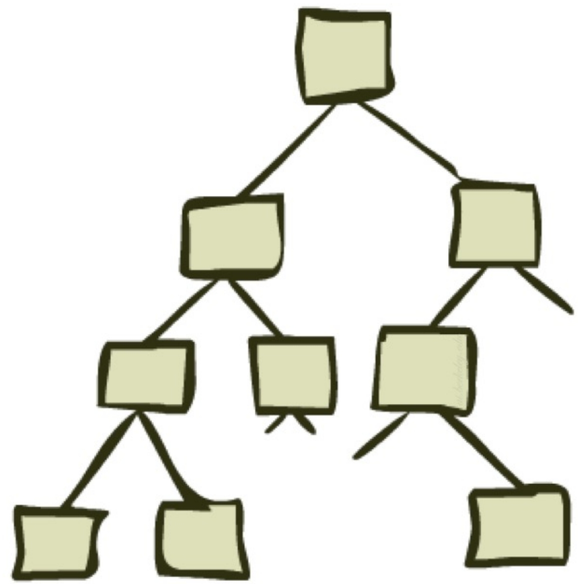
*;; general-search returns either a goal node or failure*

```
function general-search (problem, QUEUEING-FUNCTION)
  nodes = MAKE-QUEUE(MAKE-NODE(problem.INITIAL-STATE))
  loop
     if EMPTY(nodes) then return "failure"
     node = REMOVE-FRONT(nodes)
     if problem.GOAL-TEST(node.STATE) succeeds
        then return node
     nodes = QUEUEING-FUNCTION(nodes, EXPAND(node,
               problem.OPERATORS))
 end
```

*;; Note: The goal test is NOT done when nodes are generated*

*;; Note: This algorithm does not detect loops*

# State Space Graphs and Search Trees

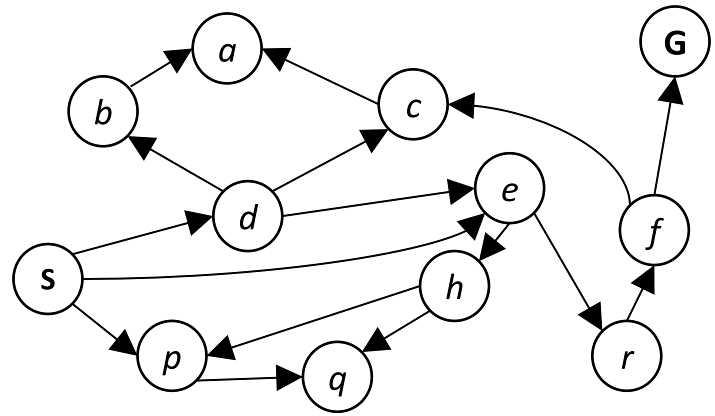


# State Space Graphs

- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent transitions/ successors (action results)
  - The goal test is a set of goal nodes (maybe only one)

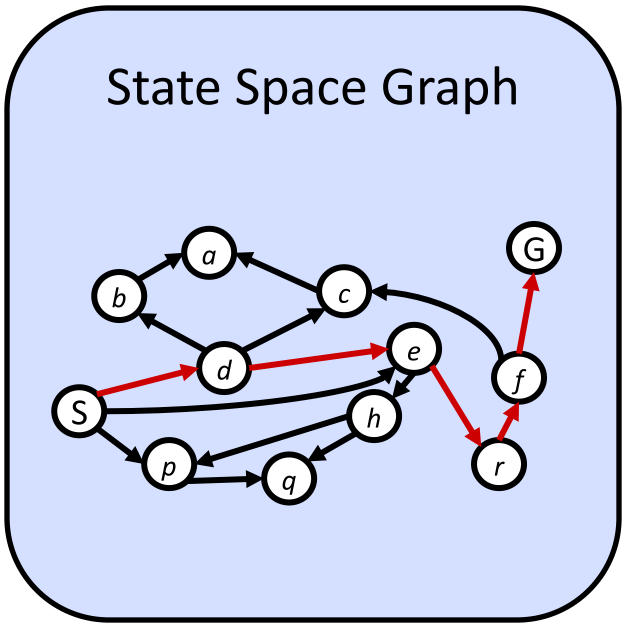
- In a state space graph, each state occurs only once!

- We can rarely build this full graph in memory (it's too big), but it's a useful idea



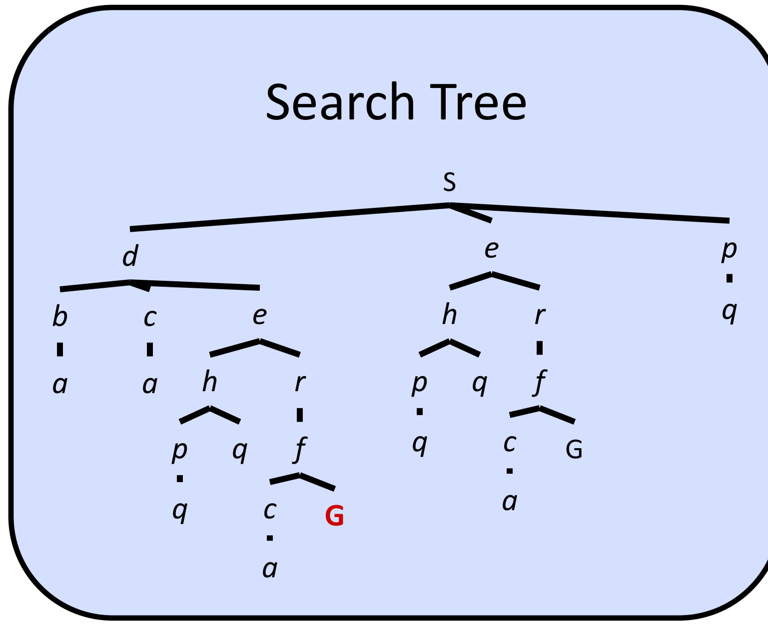

*Tiny state space graph for a tiny search problem*

# State Space Graphs vs. Search Trees




*Each NODE in in the search tree is an entire PATH in the state space graph.*

*We construct the tree on demand – and we construct as little as possible.*



# Informed vs. uninformed search

**Uninformed search strategies (blind search)**

- Use no information about likely *direction* of a goal
- Methods: breadth-first, depth-first, depth-limited, uniform-cost, depth-first iterative deepening, bidirectional

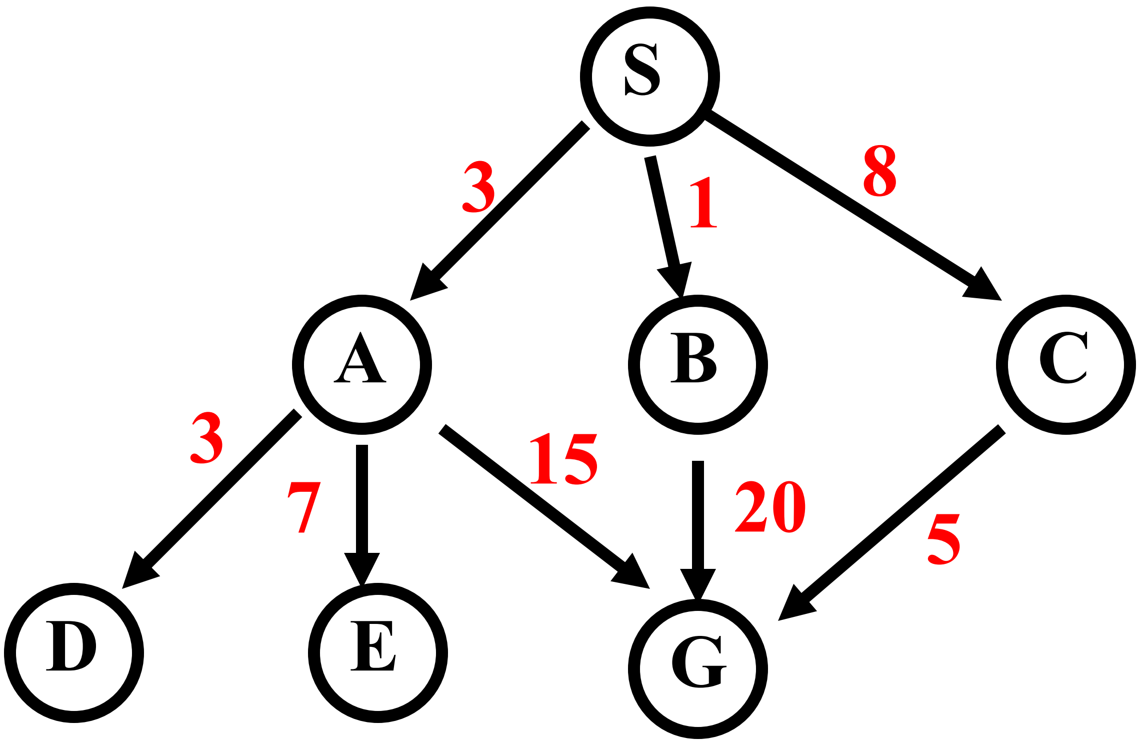
**Informed search strategies (heuristic search)**

- Use information about domain to (try to) (usually) head in the general direction of goal node(s)
- Methods: hill climbing, best-first, greedy search, beam search, algorithm A, algorithm A*

# Evaluating search strategies

- **Completeness**
  - Guarantees finding a solution whenever one exists
- **Time complexity** (worst or average case)
  - Usually measured by *number of nodes expanded*
- **Space complexity**
  - Usually measured by maximum size of graph/tree during the search
- **Optimality/Admissibility**
  - If a solution is found, is it **guaranteed** to be an optimal one, i.e., one with minimum cost

# Example of uninformed search strategies




Consider this search space where S is the start node and G is the goal. Numbers are arc costs.

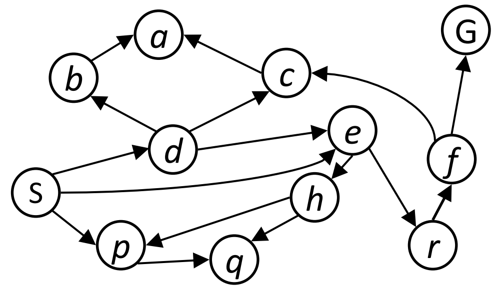
# Classic uninformed search methods

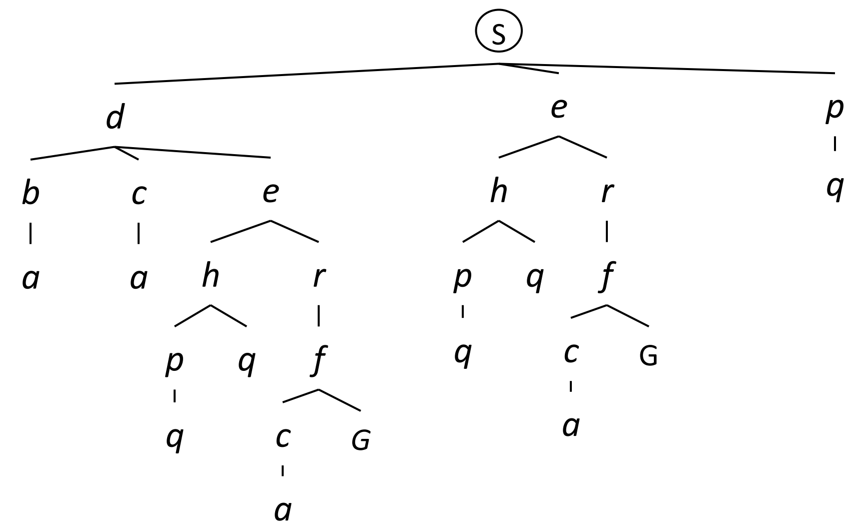
- The four classic uninformed search methods
  - Breadth first search (BFS)
  - Depth first search (DFS)
  - Uniform cost search *(generalization of BFS)*
  - Iterative deepening *(blend of DFS and BFS)*
- To which we can add another technique
  - Bi-directional search *(hack on BFS)*

# Breadth-First Search

*Strategy: expand a shallowest node first*

*Implementation: Frontier is a FIFO queue*



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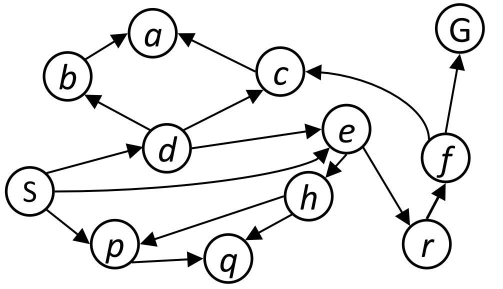


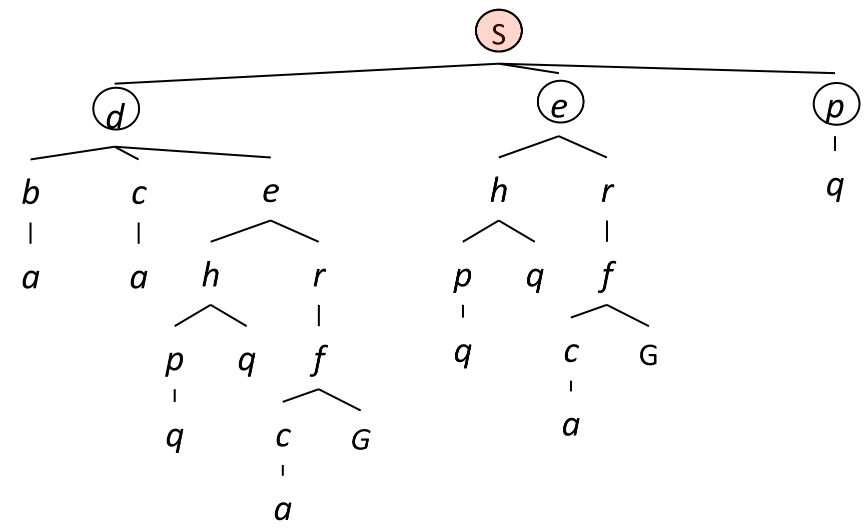

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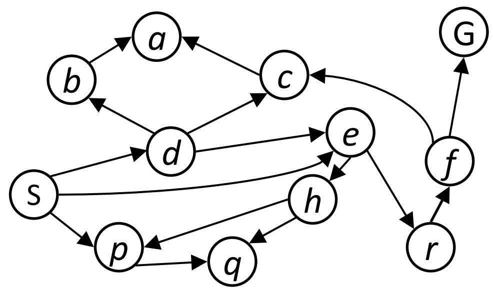


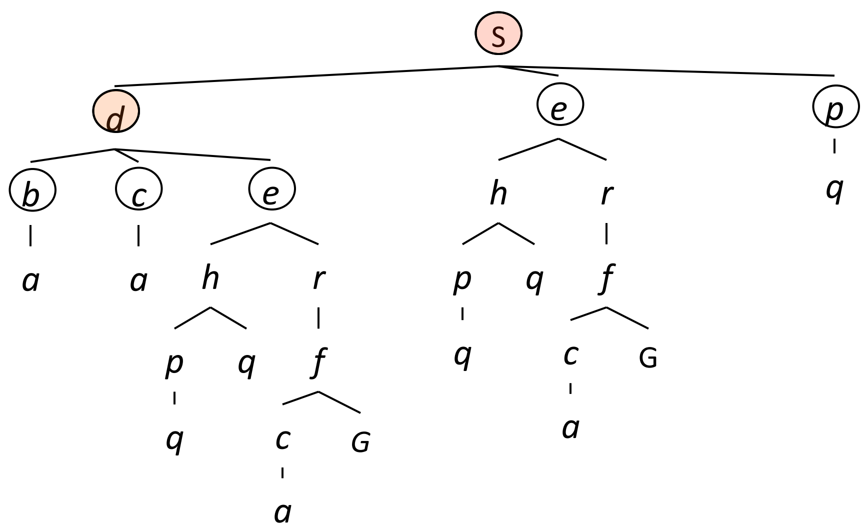

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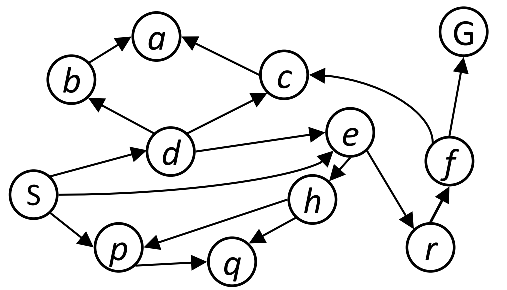


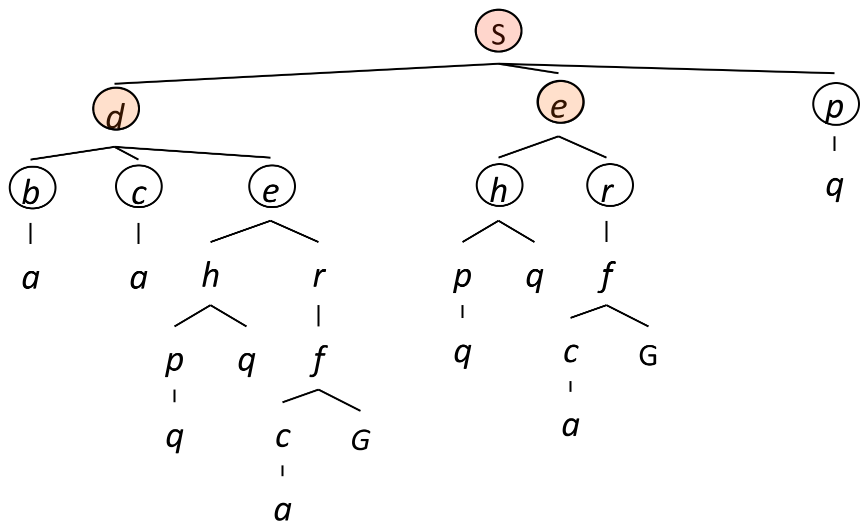

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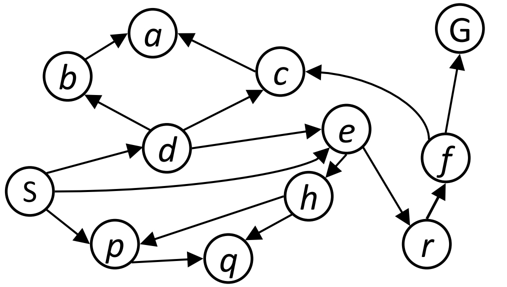


