CMSC 471: Artificial Intelligence
Fall 2023

Propositional and First-Order Logic

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Many slides courtesy Tim Finin from UMBC and Percy Liang from Stanford University

First-order logic

- First-order logic (FOL) models the world in terms of
  - **Objects,** which are things with individual identities
  - **Properties** of objects that distinguish them from others
  - **Relations** that hold among sets of objects
  - **Functions,** a subset of relations where there is only one "value" for any given "input"
- Examples:
  - Objects: students, lectures, companies, cars ...
  - Relations: brother-of, bigger-than, outside, part-of, has-color, occurs-after, owns, visits, precedes, ...
  - Properties: blue, oval, even, large, ...
  - Functions: father-of, best-friend, more-than ...

Quantifiers: ∀ and ∃

- **Universal quantification**
  - $(\forall x)P(X)$ means P holds for **all** values of X in the domain associated with variable[1]
  - E.g., $(\forall X)$ dolphin(X) $\rightarrow$ mammal(X)
- **Existential quantification**
  - $(\exists x)P(X)$ means P holds for **some** value of X in domain associated with variable
  - E.g., $(\exists$**X**$)$ mammal(X) $\wedge$ lays_eggs(X)
  - This lets us make statements about an object without identifying it

[1] a variable's domain is often not explicitly stated and is assumed by the context

Universal Quantifier: ∀

- Universal quantifiers typically used with *implies* to form *rules*:

  Logic: $(\forall X)\ student(X) \rightarrow smart(X)$

  Means: All students are smart

- Universal quantification *rarely* used without implies:

  Logic: $(\forall X)\ student(X) \wedge smart(X)$

  Means: Everything is a student and is smart

Existential Quantifier: $\exists$

- Existential quantifiers usually used with **and** to specify a list of properties about an individual

  Logic: $(\exists X)\ student(X) \wedge smart(X)$

  Meaning: There is a student who is smart

- Common mistake: represent this in FOL as:

  Logic: $(\exists X)\ student(X) \rightarrow smart(X)$

  Meaning: ?

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  Logic: $(\exists X)\ student(X) \rightarrow smart(X)$

  $P \rightarrow Q = \sim P \vee Q$

  $\exists X\ student(X) \rightarrow smart(X) = \exists X \sim student(X) \vee smart(X)$

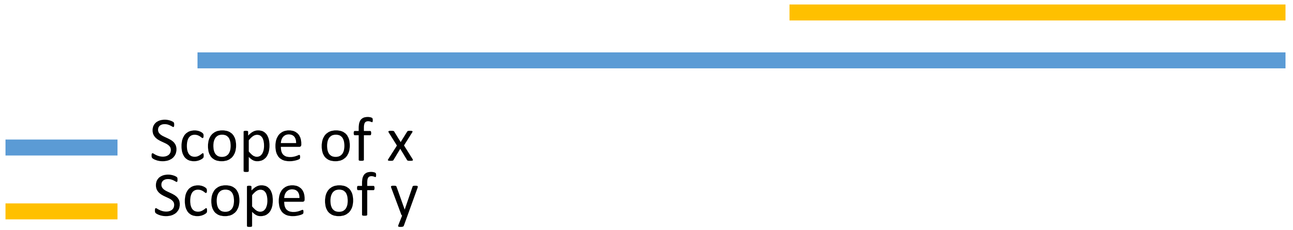
  Meaning: There's something that is either not a student or is smart

Quantifier Scope

- FOL sentences have structure, like programs
- In particular, variables in a sentence have a **scope**
- Suppose we want to say "everyone who is alive loves someone"

