# CMSC 471 Artificial Intelligence 

## Search

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## Today

- Agents that Plan Ahead
- Goal-based agents
- Search Problems
- Generic state-space search algorithm
- Uninformed Search Methods
- Depth-First Search
- Breadth-First Search
- Uniform-Cost Search



## How do you design an intelligent agent?

- An agent is an entity that perceives and acts
- Intelligent agents perceive environment via sensors and act rationally on them with their effectors
- Discrete agents receive percepts one at a time, and map them to a sequence of discrete actions



## Characteristics of environments

|  | Fully <br> observable? | Deterministic? | Episodic? | Static? | Discrete? | Single <br> agent? |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Solitaire | No | Yes | Yes | Yes | Yes | Yes |
| Backgammon | Yes | No | No | Yes | Yes | No |
| Taxi driving | No | No | No | No | No | No |
| Internet <br> shopping | No | No | No | No | Yes | No |
| Medical <br> diagnosis | No | No | No | No | No | Yes |

A Yes in a cell means that aspect is simpler; a No more complex

## Characteristics of environments

$\rightarrow$ Lots of real-world domains fall into the hardest case!

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## Agents that Plan



## (0) Table-driven agents

Use percept sequence/action table to find next action. Implemented by a lookup table

## (1) Simple reflex agents



Based on condition-action rules, stateless devices with no memory of past world states

## (2) Agents with memory

represent states and keep track of past world states

## (3) Agents with goals

Have a state and goal information describing desirable situations; can take future events into consideration

## (4) Utility-based agents

base decisions on utility theory in order to act rationally

## (3) Architecture for goal-based agent

 state and goal information describe desirable situations allowing agent to take future events into consideration

## Planning Agents

- Planning agents:
- Ask "what if"
- Decisions based on (hypothesized) consequences of actions
- Must have a model of how the world evolves in response to actions
- Must formulate a goal (test)
- Consider how the world WOULD BE
- Optimal vs. complete planning



## Big Idea

Allen Newell and Herb Simon developed the problem space principle as an Al approach in the late 60s/early 70s
"The rational activity in which people engage to solve a problem can be described in terms of (1) a set of states of knowledge, (2) operators for changing one state into another, (3) constraints on applying operators and (4) control knowledge for deciding which operator to apply next."

Newell A \& Simon H A. Human problem solving. Englewood Cliffs, NJ: Prentice-Hall. 1972.

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"The rat problem of know another control next."

We'll achieve this by formulating an appropriate graph and then applying graph search algorithms to it

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## Search Problems: Key Terms

- A search problem consists of:


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## Search Problems: Key Terms

- A search problem consists of:
- A state space

- A successor function (with actions, costs)

- A start state and a goal test
- A solution is a sequence of actions (a plan) which transforms the start state to a goal state


## Search Problems Are Models

## Search Problems Are Models



## Example: 8-Puzzle

Given an initial configuration of 8 numbered tiles on a $3 \times 3$ board, move the tiles to produce a desired goal configuration


Start State


Goal State

## Simpler: 3-Puzzle



## Building goal-based agents

## We must answer the following questions

-How do we represent the state of the "world"?
-What is the goal and how can we recognize it?
-What are the possible actions?
-What relevant information do we encoded to describe states, actions and their effects and thereby solve the problem?


## Characteristics of 8-puzzle ?

|  | Fully <br> observable? | Deterministic? | Episodic? | Static? | Discrete? | Single <br> agent? |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8-puzzle |  |  |  |  |  |  |

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| 8-puzzle | Yes | Yes | Yes | Yes | Yes | Yes |

- All the Yes's mean it may be relatively easy!
- This is typical of the problems worked on in the 60s and 70s
- And the algorithms for solving them a statespace search model


## Representing states / State-space

- State of an 8-puzzle?

| 5 | 4 |  |
| :---: | :---: | :---: |
| 6 | 1 | 8 |
| 7 | 3 | 2 |
|  |  |  |

## Representing states / State-space

- State of an 8-puzzle?
- A $3 \times 3$ array of integer in $\{0 . .8\}$
- No integer appears twice
- 0 represents the empty space
- In Python, we might implement this using a ninecharacter string: "540681732"
- And write functions to map the 2D coordinates to an index


## What's the goal to be achieved?

- Describe situation we want to achieve, a set of properties that we want to hold, etc.
- Defining a goal test function that when applied to a state returns True or False
- For our problem:

$$
\begin{aligned}
\text { def } & \text { isGoal(state): } \\
& \text { return state }==" 123405678 "
\end{aligned}
$$