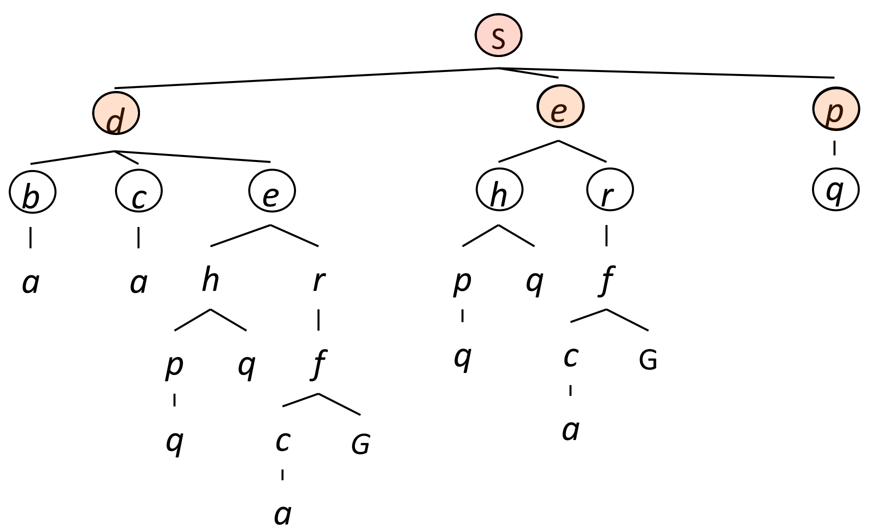

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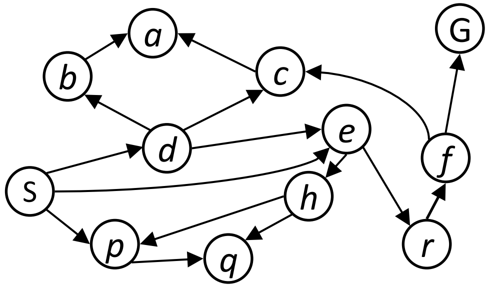


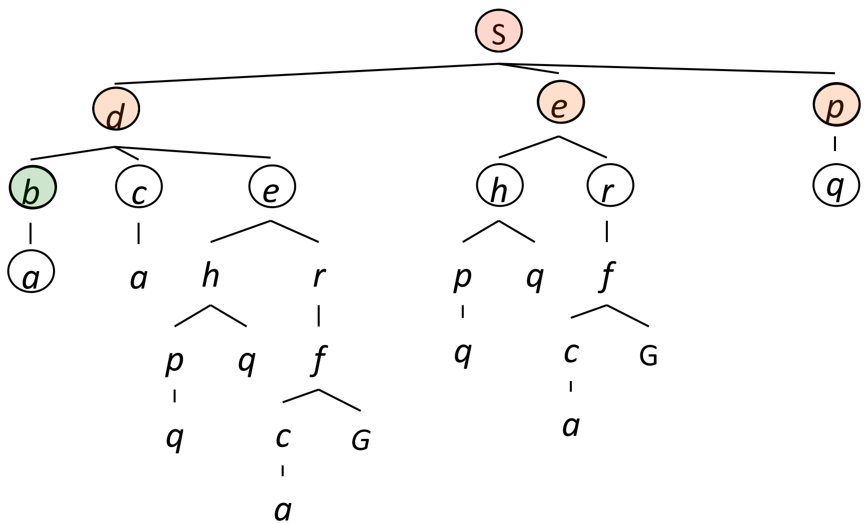

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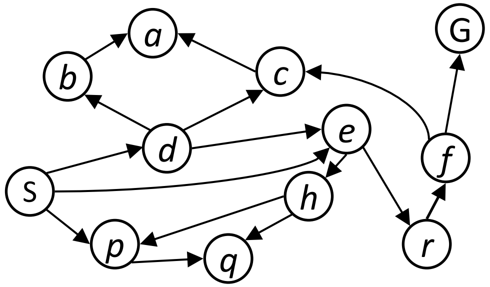
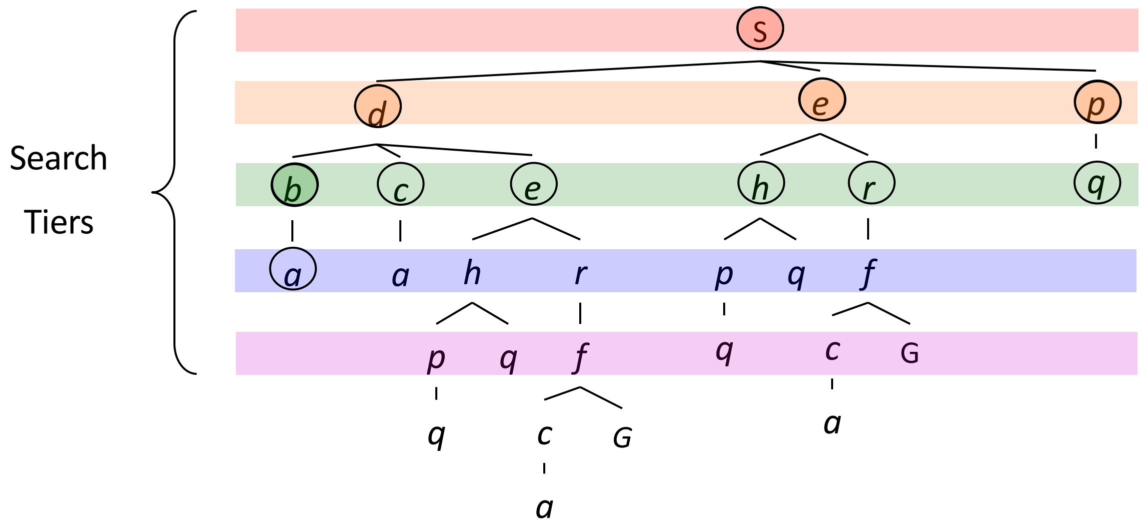
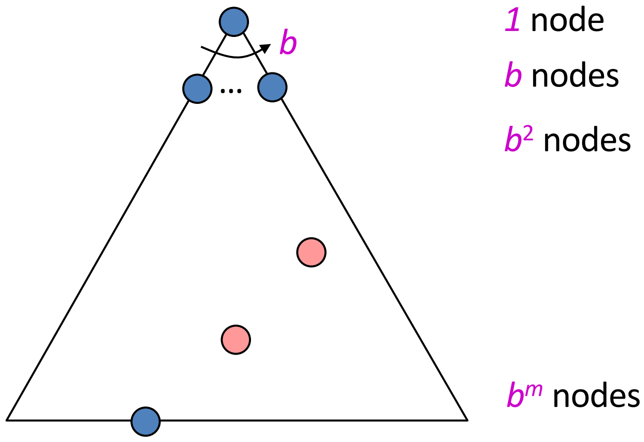


# Breadth-First Search (BFS) Properties

- What nodes does BFS expand?



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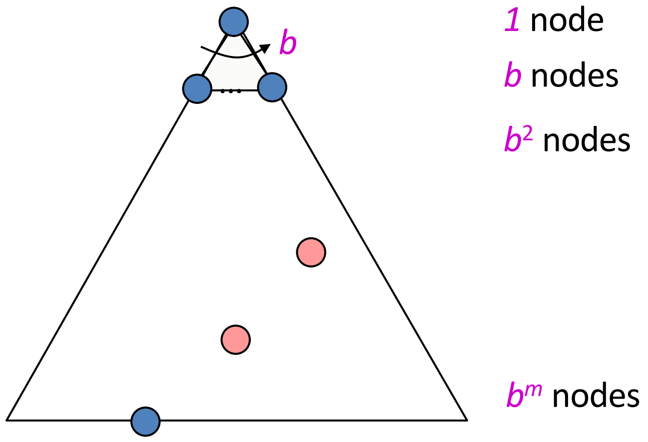

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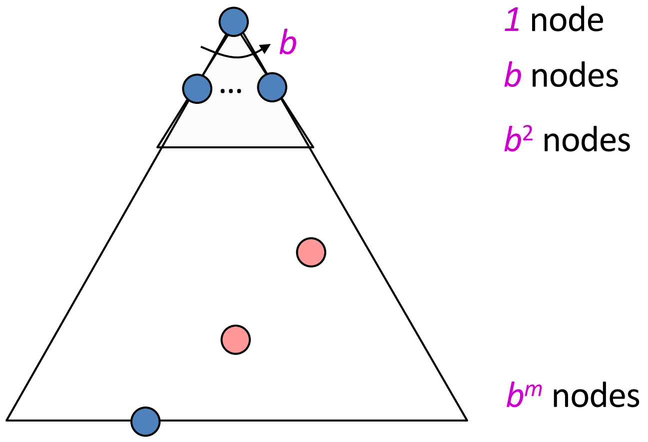

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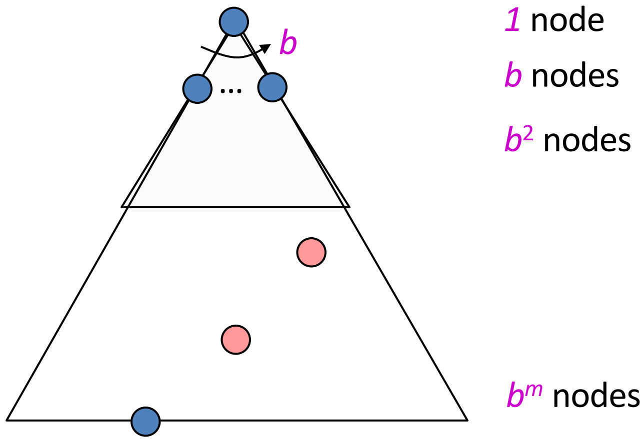

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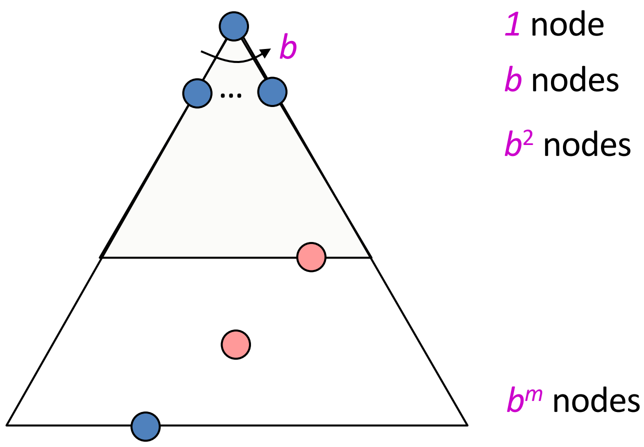

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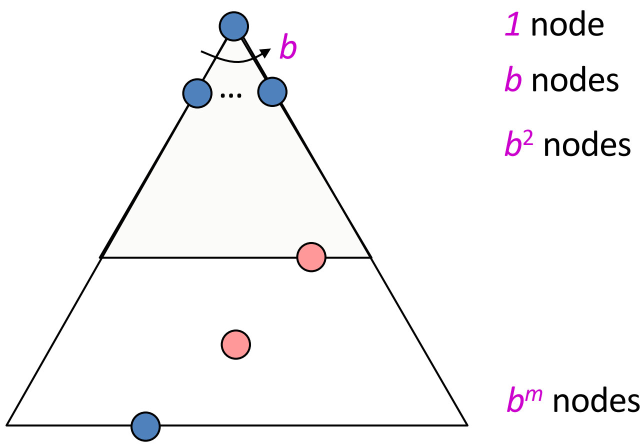

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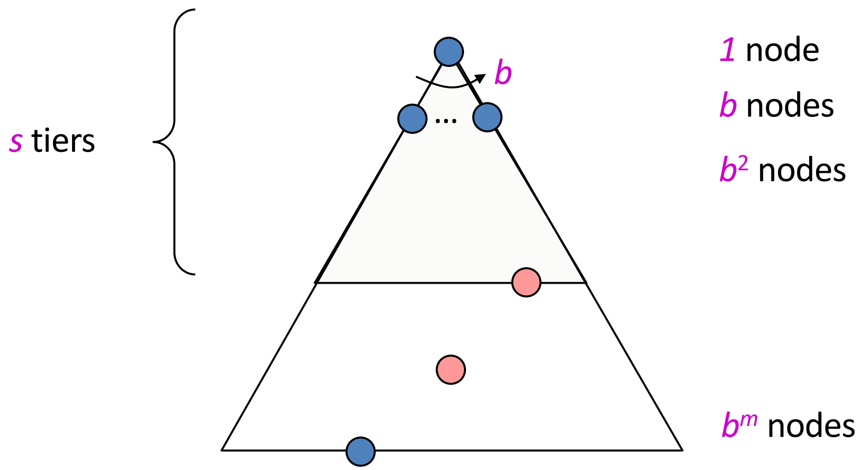

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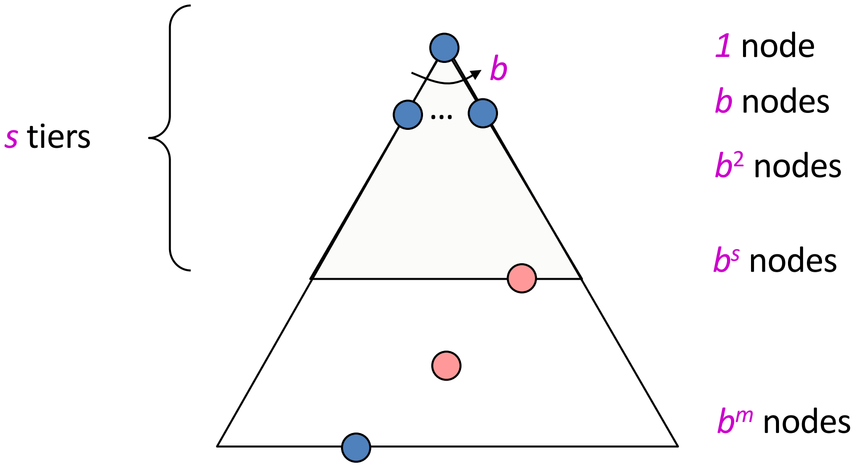

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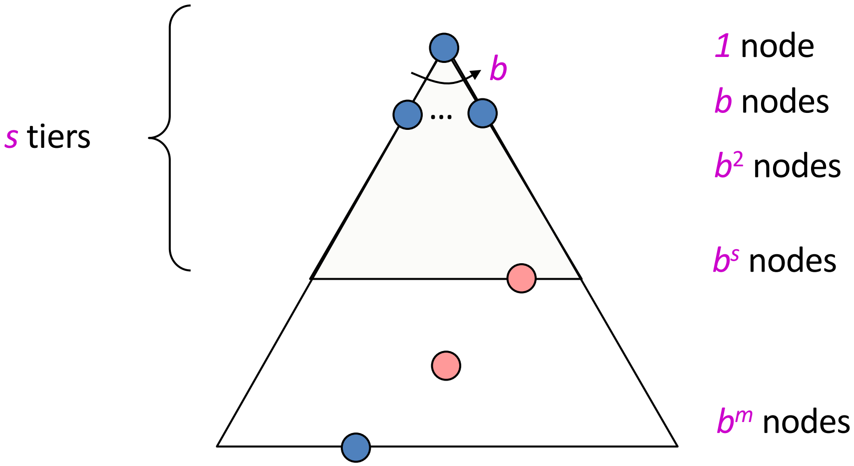

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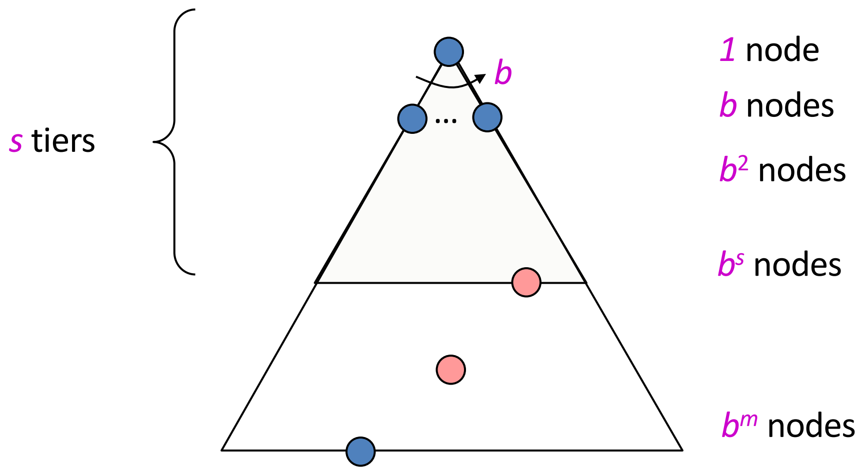

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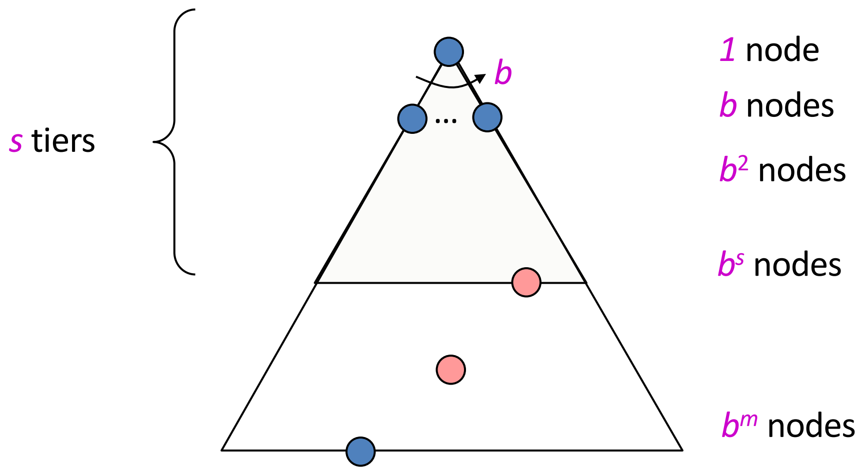