  $(\forall X)\ alive(X) \rightarrow (\exists\ Y)\ loves(X, Y)$
- Here's how we scope the variables

$$(\forall X)\ alive(X) \rightarrow (\exists Y)\ loves(X, Y)$$



Quantifier Scope

- **Switching order of universal quantifiers *does not* change the meaning**
  - $(\forall X)(\forall Y)P(X,Y) \leftrightarrow (\forall Y)(\forall X)\ P(X,Y)$
  - Dogs hate cats (i.e., all dogs hate all cats)
- **You can switch order of existential quantifiers**
  - $(\exists X)(\exists Y)P(X,Y) \leftrightarrow (\exists Y)(\exists X)\ P(X,Y)$
  - A cat killed a dog
- **Switching order of universal and existential quantifiers *does* change meaning:**
  - Everyone likes someone: $(\forall X)(\exists Y)\ likes(X,Y)$
  - Someone is liked by everyone: $(\exists Y)(\forall X)\ likes(X,Y)$

Procedural example 1 (Illustrative only!)

```
def verify1():
    # Everyone likes someone: (∀x)(∃y) likes(x,y)
    for p1 in people():
        foundLike = False
        for p2 in people():
            if likes(p1, p2):
                foundLike = True
                break
        if not foundLike:
            print(p1, 'does not like anyone ☹')
            return False
    return True
```

Every person has at least one individual that they like.

Procedural example 2 (Illustrative only!)

```
def verify2():
  # Someone is liked by everyone: (∃y)( ∀x) likes(x,y)
  for p2 in people():
    foundHater = False
    for p1 in people():
      if not likes(p1, p2):
        foundHater = True
        break
    if not foundHater
      print(p2, ‘is liked by everyone ☺’)
      return True
  return False
```

There is a person who is liked by every person in the universe.

Connections between ∀ and ∃

- We can relate sentences involving ∀ and ∃ using extensions to **De Morgan's laws**:
  1. $(\forall x)\ P(x) \leftrightarrow \neg(\exists x)\ \neg\ P(x)$
  2. $\neg(\forall x)\ P(x) \leftrightarrow (\exists x)\ \neg P(x)$
  3. $(\exists\ x)\ P(x) \leftrightarrow \neg\ (\forall\ x)\ \neg P(x)$
  4. $\neg(\exists x)\ P(x) \leftrightarrow (\forall x)\ \neg P(x)$
- Examples
  1. All dogs don't like cats ↔ No dog likes cats
  2. Not all dogs bark ↔ There is a dog that doesn't bark
  3. All dogs sleep ↔ There is no dog that doesn't sleep
  4. There is a dog that talks ↔ Not all dogs can't talk

Notational differences

- **Different symbols** for *and, or, not, implies, ...*
  - $\forall$ $\exists$ $\Rightarrow$ $\Leftrightarrow$ $\wedge$ $\vee$ $\neg$ $\bullet$ $\supset$
  - p v (q ^ r)
  - p + (q * r)
- **Prolog**

  cat(X) :- furry(X), meows (X), has(X, claws)
- **Lisp notations**

```
(forall ?x (implies (and (furry ?x)
                         (meows ?x)
                         (has ?x claws))
                    (cat ?x)))
```

Translating English to FOL

Every gardener likes the sun

All purple mushrooms are poisonous

Translating English to FOL

Every gardener likes the sun

$\forall$x gardener(x) $\rightarrow$ likes(x,Sun)

All purple mushrooms are poisonous

Translating English to FOL

Every gardener likes the sun

$\forall x\ gardener(x) \rightarrow likes(x,Sun)$

All purple mushrooms are poisonous

$\forall x\ (mushroom(x) \land purple(x)) \rightarrow poisonous(x)$

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No purple mushroom is poisonous (two ways)

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$\neg\exists$x purple(x) $\wedge$ mushroom(x) $\wedge$ poisonous(x)

$\forall$x (mushroom(x) $\wedge$ purple(x)) $\rightarrow$ $\neg$poisonous(x)

English to FOL: Counting

Use = predicate to identify different individuals

- **There are <u>at least</u> two purple mushrooms**

  ∃x ∃y mushroom(x) ∧ purple(x) ∧ mushroom(y) ∧ purple(y) ∧ ¬**(x=y)**

- **There are <u>exactly</u> two purple mushrooms**

  ∃x ∃y mushroom(x) ∧ purple(x) ∧ mushroom(y) ∧ purple(y) ∧ ¬**(x=y)** ∧
  ∀z (mushroom(z) ∧ purple(z)) → **((x=z)** ∨ **(y=z))**

Saying there are 802 different Pokemon will be hard!