## What are the actions?

- Primitive actions for changing the state In a deterministic world: no uncertainty in an action's effects (simple model)
- Given action and description of current world state, action completely specifies
- Whether action can be applied to the current world (i.e., is it applicable and legal?) and
- What state results after action is performed in the current world (i.e., no need for history information to compute the next state)


## Representing actions

- Actions ideally considered as discrete events that occur at an instant of time
- Example, in a planning context
- If state:inClass and perform action:goHome, then next state is state:atHome
- There's no time where you're neither in class nor at home (i.e., in the state of "going home")


## Representing actions

- Actions for 8-puzzle?

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## Representing actions

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| 7 | 3 | 2 |
| 7 |  |  |

- Number of actions/operators depends on the representation used in describing a state
- Specify 4 possible moves for each of the 8 tiles, resulting in a total of $4 * 8=32$ operators
- Or: Specify four moves for "blank" square and we only need 4 operators
- Representational shift can simplify a problem!


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The world state includes every last detail of the environment


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- Problem: Pathing
- Problem: Eat-All-Dots
- States: ( $\mathrm{x}, \mathrm{y}$ ) location
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- Successor: update location only
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- States: $\{(x, y)$, dot booleans $\}$


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- Actions: NSEW
- Successor: update location and possibly a dot boolean
- Goal test: dots all false


## State Space Sizes?

- World state:
- Agent positions: 120
- Food count: 30
- Ghost positions: 12
- Agent facing: NSEW



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120


- States for eat-all-dots?


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- States for pathing? 120

- States for eat-all-dots?
$120 \times\left(2^{30}\right)$


## State Space Sizes?

- Size of a problem usually described in terms of possible number of states
- Tic-Tac-Toe has about $3^{9}$ states $\left(19,683 \approx 2 * 10^{4}\right)$
- Checkers has about $10^{40}$ states
- Rubik's Cube has about $10^{19}$ states
- Chess has about $10^{120}$ states in a typical game
- Go has 2*10 ${ }^{170}$
- Theorem provers may deal with an infinite space
- State space size $\approx$ solution difficulty


## State Space Sizes?

- Our estimates were loose upper bounds
- How many possible, legal states does tic-tac-toe really have?
- Simple upper bound: nine board cells, each of which can be empty, O or X , so $3^{9}$
- Only 593 states after eliminating
- impossible states $\stackrel{x|x|}{\mp}$
- Rotations and reflections
 \#


## Some example problems

- Toy problems and micro-worlds
-8-Puzzle
- Missionaries and Cannibals
- Cryptarithmetic
- 8-Queens Puzzle
- Remove 5 Sticks
- Water Jug Problem
- Real-world problems


## Example: The 8-Queens Puzzle

Place eight queens on a chessboard such that no queen attacks any other

We can generalize the problem to a NxN chessboard


## Some more real-world problems

- Route finding
- Touring (traveling salesman)
- Logistics
- VLSI layout
- Robot navigation
- Theorem proving
- Learning


## Route Planning

## Find a route from Arad to Bucharest



A simplified map of major roads in Romania used in our text

## Example: Traveling in Romania

- State space:
- Cities

- Successor function:
- Roads: Go to adjacent city with cost = distance
- Start state:
- Arad
- Goal test:
- Is state == Bucharest?
- Solution?


## State Space Graphs and Search Trees



## State Space Graphs

- State space graph: A mathematical representation of a search problem
- Nodes are (abstracted) world configurations
- Arcs represent transitions/ successors (action results)
- The goal test is a set of goal nodes (maybe only one)


Tiny state space graph for a tiny search problem

- In a state space graph, each state occurs only once!
- We can rarely build this full graph in memory (it's too big), but it's a useful idea


## Search Trees

## Search Trees

 1
## Search Trees



This is now / start

Possible futures

## Search Trees



This is now / start

Possible futures

## Search Trees



This is now / start

Possible futures

- A search tree:
- A "what if" tree of plans and their outcomes
- The start state is the root node
- Children correspond to successors
- Nodes show states, but correspond to PLANS that achieve those states
- For most problems, we can never actually build the whole tree


## State Space Graphs vs. Search Trees



Each NODE in in the search tree is an entire PATH in the state space graph.

We construct the tree on demand and we construct as little as possible.