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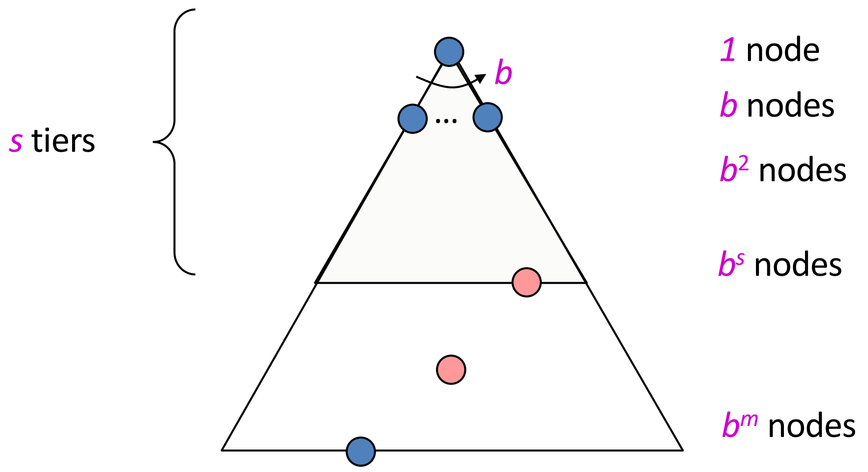

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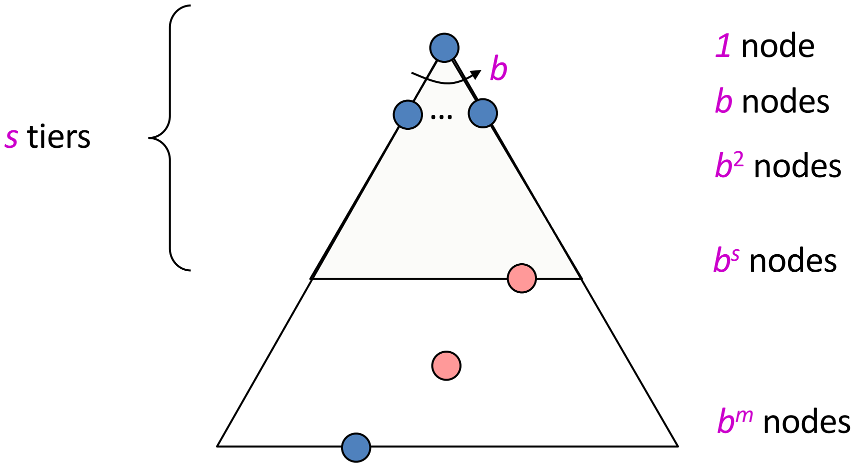

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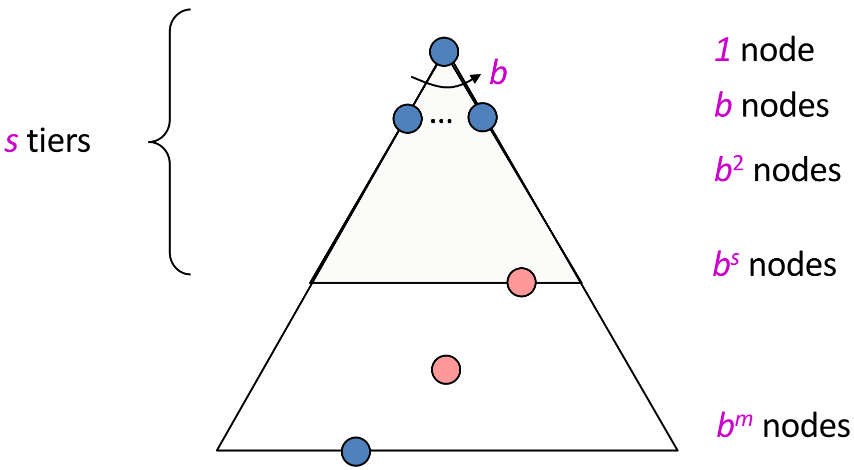

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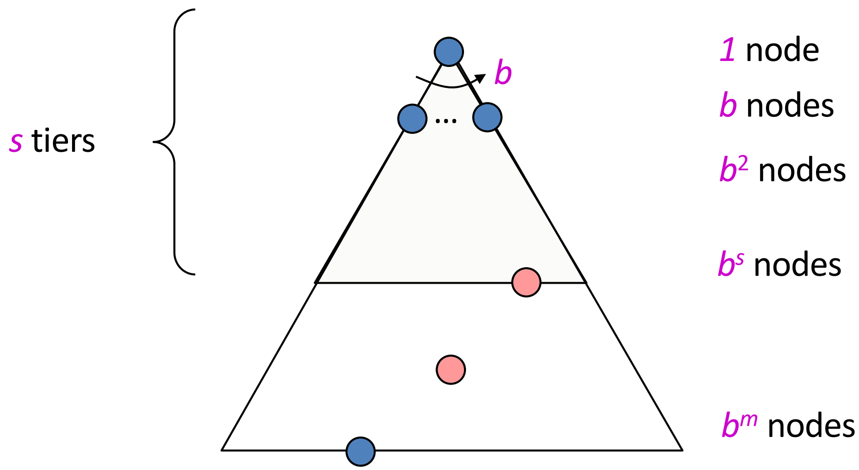

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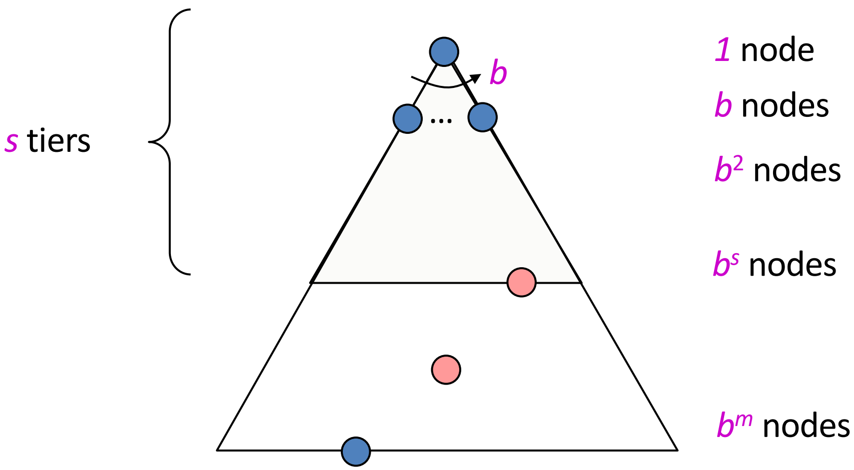



Potential issues??

# Breadth-First Search

- Takes a **long time to find solutions** with large number of steps because must explore all shorter length possibilities first

# Breadth-First Search

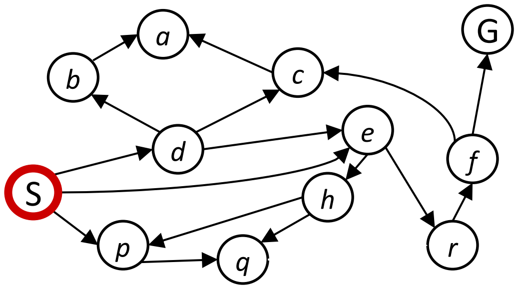
**Long time to find solutions** with many steps: we must look at all shorter length possibilities first

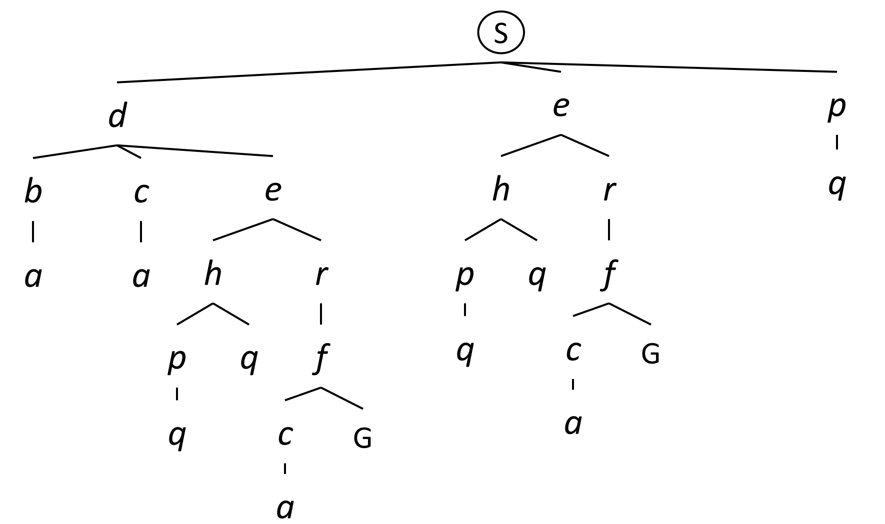
- Complete search tree of depth s where nodes have b children has $1 + b + b^2 + ... + b^s = (b^{(s+1)} - 1)/(b-1)$ nodes = $0(b^s)$
- Tree of depth 12 with branching 10 has more than a trillion nodes
- If BFS expands 1000 nodes/sec and nodes uses 100 bytes, then it may take 35 years to run and uses 111 terabytes of memory!

# Depth-First Search

*Strategy: expand a deepest node first*

*Implementation: Frontier is a LIFO stack*



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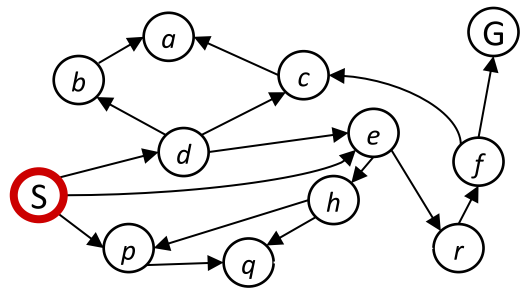


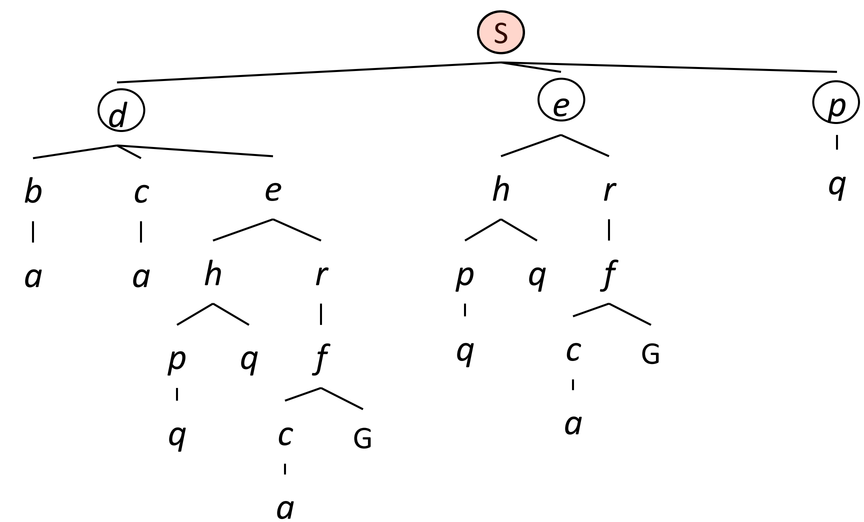

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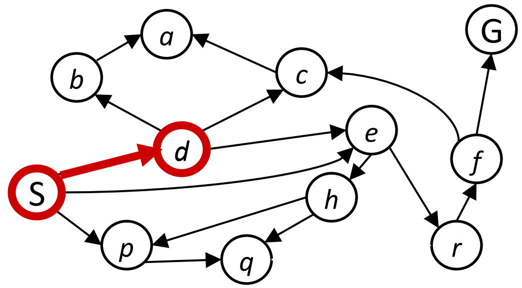


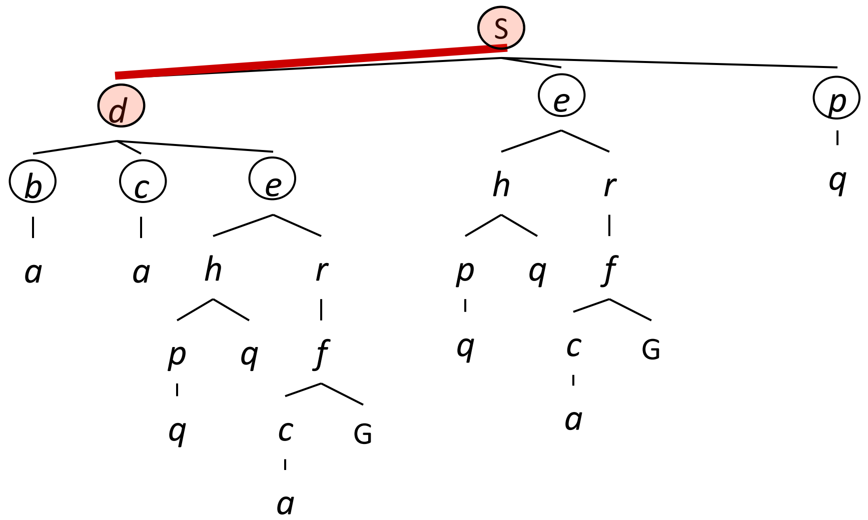

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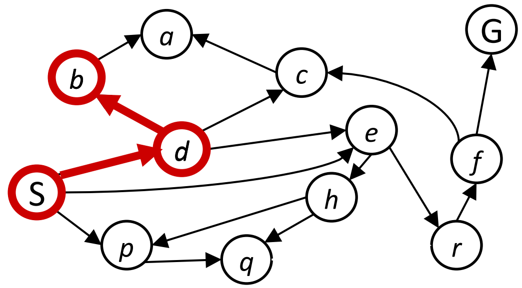


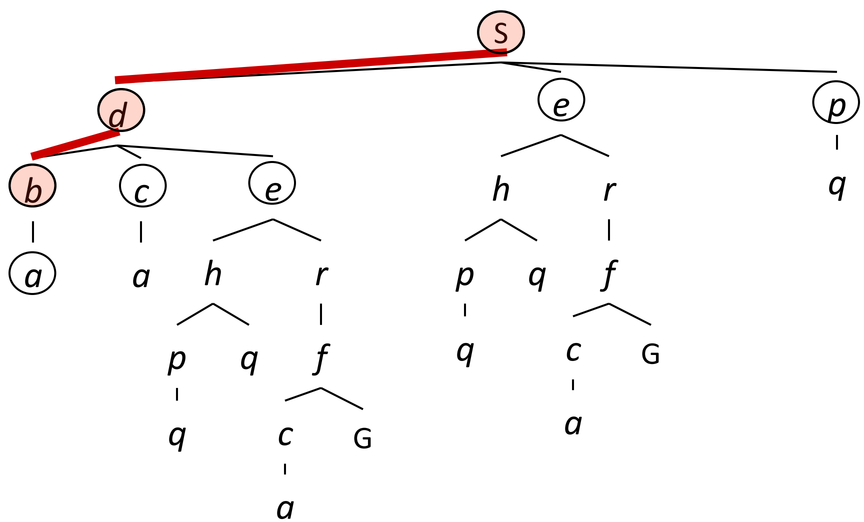

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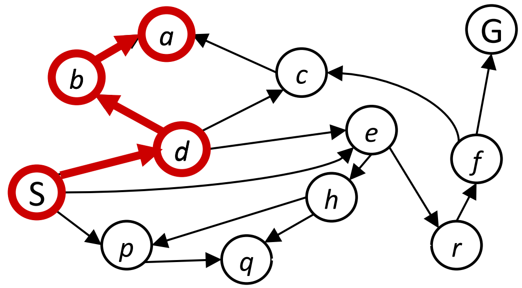


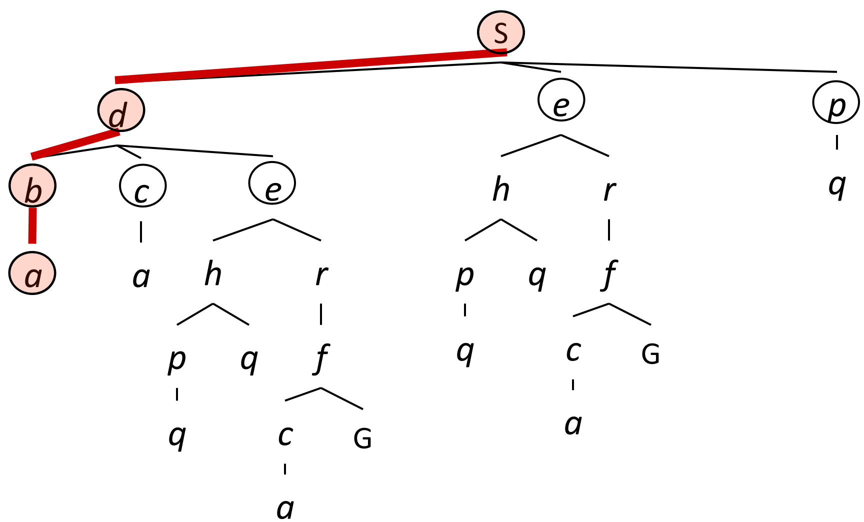

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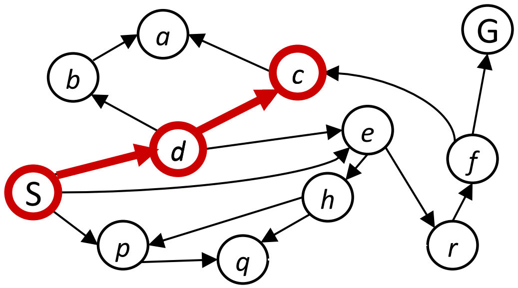


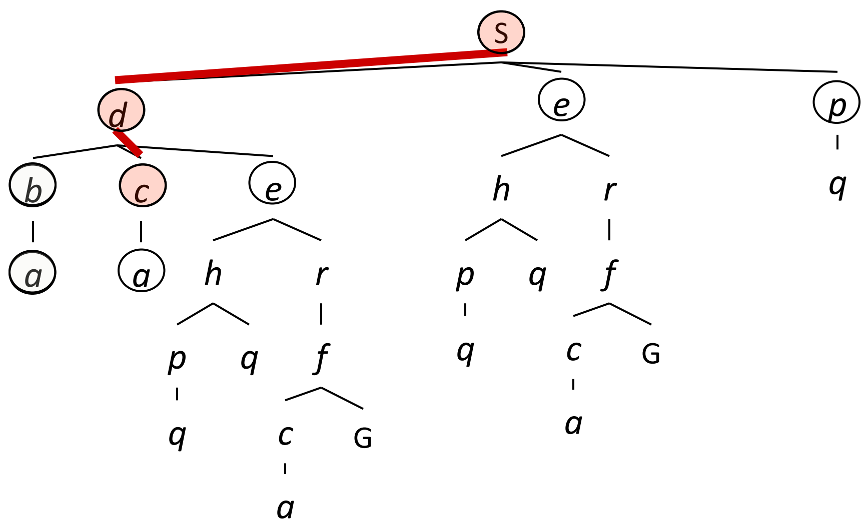

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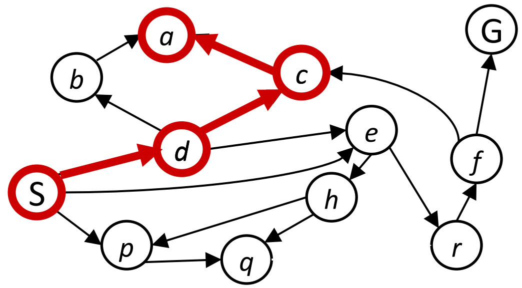