Translating English to FOL

What do these mean?

- **You can fool some of the people all of the time**
- **You can fool all of the people some of the time**

Translating English to FOL

What do these mean?

Both English statements are ambiguous

- **You can fool some of the people all of the time**
  - There is a nonempty subset of people so easily fooled that you can fool that subset every time*
  - For any given time, there is a non-empty subset at that time that you can fool
- **You can fool all of the people some of the time**
  - There are one or more times when it's possible to fool everyone*
  - Everybody can be fooled at some point in time

* Most common interpretation, I think

Some terms we will need

- **person(x):** True iff x is a person
- **time(t):** True iff t is a point in time
- **canFool(x, t):** True iff x can be fooled at time t

Note: *iff* = *if and only if* = $\leftrightarrow$

Translating English to FOL

You can fool some of the people all of the time

There is a nonempty group of people so easily fooled that you can fool that group every time*

≡ There's (at least) one person you can fool every time

$\exists x \; \forall t \;\; person(x) \wedge time(t) \rightarrow canFool(x, t)$

For any given time, there is a non-empty group at that time that you can fool

≡ For every time, there's a person at that time that you can fool

$\forall t \; \exists x \;\; person(x) \wedge time(t) \rightarrow canFool(x, t)$

* Most common interpretation, I think

Translating English to FOL

You can fool all of the people some of the time

There's at least one time when you can fool everyone*

$\exists t\ \forall x\ time(t) \land person(x) \rightarrow canFool(x, t)$

Everybody can be fooled at some point in time

$\forall x\ \exists t\ person(x) \land time(t) \rightarrow canFool(x, t)$

* Most common interpretation, I think

Limits of classical logic

- Note that **there's no easy, natural way** to talk about a few, many, most, almost all …
- This is natural in human languages
  - There are **many** people you can fool **most** of the time
  - There are a **few** people you can fool **almost every** time
- We also can't have exceptions naturally as in human languages
  - All birds can fly, **except for** penguins, ostriches and a few other species
  - This can be represented in FOL, but it may be challenging – lot of new relations, paraphrasing, and conditions needed.
  - "For all entities *x*, if *x* is a bird and *x* is not a penguin, not an ostrich, and not one of the specified other species, then *x* can fly."
- There are non-classical logic systems that can handle these problems

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- There are non-classical logic systems that can handle these problems

$$\forall x\,(B(x) \wedge \neg(\mathrm{Penguin}(x) \vee \mathrm{Ostrich}(x) \vee \mathrm{OtherSpecies}(x)) \rightarrow F(x))$$

Representation Design

- Many options for representing even a simple fact, e.g., something's color as red, green or blue, e.g.:
  - green(kermit)
  - color(kermit, green)
  - hasProperty(kermit, color, green)
- **Choice can influence how easy it is to use**
- Last option of representing properties & relations as triples used by modern knowledge graphs
  - Easy to ask: What color is Kermit? What are Kermit's properties?, What green things are there? Tell me everything you know, …

Simple genealogy KB in FOL

Design a knowledge base using FOL that

- Has facts of immediate family relations, e.g., spouses, parents, etc.
- Defines more complex relations (ancestors, relatives)
- Detect conflicts, e.g., you are your own parent
- Infers relations, e.g., grandparent from parent
- Answers queries about relationships between people

How do we approach this?