# Quiz: State Space Graphs vs. Search Trees 

Consider this 4-state graph:


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Consider this 4-state graph:
How big is its search tree (from S )?


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Important: Those who don't know history are doomed to repeat it!

## Formalizing search

- Solution: sequence of actions associated with a path from a start node to a goal node
- Solution cost: sum of the arc costs on the solution path
- If all arcs have same (unit) cost, then solution cost is length of solution (number of steps)
- Algorithms generally require that arc costs cannot be negative (why?)


## A General Searching Algorithm

## Core ideas:

1. Maintain a list of frontier (fringe) nodes
2. Nodes coming into the frontier have been explored
3. Nodes going out of the frontier have not been explored
4. Iteratively select nodes from the frontier and explore unexplored nodes from the frontier
5. Stop when you reach your goal


## State-space search algorithm

;; problem describes the start state, operators, goal test, and operator costs
;; queueing-function is a comparator function that ranks two states
;; general-search returns either a goal node or failure
function general-search (problem, QUEUEING-FUNCTION) nodes = MAKE-QUEUE (MAKE-NODE (problem.INITIAL-STATE))
loop
if EMPTY(nodes) then return "failure"
node $=$ REMOVE-FRONT (nodes)
if problem.GOAL-TEST (node.STATE) succeeds then return node
nodes = QUEUEING-FUNCTION (nodes, EXPAND (node, problem. OPERATORS))
end
;; Note: The goal test is NOT done when nodes are generated
;; Note: This algorithm does not detect loops

## Key procedures to be defined

- EXPAND
- Generate a node's successor nodes, adding them to the graph if not already there
- GOAL-TEST
- Test if state satisfies all goal conditions
- QUEUEING-FUNCTION
- Maintain ranked list of nodes that are candidates for expansion
- Changing definition of the QUEUEING-FUNCTION leads to different search strategies


## Informed vs. uninformed search

Uninformed search strategies (blind search)
-Use no information about likely direction of a goal
-Methods: breadth-first, depth-first, depth-limited, uniform-cost, depth-first iterative deepening, bidirectional
Informed search strategies (heuristic search)
-Use information about domain to (try to) (usually) head in the general direction of goal node(s)
-Methods: hill climbing, best-first, greedy search, beam search, algorithm $A$, algorithm $A^{*}$

## Evaluating search strategies

- Completeness
- Guarantees finding a solution whenever one exists
- Time complexity (worst or average case)
- Space complexity
- Optimality/Admissibility


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- Space complexity
- Usually measured by maximum size of graph/tree during the search
- Optimality/Admissibility
- If a solution is found, is it guaranteed to be an optimal one, i.e., one with minimum cost

NEXT CLASS UNINFORMED SEARCH

## EXTRA HELPER SLIDES

## PRIMER ON GRAPHS

## Remember: Graphs

- A graph $G=(E, V)$
- $V=$ set of vertices (nodes)
- $\mathrm{E}=$ set of edges between pairs of nodes, $(x, y)$

G can be:

- Undirected: order of $(x, y)$ doesn't matter
- These are symmetric
- Directed: order of $(x, y)$ does matter
- Weighted: cost function $g(x, y)$
- (among other qualities)


## Remember: Graphs

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$$
\begin{aligned}
& V=\{? ? ?\} \\
& E=\{? ? ?\}
\end{aligned}
$$

## Remember: Graphs

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$$
\begin{gathered}
V=\{a, b, c\} \\
E=\{(a, c),(b, c)\}
\end{gathered}
$$

undirected

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$$
\begin{gathered}
V=\{a, b, c\} \\
E=\{(a, c),(b, c),(c, b)\} \\
g=\{(a, c): 4,(b, c): 5,(c, b): 1\}
\end{gathered}
$$

weighted, directed

## MORE EXAMPLES

## Water Jug Problem

- Two jugs J1 \& J2 with capacity C1 \& C2
- Initially J1 has W1 water and J2 has W2 water
- e.g.: full 5 gallon jug and empty 2 gallon jug
- Possible actions:
- Pour from jug $X$ to jug $Y$ until $X$ empty or $Y$ full
- Empty jug X onto the floor
- Goal: J1 has G1 water and J2 G2
- G1 or G2 can be -1 to represent any amount
- E.g.: initially full jugs with capacities 3 and 1 liters, goal is to have 1 liter in each


## Example: Water Jug Problem



- Two jugs J1 and J2 with capacity C1 and C2
- Initially J1 has W1 water and J2 has W2 water
- e.g.: a full 5-gallon jug and an empty 2-gallon jug
- Possible actions:
- Pour from jug $X$ to jug $Y$ until $X$ empty or $Y$ full
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- Goal: J1 has G1 water and J2 G2
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## Example: Water Jug Problem

Given full 5-gal. jug and empty 2-gal. jug, fill 2gal jug with one gallon

- State representation?
-General state?
-Initial state?
-Goal state?
- Possible actions?
-Condition?
-Resulting state?