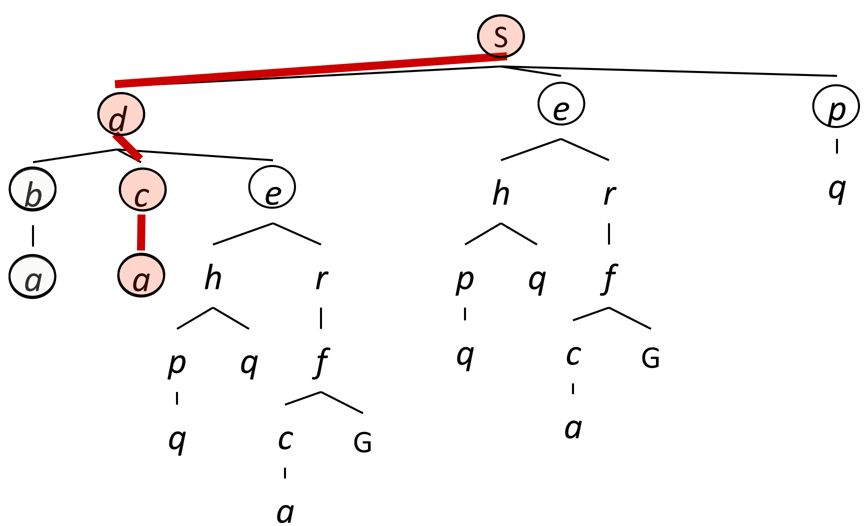

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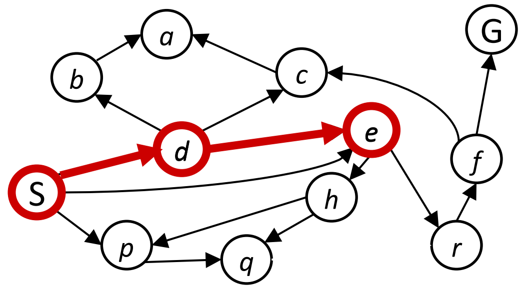


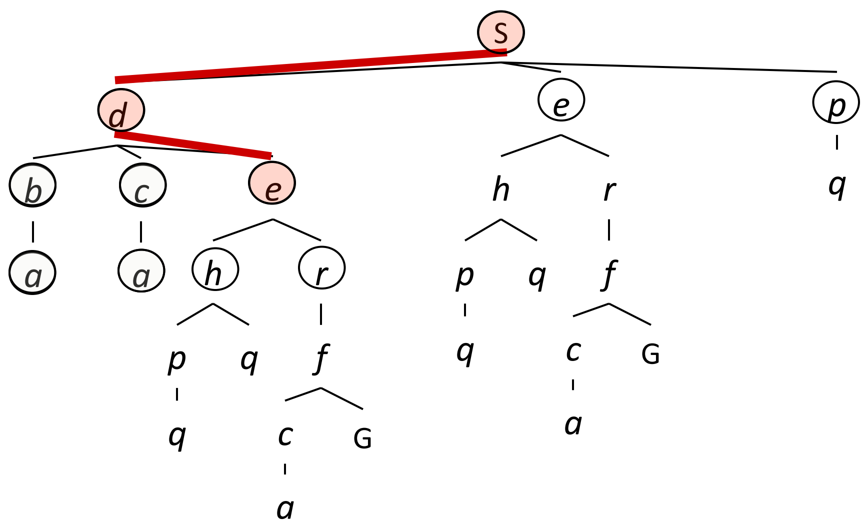

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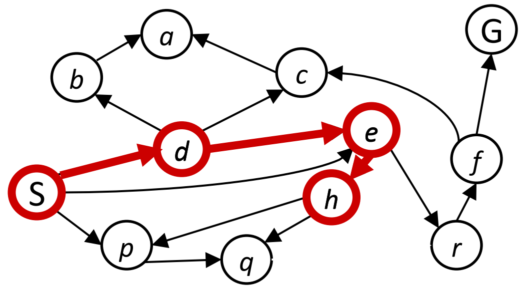


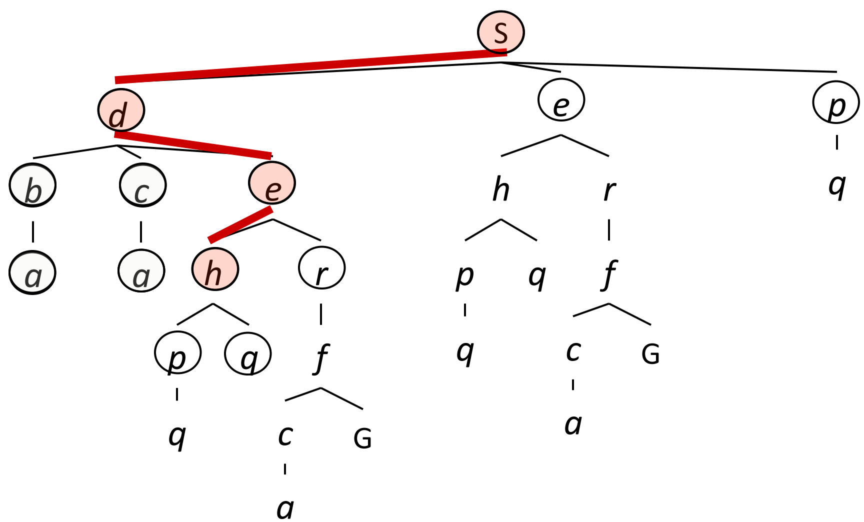

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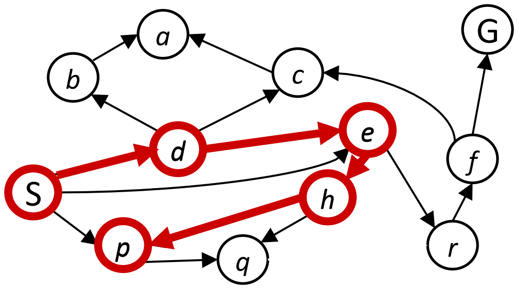


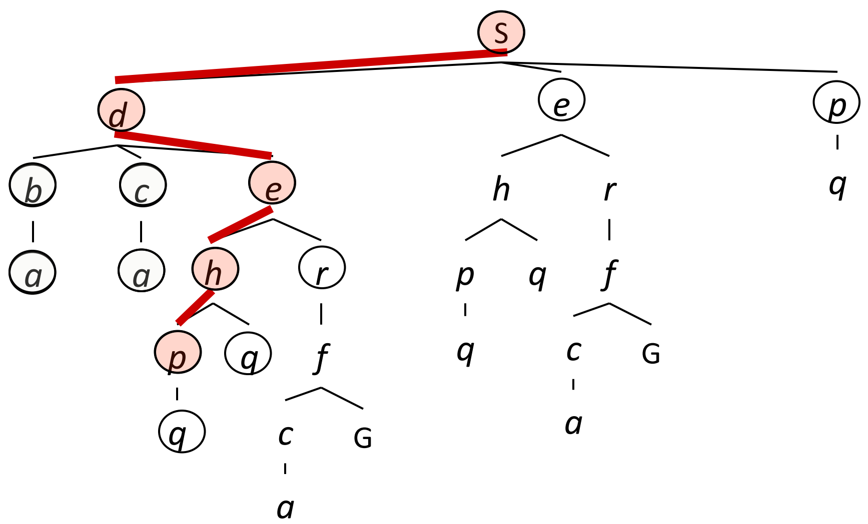

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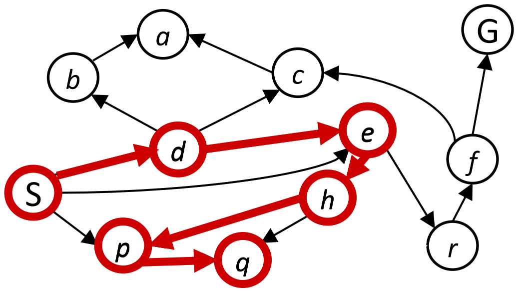


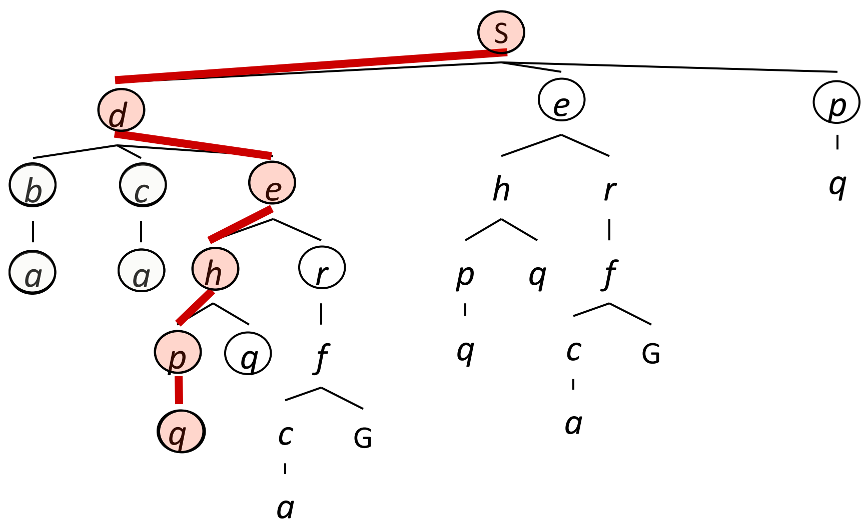

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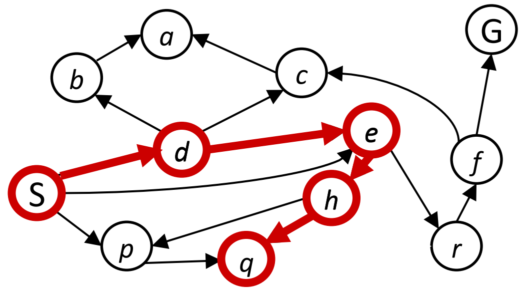


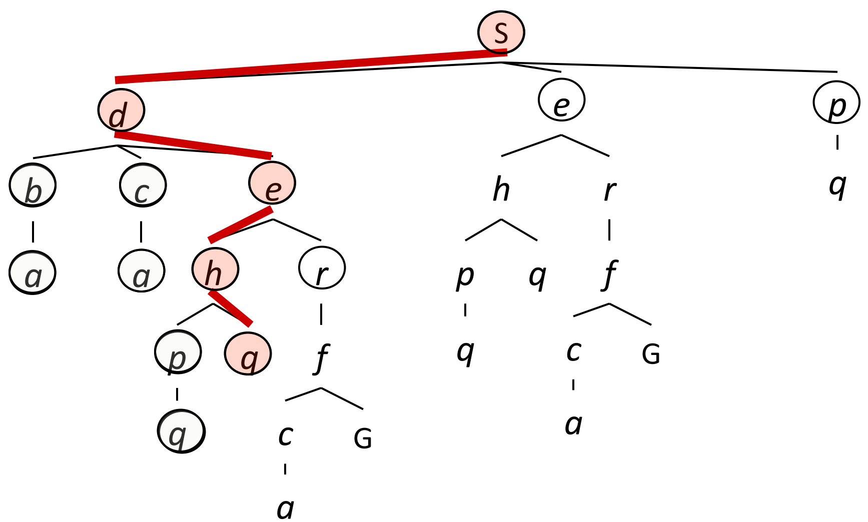

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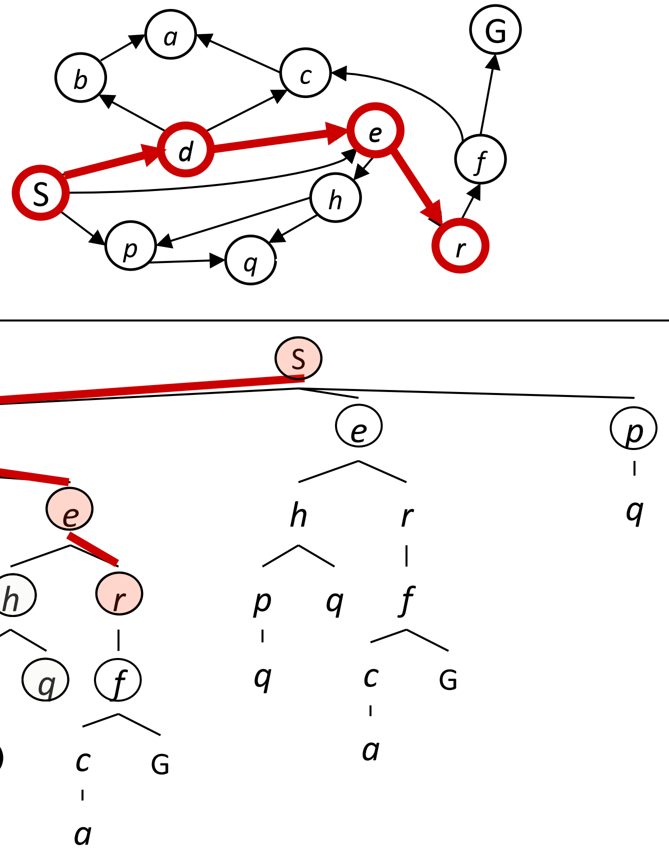

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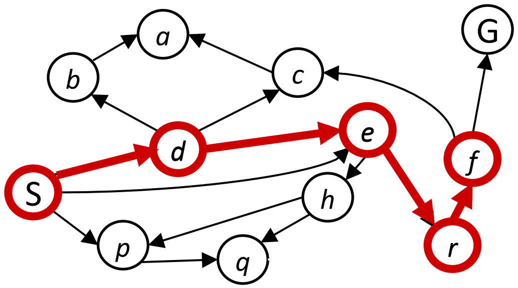


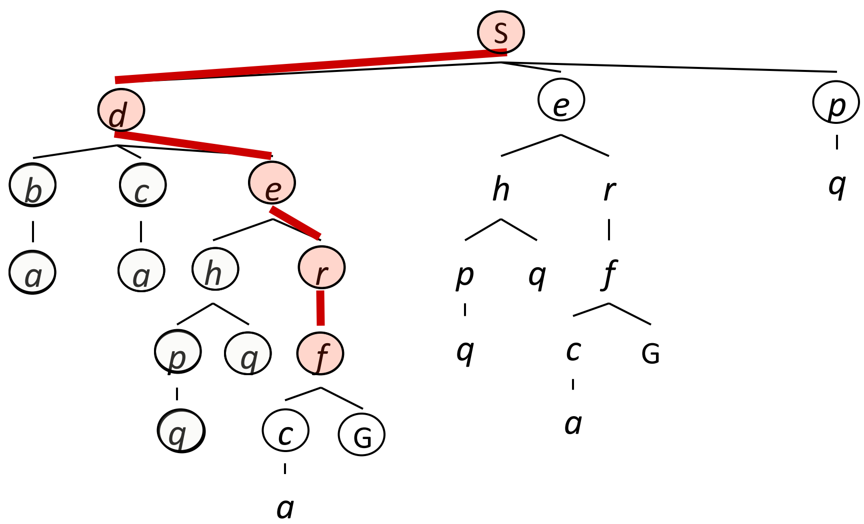

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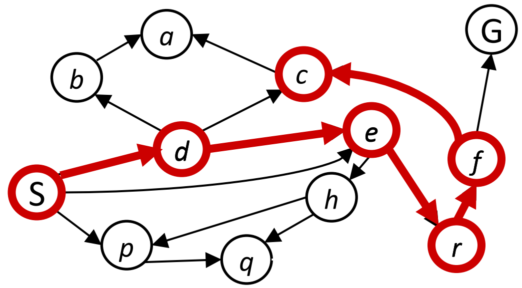


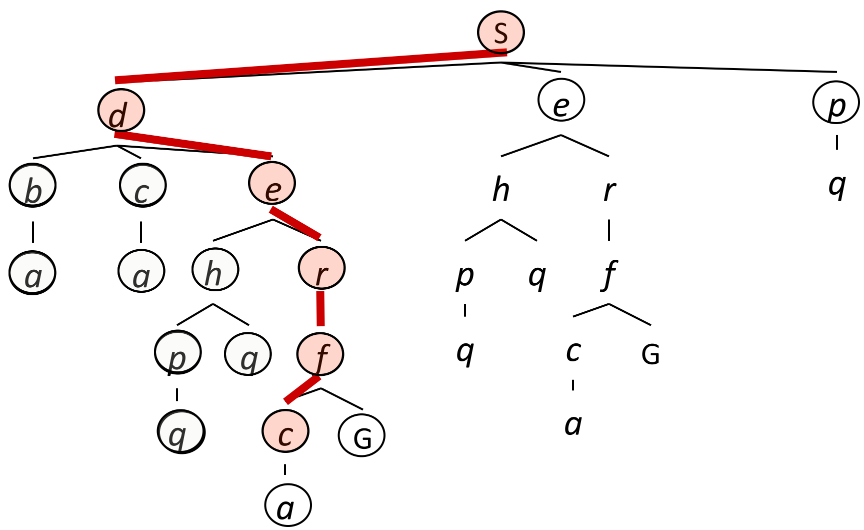

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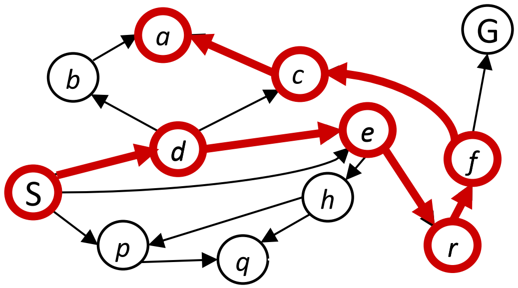


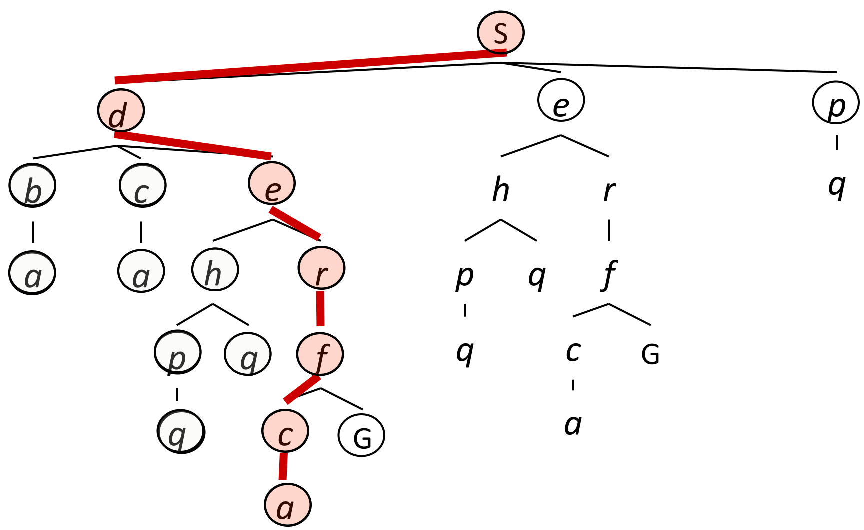

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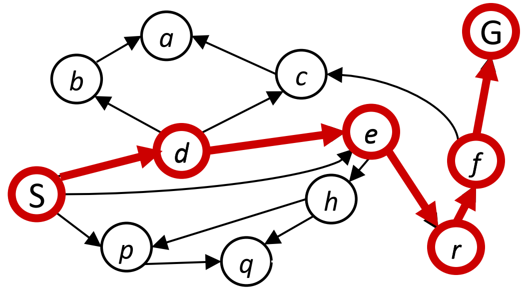


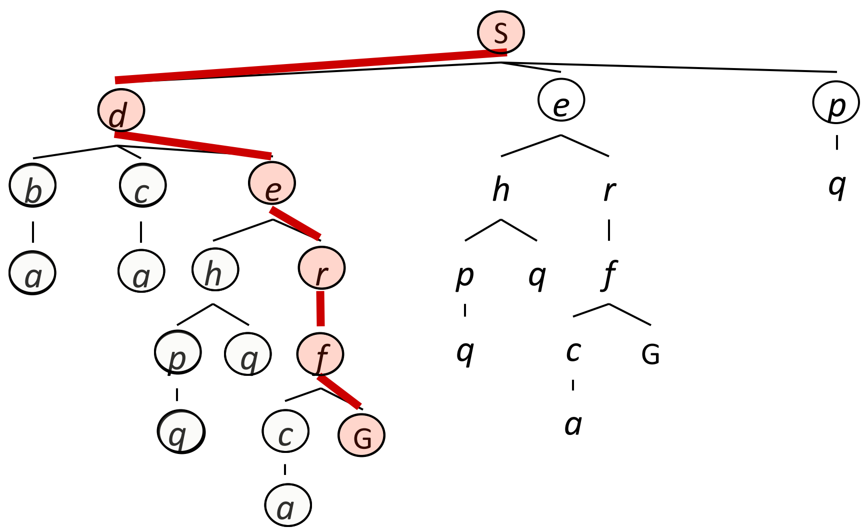

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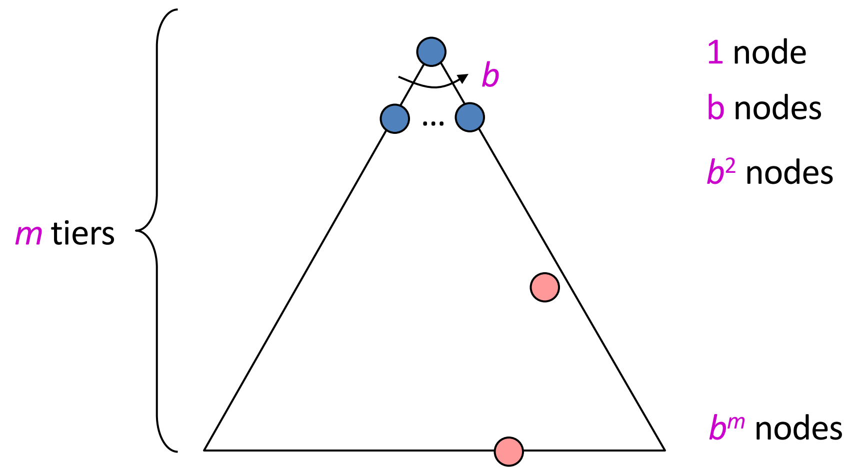


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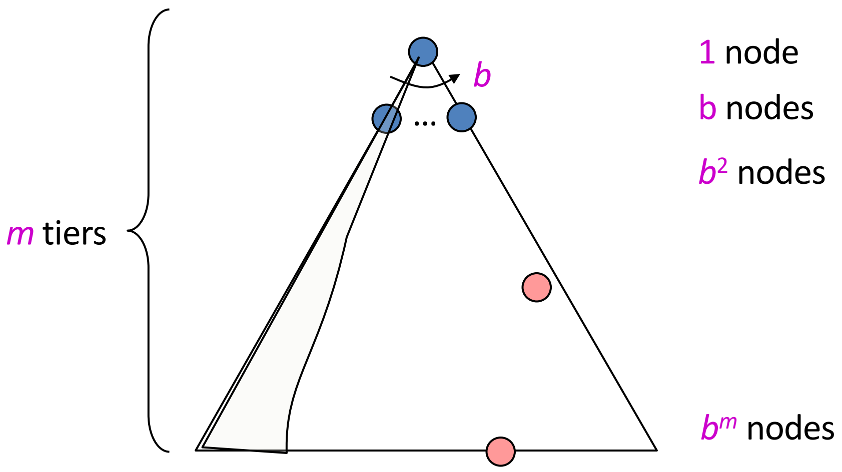


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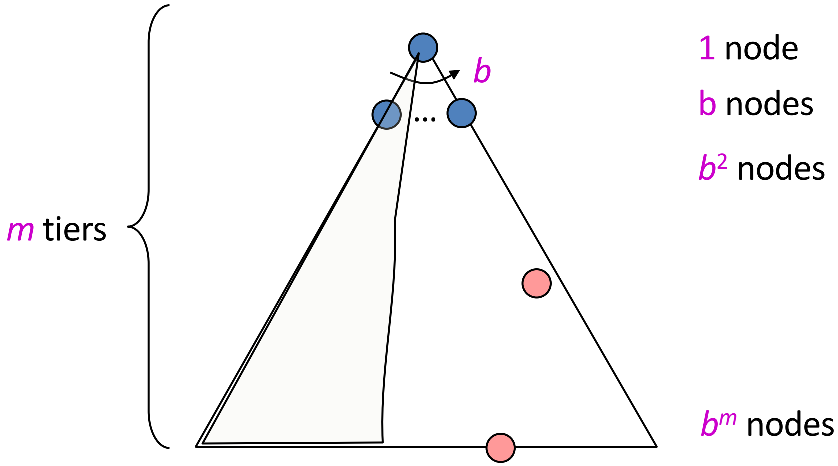


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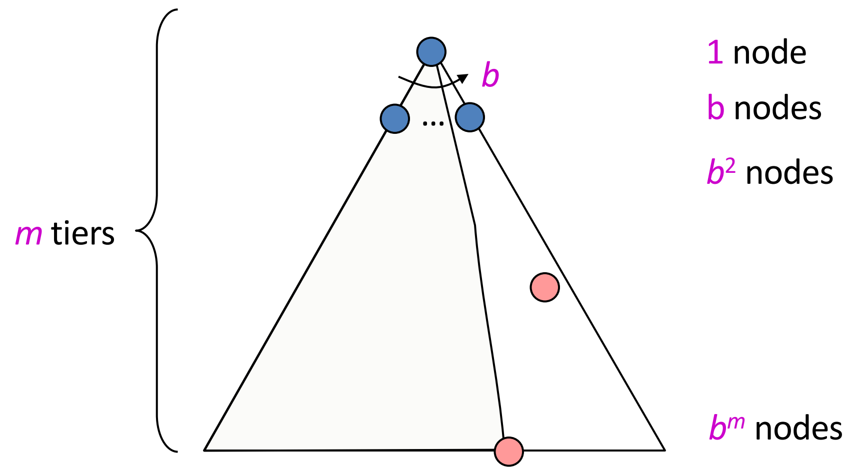


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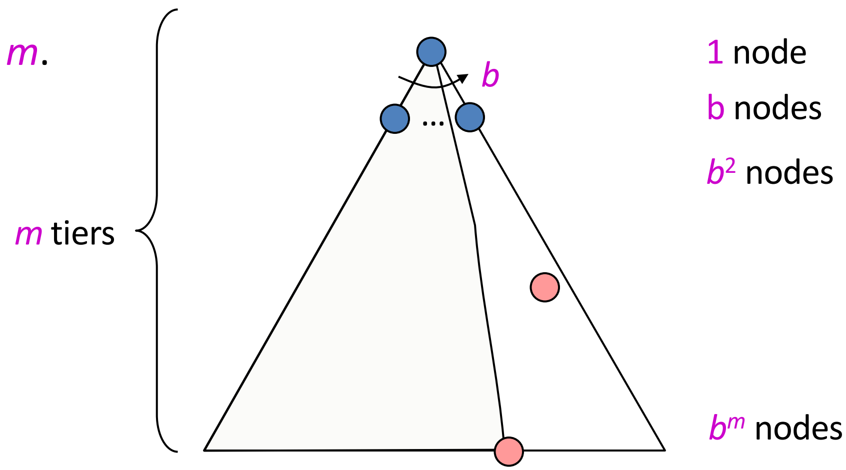


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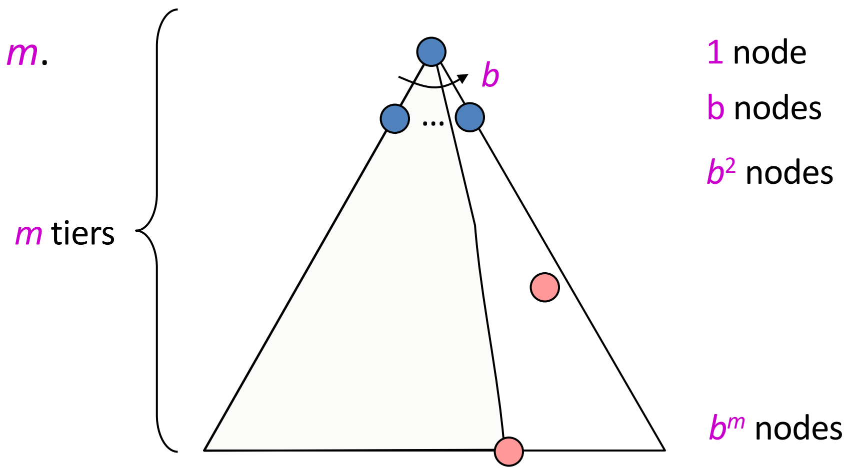


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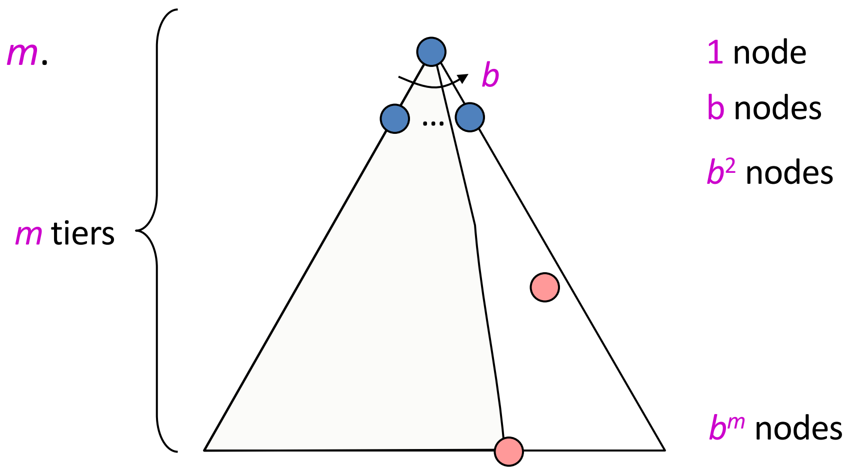


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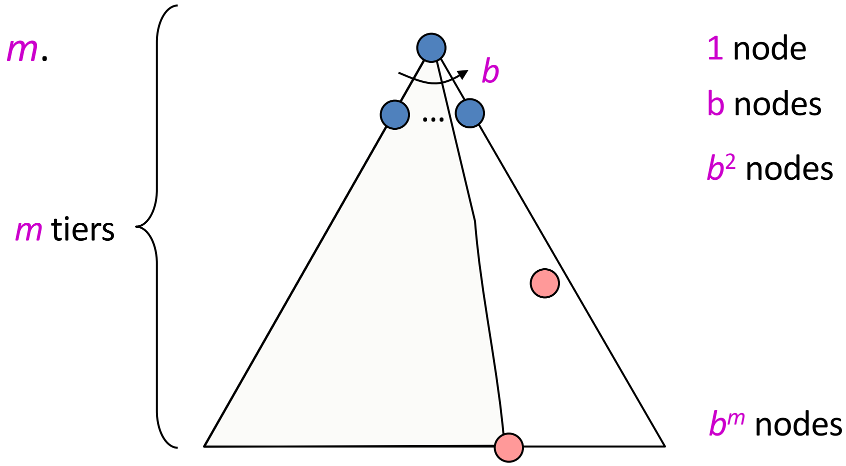


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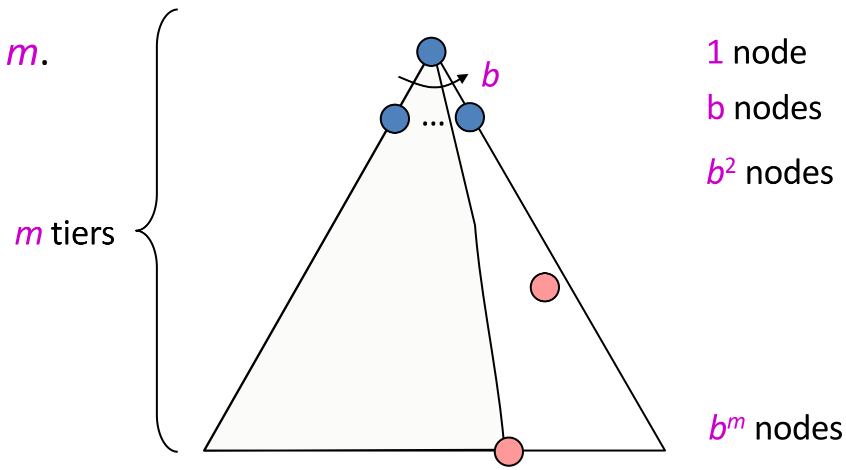


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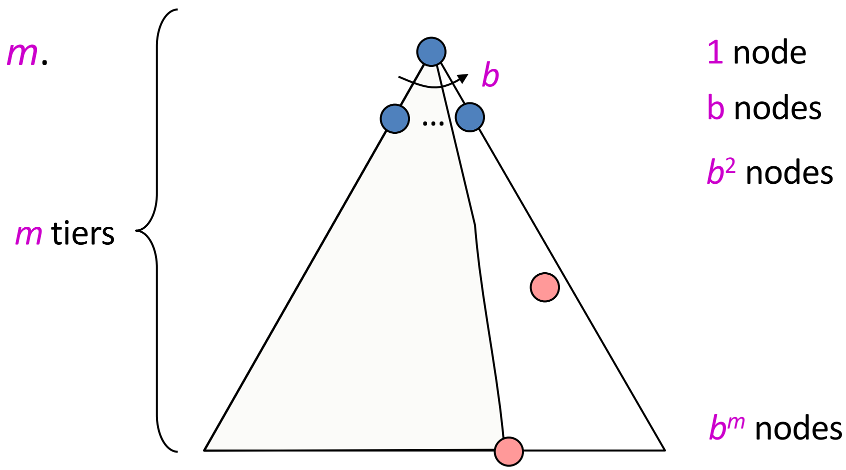


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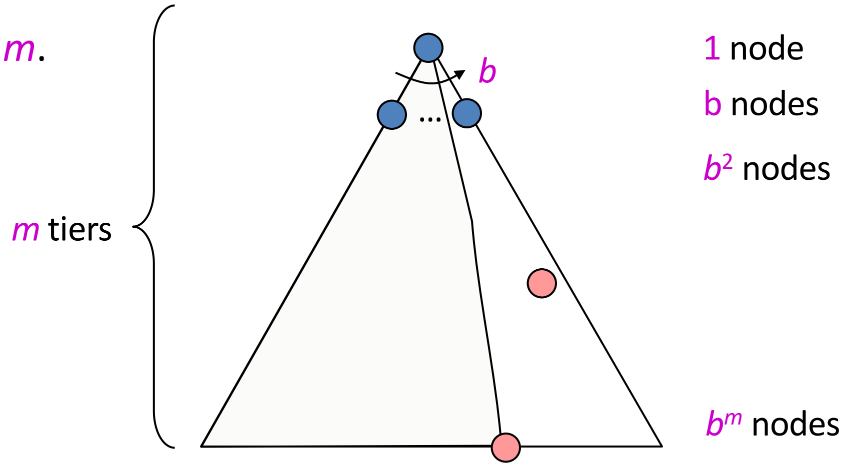