Action table

| Name | Cond. | Transition | Effect |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

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Action table

| Name | Cond. | Transition | Effect |
| :--- | :---: | :--- | :--- |
| dump1 | $\mathrm{x}>0$ | $(\mathrm{x}, \mathrm{y}) \rightarrow(0, \mathrm{y})$ | Empty Jug 1 |
| dump2 | $\mathrm{y}>0$ |  | Empty Jug 2 |
| pour_1_2 |  <br> $\mathrm{y}<\mathrm{C} 2$ |  | Pour from Jug <br> 1 to Jug 2 |
| pour_2_1 |  <br> $\mathrm{X}<\mathrm{C} 1$ |  | Pour from Jug <br> 2 to Jug 1 |
|  |  |  |  |

## So...

- How can we represent the states?
- What's an initial state
- How do we recognize a goal state
- What are the actions; how can we tell which ones can be performed in a given state; what is the resulting state
- How do we search for a solution from an initial state given a goal state
- What is a solution? The goal state achieved or a path to it?


## Search in a state space

- Basic idea:
-Create representation of initial state
-Try all possible actions \& connect states that result
-Recursively apply process to the new states until we find a solution or dead ends
- We need to keep track of the connections between states and might use a
-Tree data structure or
-Graph data structure
- A graph structure is best in general...


## Formalizing state space search

- A state space is a graph ( $V, E$ ) where $V$ is a set of nodes and $E$ is a set of arcs, and each arc is directed from a node to another node
- Nodes: data structures with state description and other info, e.g., node's parent, name of action that generated it from parent, etc.
- Arcs: instances of actions, head is a state, tail is the state that results from action


## Formalizing search in a state space

- Each arc has fixed, positive cost associated with it corresponding to the action cost
- Simple case: all costs are 1
- Each node has a set of successor nodes corresponding to all legal actions that can be applied at node's state
- Expanding a node = generating its successor nodes and adding them and their associated arcs to the graph
- One or more nodes are marked as start nodes
- A goal test predicate is applied to a state to determine if its associated node is a goal node

What does "search" look like for a particular problem?

Start $\quad$| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 8 |  |
| 7 | 6 | 5 |

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |  |
| $\mathbf{7}$ | $\mathbf{8}$ |  |  |

Expanding a node on the fringe (taking a certain action)

| 1 | 2 |  |
| :--- | :--- | :--- |
| 4 | 8 | 3 |
| 7 | 6 | 5 |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 8 | 5 |
| 7 | 6 |  |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 5 | 6 |
| 7 | 8 |  |



| 1 | 2 |  |
| :--- | :--- | :--- |
| 4 | 8 | 3 |
| 7 | 6 | 5 |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 8 | 5 |
| 7 | 6 |  |
|  |  |  |

Expanding a node on the fringe (taking a certain action). Not all actions shown.

| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 8 | 5 |
| 7 |  | 6 |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 5 | 6 |
| 7 | 8 |  |



| 1 | 2 |  |
| :--- | :--- | :--- |
| 4 | 8 | 3 |
| 7 | 6 | 5 |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 8 | 5 |
| 7 | 6 |  |
|  |  |  |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 8 | 5 |
| 7 |  | 6 |

Expanding a node on the fringe (taking a certain action). Not all actions shown.

| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 8 | 5 |
|  | 7 | 6 |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 |  | 5 |
| 7 | 8 | 6 |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 5 | 6 |
| 7 | 8 |  |



| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 8 | 5 |
|  | 7 | 6 |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 |  | 5 |
| 7 | 8 | 6 |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 5 |  |
| 7 | 8 | 6 |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
|  | 4 | 5 |
| 7 | 8 | 6 |


| 1 |  | 3 |
| :--- | :--- | :--- |
| 4 | 2 | 5 |
| 7 | 8 | 6 |


| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 5 | 6 |
| 7 | 8 |  |

start