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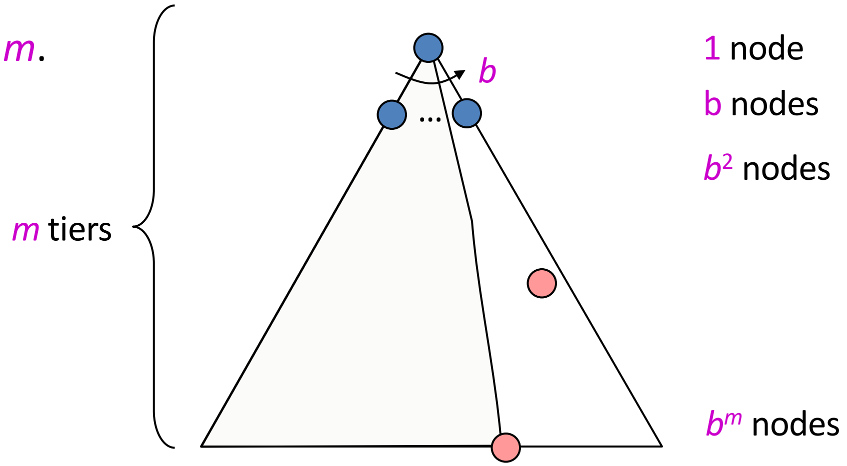


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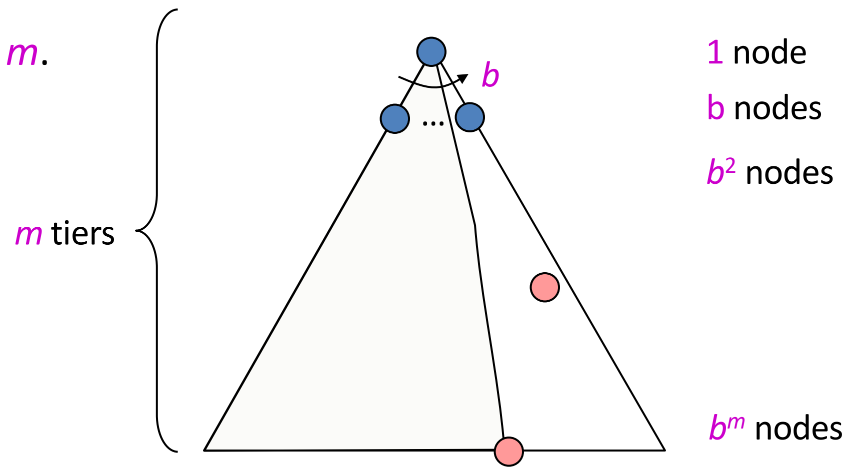


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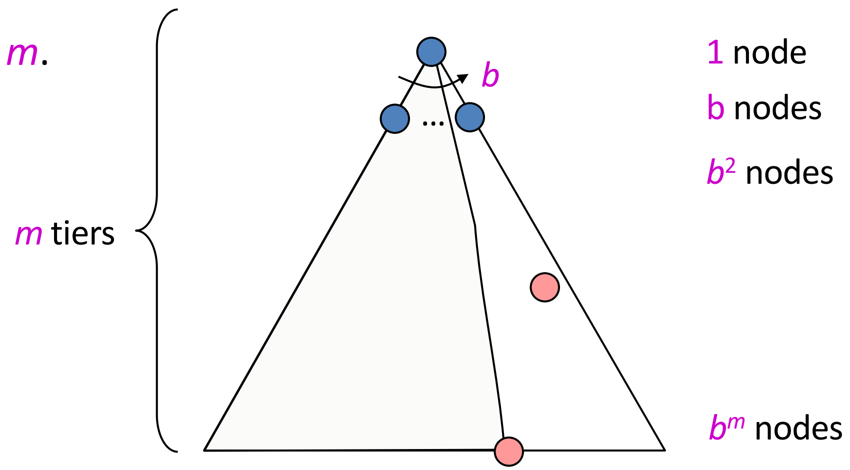


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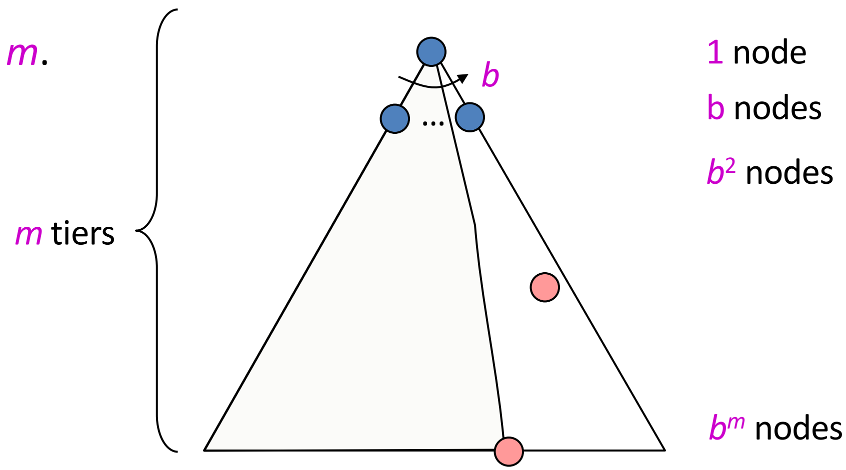
- Is it optimal?
  - No, it finds the "leftmost" solution, regardless of depth or cost

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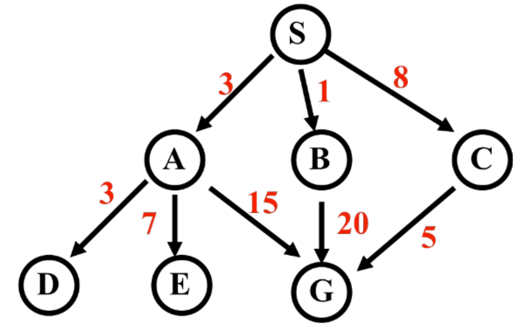
# Breadth-First Search

weighted arcs




| **Expanded node** | **Nodes list (aka Fringe)** |
|---|---|
| | { $S^0$ } |
| $S^0$ | { $A^3$ $B^1$ $C^8$ } |
| $A^3$ | { $B^1$ $C^8$ $D^6$ $E^{10}$ $G^{18}$ } |
| $B^1$ | { $C^8$ $D^6$ $E^{10}$ $G^{18}$ $G^{21}$ } |
| $C^8$ | { $D^6$ $E^{10}$ $G^{18}$ $G^{21}$ $G^{13}$ } |
| $D^6$ | { $E^{10}$ $G^{18}$ $G^{21}$ $G^{13}$ } |
| $E^{10}$ | { $G^{18}$ $G^{21}$ $G^{13}$ } |
| $G^{18}$ | { $G^{21}$ $G^{13}$ } |

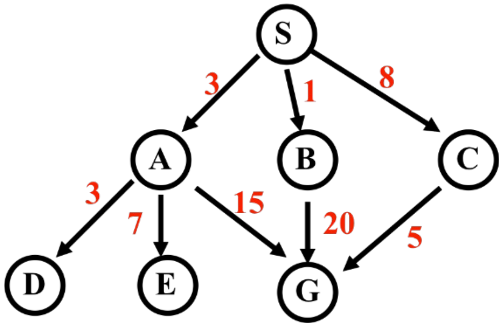
*Notation*

$G^{18}$

G is node; 18 is cost of shortest known path from start node S

Note: we typically don't check for goal until we expand node
Solution path found is S A G , cost 18
Number of nodes expanded (including goal node) = 7

# Depth-First Search

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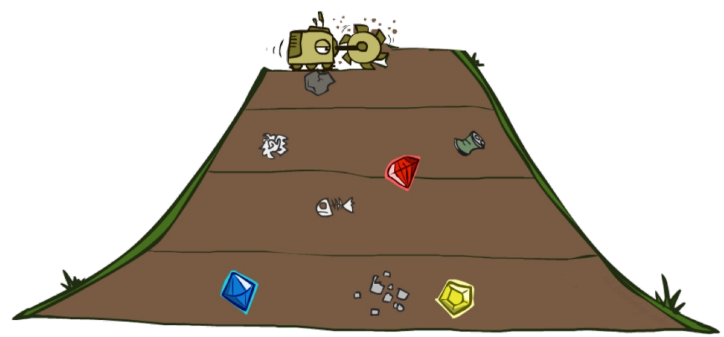
| $E^{10}$ | { $G^{18}$ $B^1$ $C^8$ } |
| $G^{18}$ | { $B^1$ $C^8$ } |

Solution path found is S A G, cost 18

Number of nodes expanded (including goal node) = 5

# Quiz: DFS vs BFS



# Quiz: DFS vs BFS

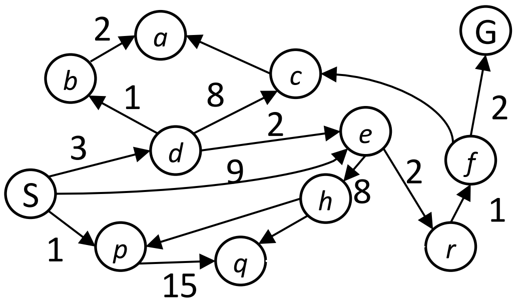
- When will BFS outperform DFS?
- When will DFS outperform BFS?

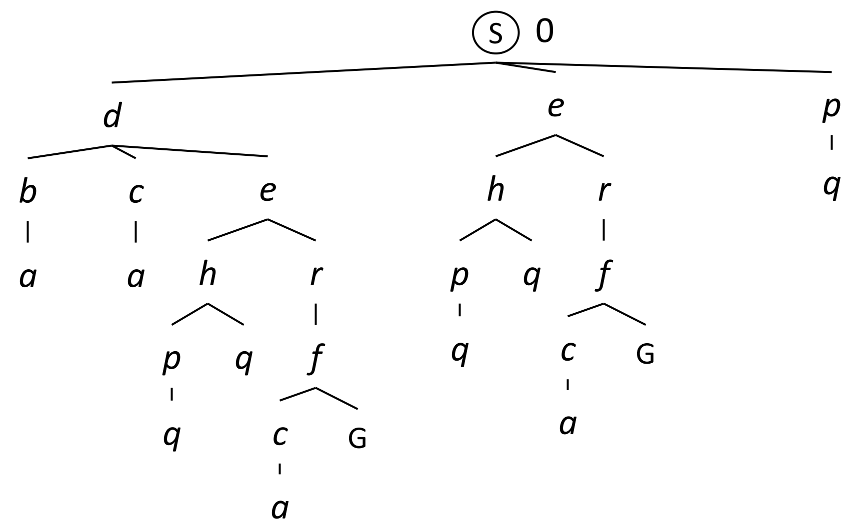
# Uniform Cost Search

*g(n) = cost from root to n*

*Strategy: expand lowest g(n)*

*Frontier is a priority queue sorted by g(n)*



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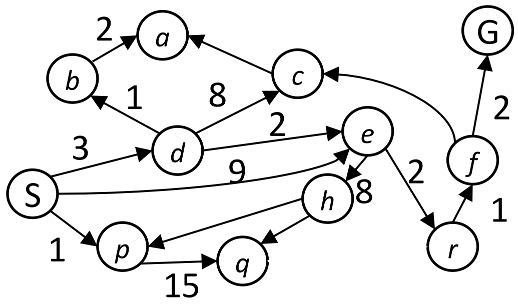


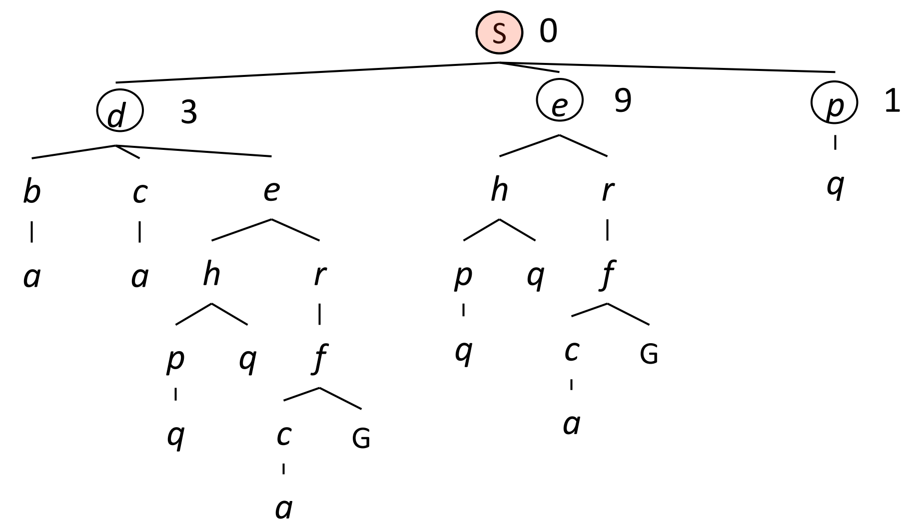

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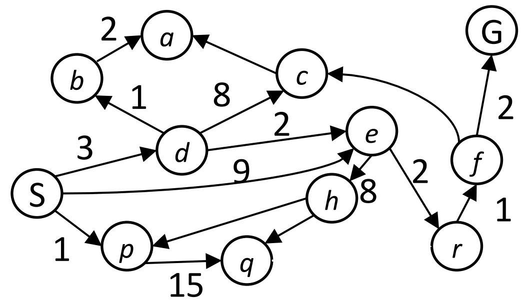


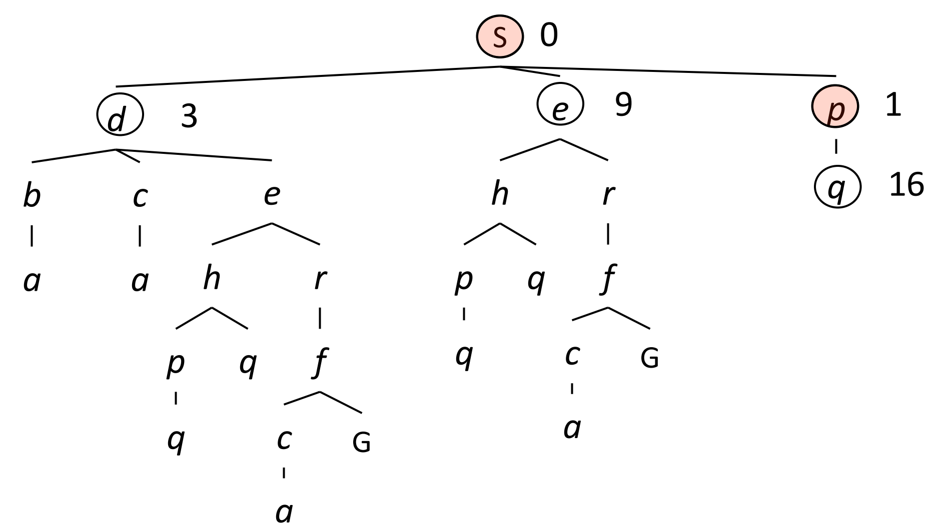

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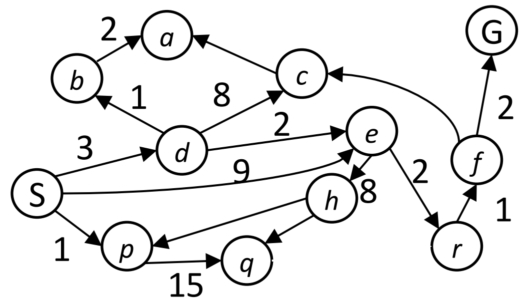


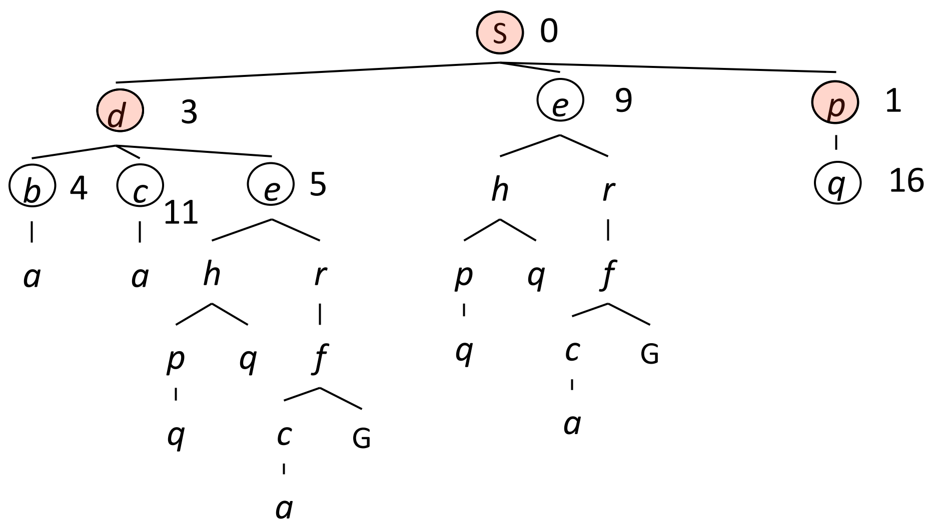

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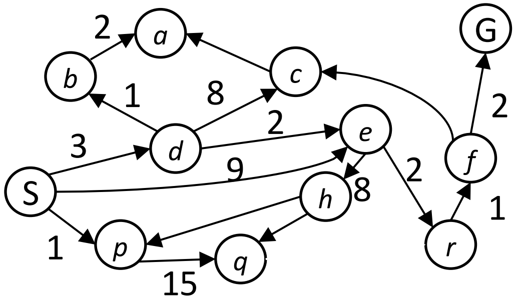


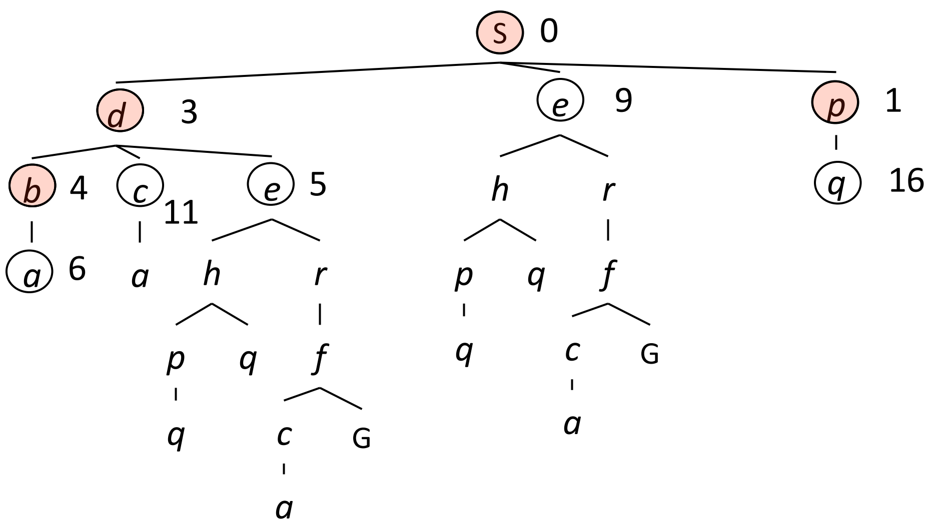

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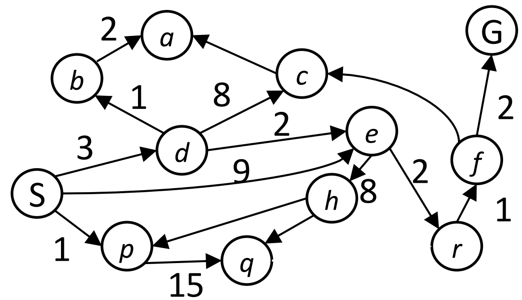


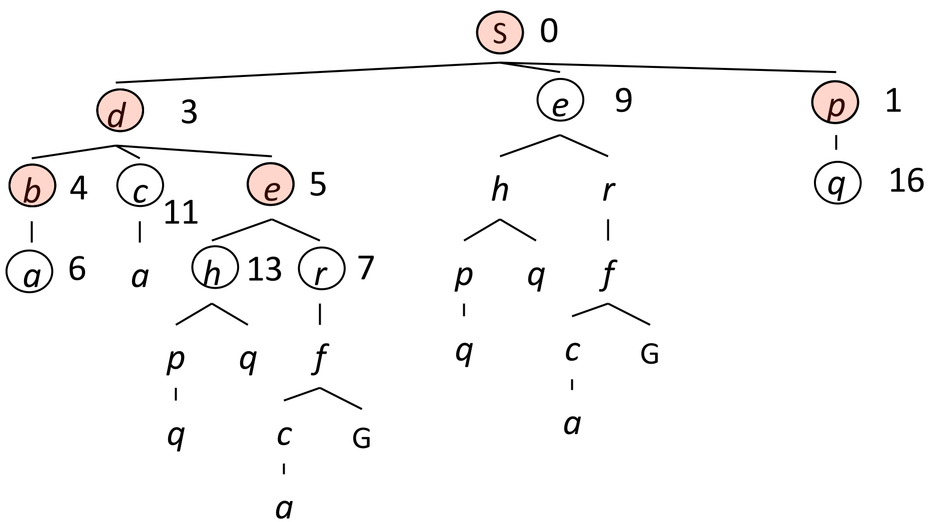

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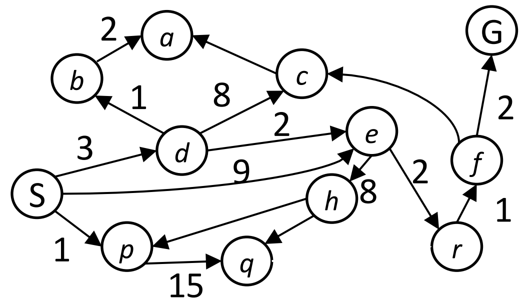


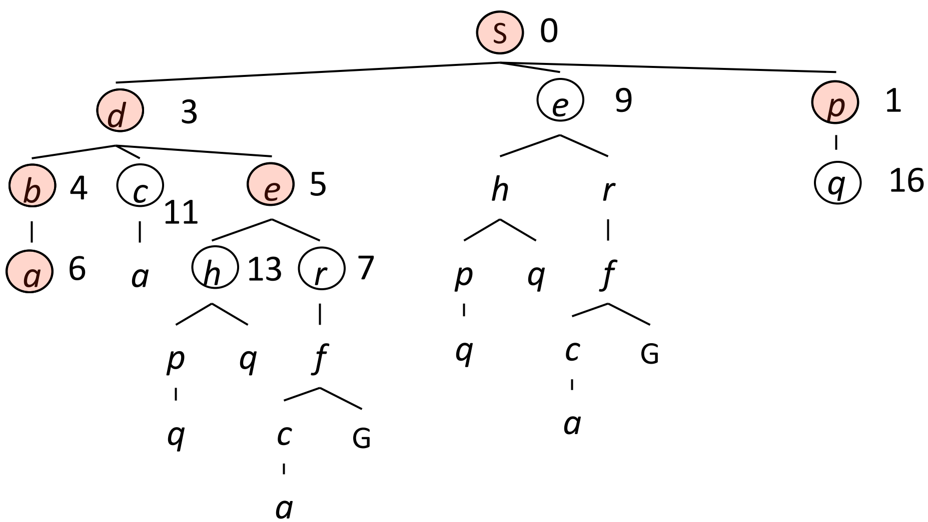

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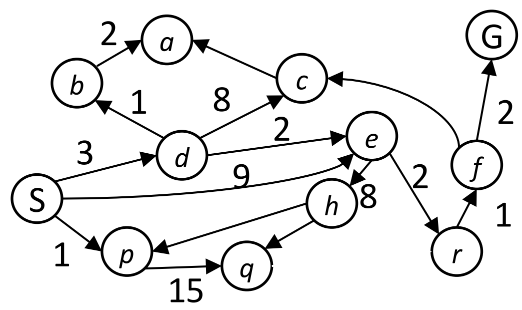


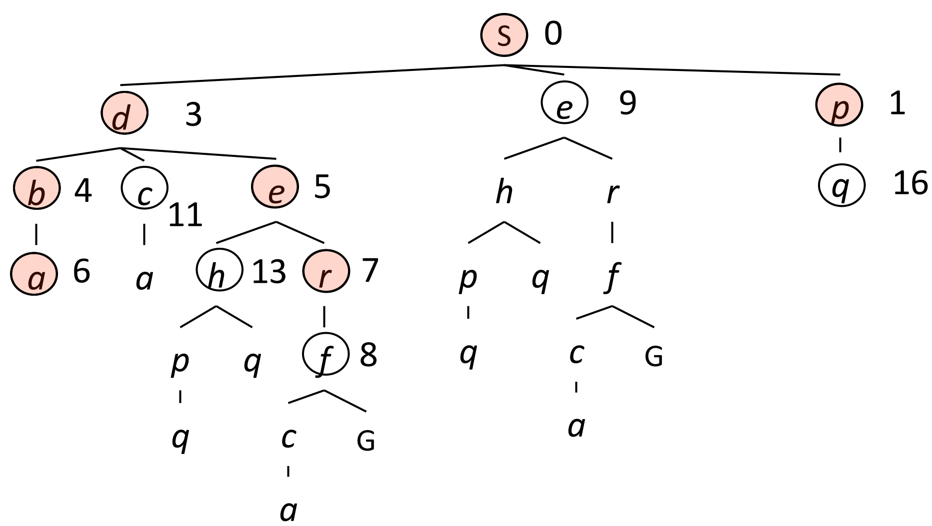

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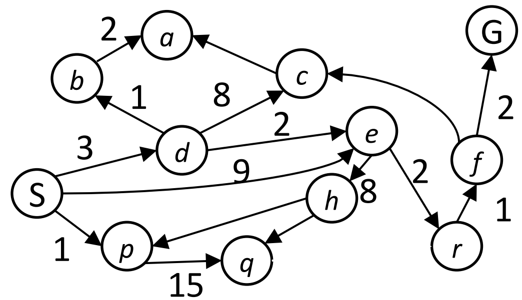


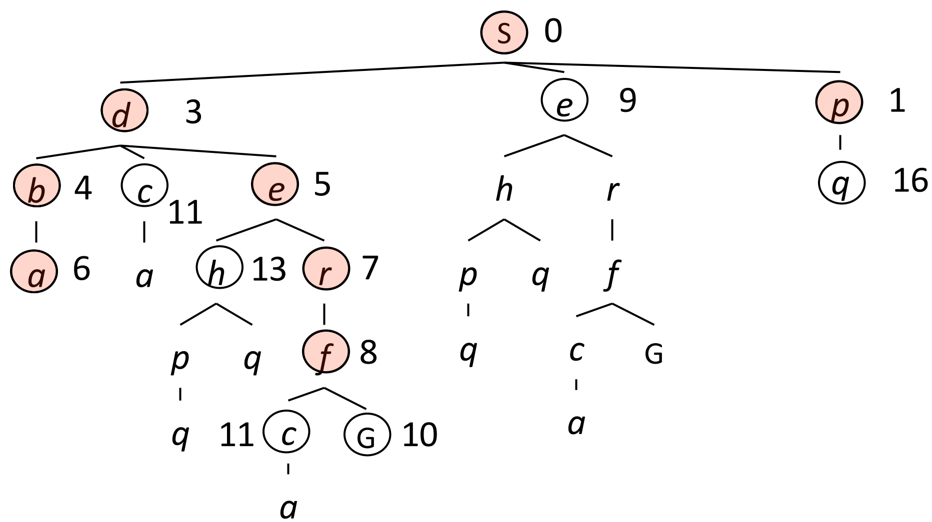

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*g(n)* = *cost from root to n*

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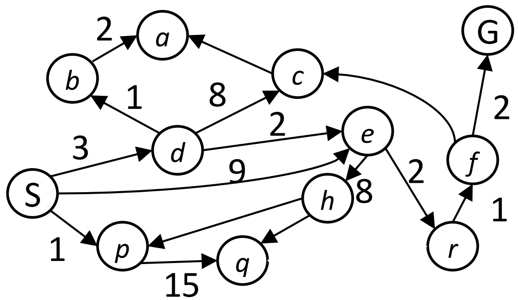


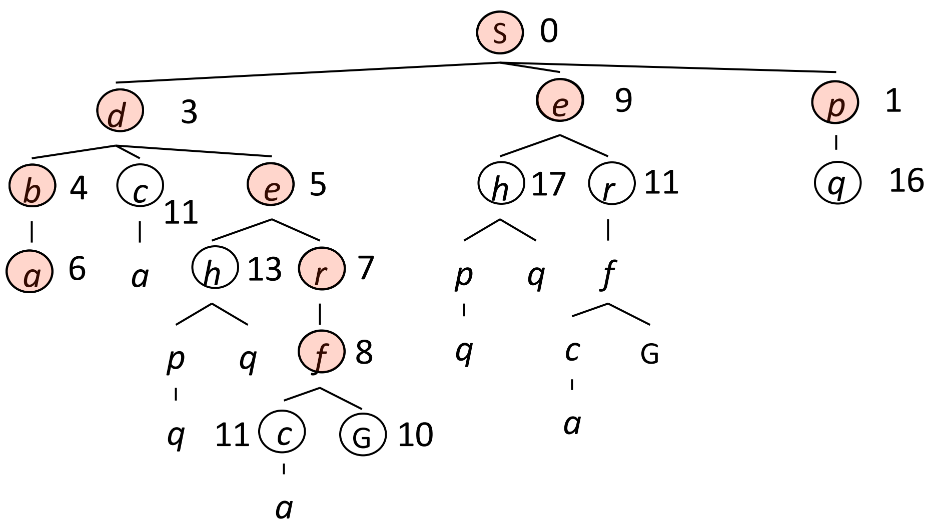

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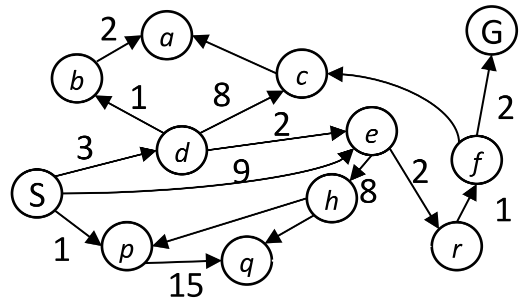


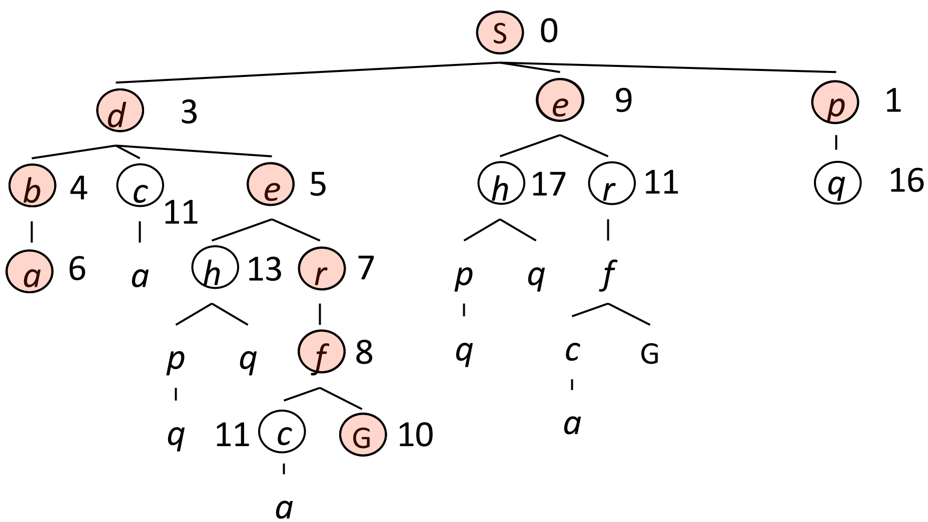

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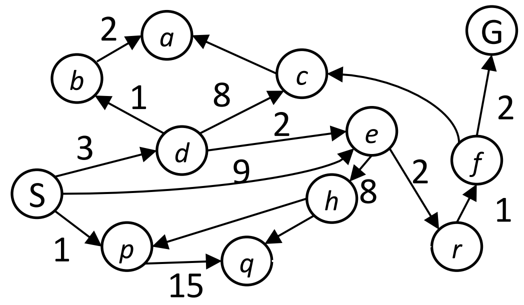


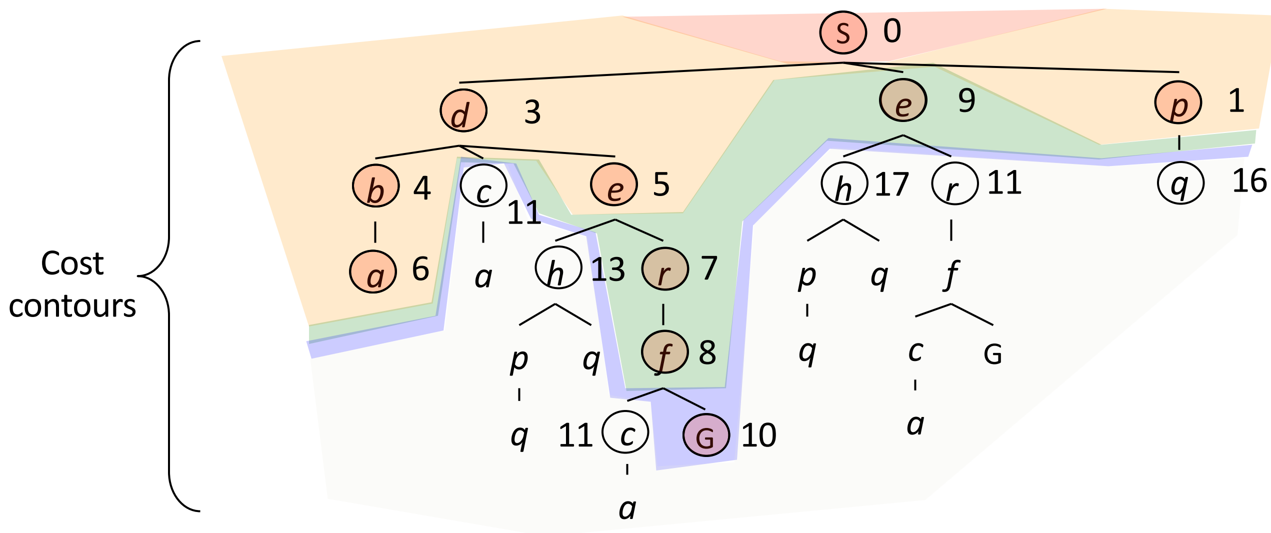

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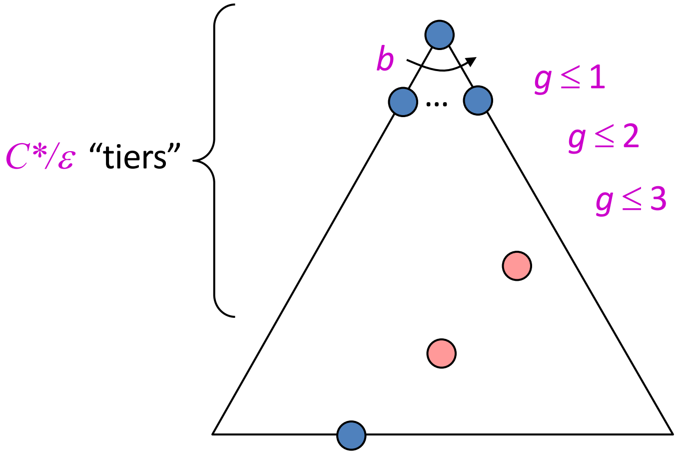
- What nodes does UCS expand?



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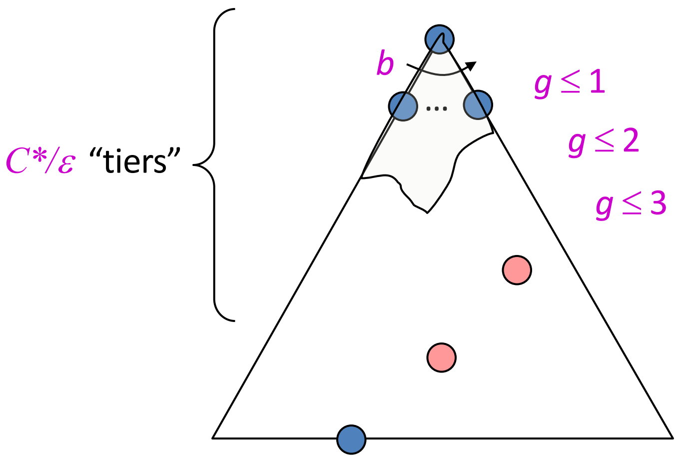

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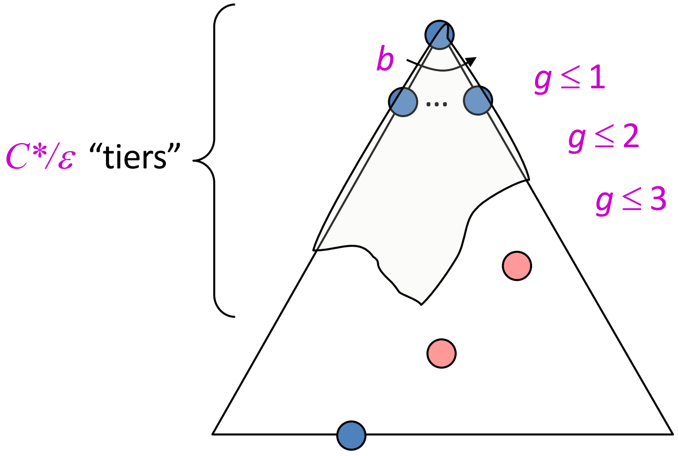

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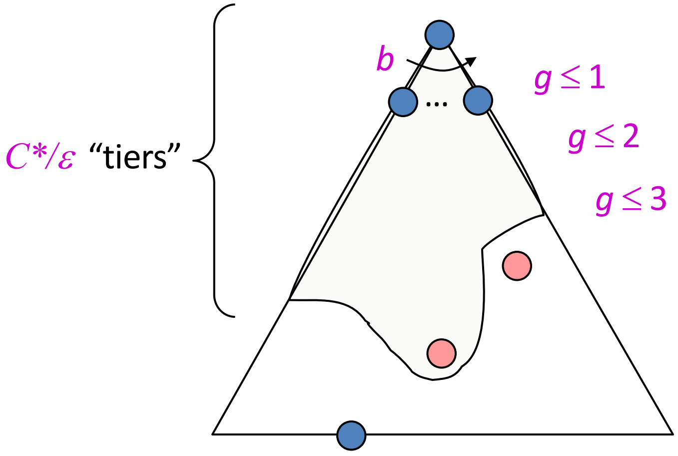

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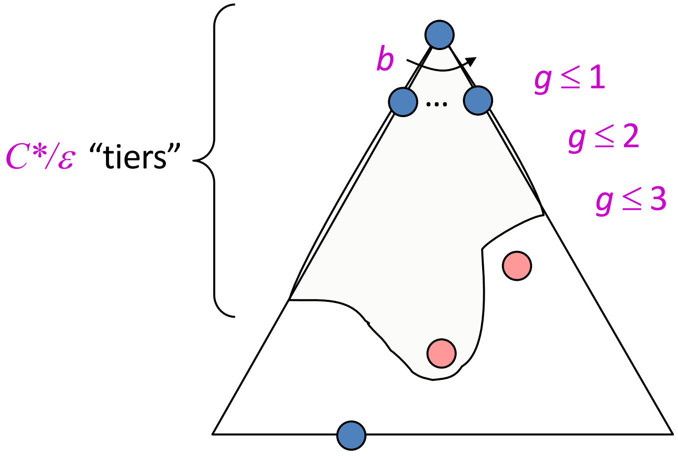

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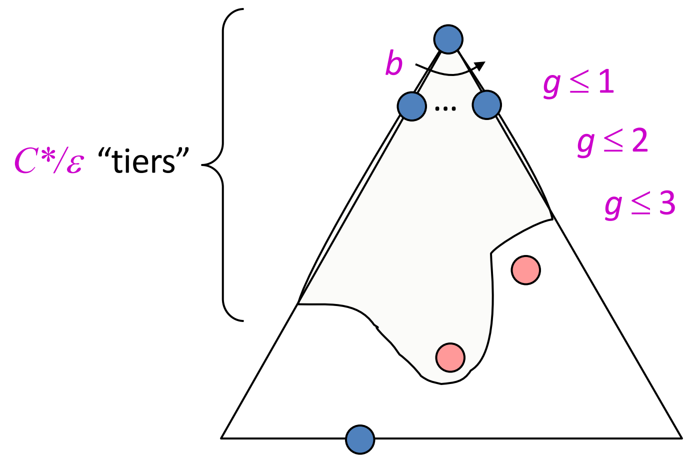

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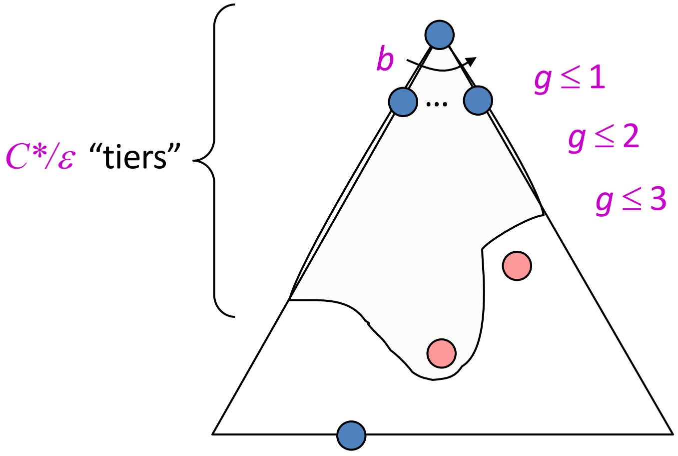

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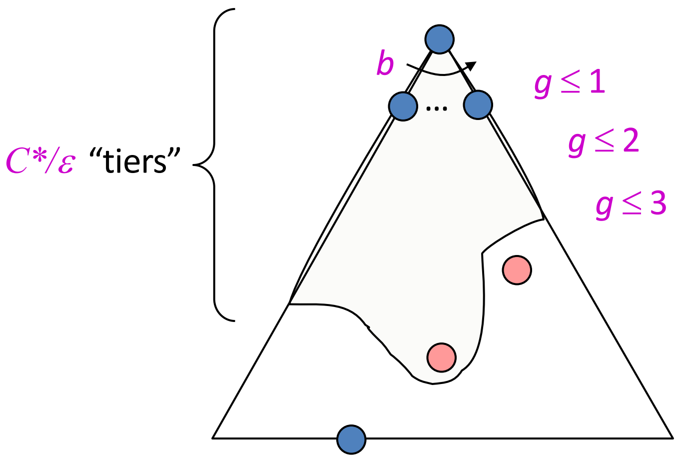

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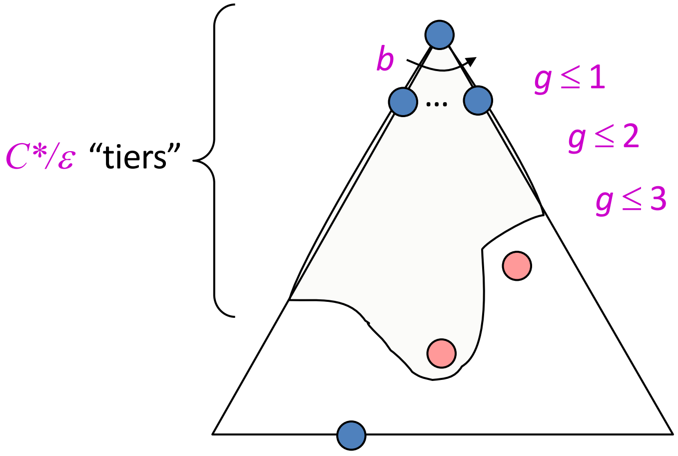


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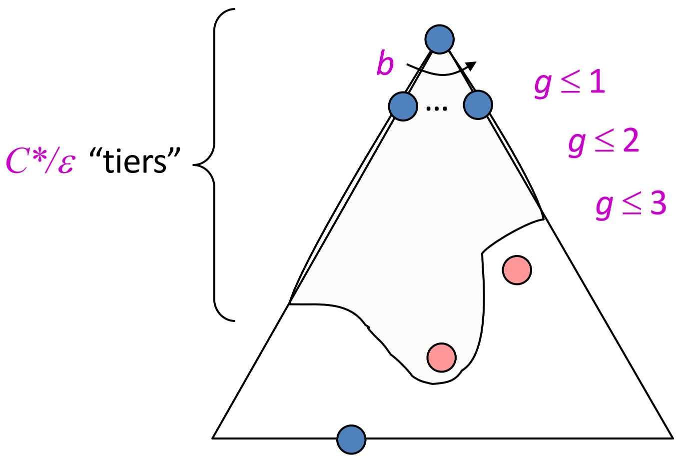

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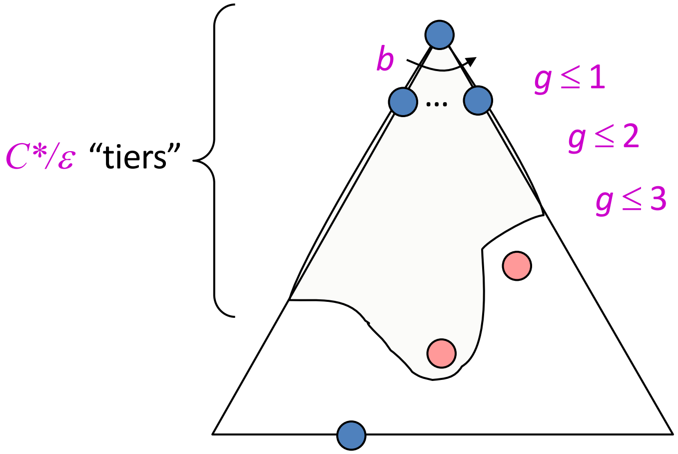
  - Assuming $C^*$ is finite and $\varepsilon > 0$, yes!



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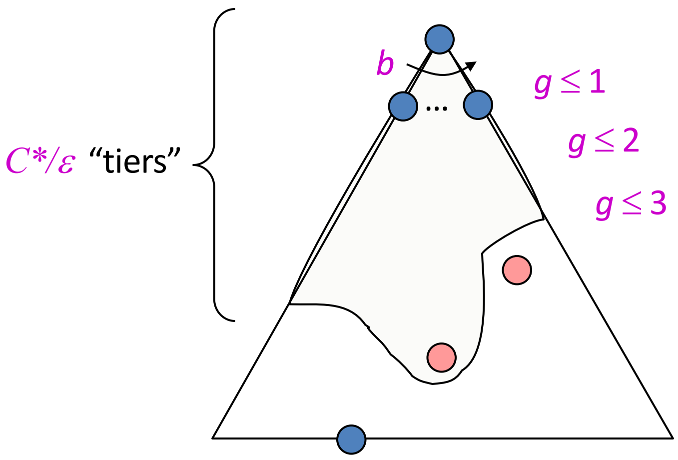

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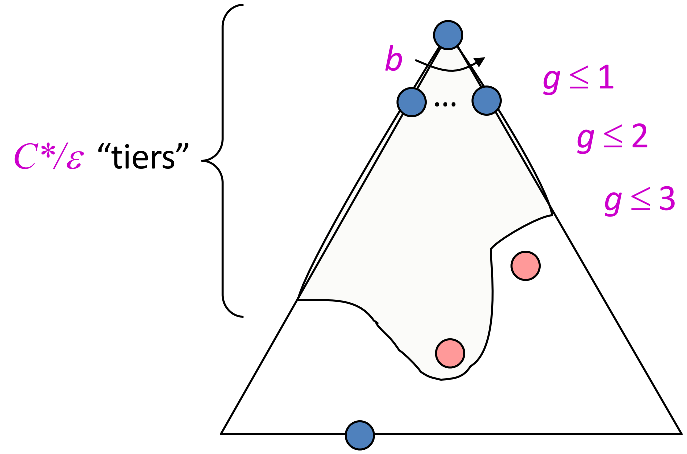
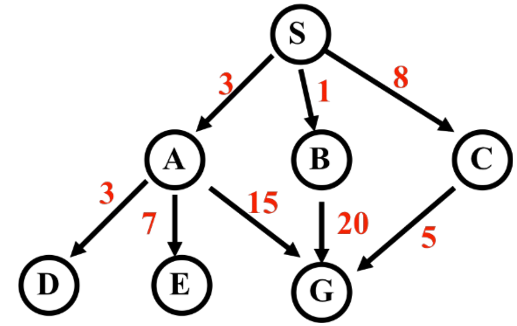



- How much space does the frontier take?
  - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$
- Is it complete?
  - Assuming $C^*$ is finite and $\varepsilon > 0$, yes!
- Is it optimal?
  - Yes! (Proof next lecture via A*)

# Uniform-Cost Search




| Expanded node | Nodes list |
|---|---|
| | { $S^0$ } |
| $S^0$ | { $B^1$ $A^3$ $C^8$ } |
| $B^1$ | { $A^3$ $C^8$ $G^{21}$ } |
| $A^3$ | { $D^6$ $C^8$ $E^{10}$ $G^{18}$ $G^{21}$ } |
| $D^6$ | { $C^8$ $E^{10}$ $G^{18}$ $G^{21}$ } |
| $C^8$ | { $E^{10}$ $G^{13}$ $G^{18}$ $G^{21}$ } |
| $E^{10}$ | { $G^{13}$ $G^{18}$ $G^{21}$ } |
| $G^{13}$ | { $G^{18}$ $G^{21}$ } |

priority queue

Solution path found is S C G, cost 13

Number of nodes expanded (including goal node) = 7

# Depth-First Iterative Deepening (DFID)

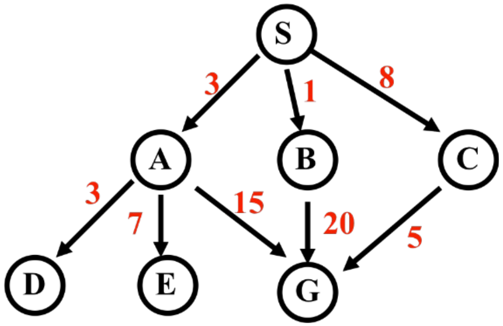
- Do DFS to depth 0, then (if no solution) DFS to depth 1, etc.
- Usually used with a tree search
- **Complete**
- **Optimal/Admissible** if all operators have unit cost, else finds shortest solution (like BFS)
- Time complexity a bit worse than BFS or DFS

  Nodes near top of search tree generated many times, but since almost all nodes are near tree bottom, worst case time complexity still exponential, $O(b^d)$

# Depth-First Iterative Deepening (DFID)

- If branching factor is b and solution is at depth d, then nodes at depth d are generated once, nodes at depth d-1 are generated twice, etc.
  - Hence $b^d + 2b^{(d-1)} + ... + db <= b^d / (1 - 1/b)^2 = O(b^d)$.
  - If b=4, worst case is $1.78 * 4^d$, i.e., 78% more nodes searched than exist at depth d (in worst case)
- **Linear space complexity**, O(bd), like DFS
- Has advantages of BFS (completeness) and DFS (i.e., limited space, finds longer paths quickly)
- Preferred for **large state spaces** where **solution depth is unknown**

# How they perform



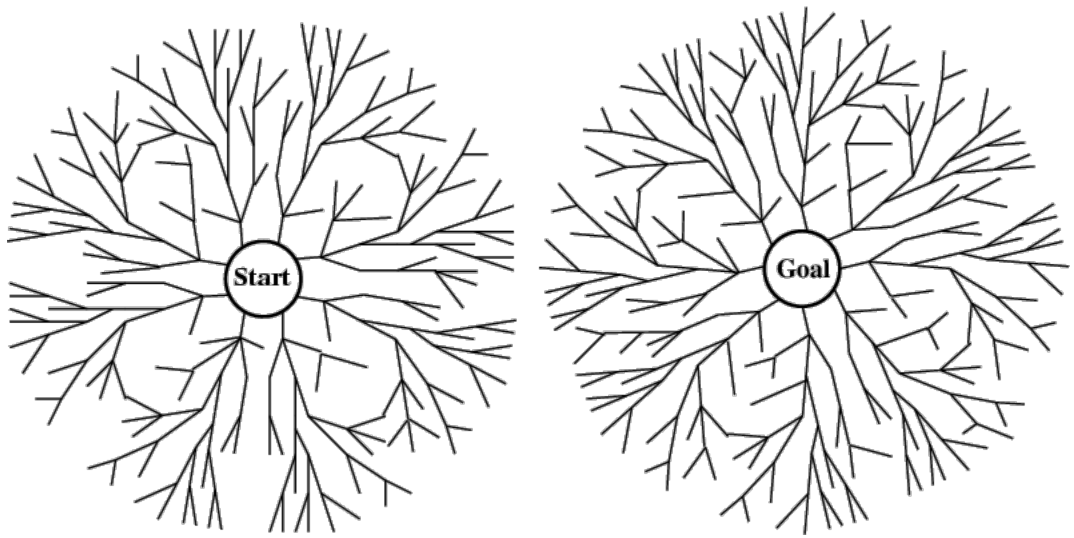

- **Depth-First Search:**
  - 4 Expanded nodes: S A D E G
  - Solution found: S A G (cost 18)
- **Breadth-First Search**:
  - 7 Expanded nodes: S A B C D E G
  - Solution found: S A G (cost 18)
- **Uniform-Cost Search**:
  - 7 Expanded nodes: S A D B C E G
  - Solution found: S C G (cost 13)

  ***Only uninformed search that worries about costs***
- **Iterative-Deepening Search**:
  - 10 nodes expanded: S S A B C S A D E G
  - Solution found: S A G (cost 18)

# Searching Backward from Goal

- Usually a successor function is reversible
  - i.e., can generate a node’s predecessors in graph
- If we know a single goal (rather than a goal’s properties), we could search backward to the initial state
- It might be more efficient
  - Depends on whether the graph fans in or out

# Bi-directional search




- Alternate searching from the start state toward the goal and from the goal state toward the start
- Stop when the frontiers intersect
- Works well only when there are unique start & goal states
- Requires ability to generate "predecessor" states
- Can (sometimes) lead to finding a solution more quickly

# Comparing Search Strategies

| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening | Bidirectional (if applicable) |
|---|---|---|---|---|---|---|
| Time | $b^d$ | $b^d$ | $b^m$ | $b^l$ | $b^d$ | $b^{d/2}$ |
| Space | $b^d$ | $b^d$ | $bm$ | $bl$ | $bd$ | $b^{d/2}$ |
| Optimal? | Yes | Yes | No | No | Yes | Yes |
| Complete? | Yes | Yes | No | Yes, if $l \geq d$ | Yes | Yes |

# Summary

- Search in a problem space is at the heart of many AI systems
- Formalizing the search in terms of **states**, **actions**, and **goals** is key
- The simple "uninformed" algorithms we examined can be augmented to heuristics to improve them in various ways
- But for some problems, a simple algorithm is best