- Design an initial ontology of types, e.g.
  - e.g., person, man, woman, male, female
- Extend ontology by defining simple two argument relations, e.g.
  - spouse, has_child, has_parent
- Add general constraints to relations, e.g.
  - spouse(X,Y) => ~ X = Y
  - spouse(X,Y) => person(X), person(Y)
- Add FOL sentences for inference, e.g.
  - spouse(X,Y) ⇔ spouse(Y,X)
  - man(X) ⇔ person(X) ∧ male(X)
- Add instance data
  - e.g., spouse(djt, mt)

Example: A simple genealogy KB by FOL

Predicates:

- parent(x, y), child(x, y), father(x, y), daughter(x, y), etc.
- spouse(x, y), husband(x, y), wife(x,y)
- ancestor(x, y), descendant(x, y)
- male(x), female(y)
- relative(x, y)

Facts:

- husband(Joe, Mary), son(Fred, Joe)
- spouse(John, Nancy), male(John), son(Mark, Nancy)
- father(Jack, Nancy), daughter(Linda, Jack)
- daughter(Liz, Linda)
- etc.

Example Axioms

$(\forall x,y)$ parent(x, y) $\leftrightarrow$ child (y, x)

$(\forall x,y)$ father(x, y) $\leftrightarrow$ parent(x, y) $\wedge$ male(x) *;similar for mother(x, y)*

$(\forall x,y)$ daughter(x, y) $\leftrightarrow$ child(x, y) $\wedge$ female(x) *;similar for son(x, y)*

$(\forall x,y)$ husband(x, y) $\leftrightarrow$ spouse(x, y) $\wedge$ male(x) *;similar for wife(x, y)*

$(\forall x,y)$ spouse(x, y) $\leftrightarrow$ spouse(y, x) *;spouse relation is symmetric*

$(\forall x,y)$ parent(x, y) $\rightarrow$ ancestor(x, y)

$(\forall x,y)(\exists z)$ parent(x, z) $\wedge$ ancestor(z, y) $\rightarrow$ ancestor(x, y)

$(\forall x,y)$ descendant(x, y) $\leftrightarrow$ ancestor(y, x)

$(\forall x,y)(\exists z)$ ancestor(z, x) $\wedge$ ancestor(z, y) $\rightarrow$ relative(x, y)

$(\forall x,y)$ spouse(x, y) $\rightarrow$ relative(x, y) ;related by marriage

$(\forall x,y)(\exists z)$ relative(z, x) $\wedge$ relative(z, y) $\rightarrow$ relative(x, y) *;transitive*

$(\forall x,y)$ relative(x, y) $\leftrightarrow$ relative(y, x) *;symmetric*

Axioms, definitions and theorems

- **Axioms**: facts and rules that capture (important) facts & concepts in a domain; axioms are used to prove **theorems**
  - Mathematicians dislike unnecessary (dependent) axioms, i.e. ones that can be derived from others
  - Dependent axioms can make reasoning faster, however
  - Choosing a good set of axioms is a design problem
- A **definition** of a predicate is of the form "$p(X) \leftrightarrow \ldots$" and can be decomposed into two parts
  - **Necessary** description: "$p(x) \rightarrow \ldots$"
  - **Sufficient** description "$p(x) \leftarrow \ldots$"
  - Some concepts have definitions (e.g., triangle) and some don't (e.g., person)

More on definitions

Example: define father(x, y) by parent(x, y) and male(x)

- **parent(x, y)** is a necessary (but not sufficient) description of father(x, y)

  father(x, y) $\rightarrow$ parent(x, y)

- **parent(x, y) ^ male(x) ^ age(x, 35)** is a sufficient (but not necessary) description of father(x, y):

  father(x, y) $\leftarrow$ parent(x, y) ^ male(x) ^ age(x, 35)

- **parent(x, y) ^ male(x)** is a necessary and sufficient description of father(x, y)

  parent(x, y) ^ male(x) $\leftrightarrow$ father(x, y)

Higher-order logic

- FOL only lets us quantify over variables, and **variables can only range over objects**
- HOL allows us to quantify over relations, e.g.
  "two functions are equal iff they produce the same value for all arguments"
  $\forall f\ \forall g\ (f = g) \leftrightarrow (\forall x\ f(x) = g(x))$
- E.g.: (quantify over predicates)
  $\forall r\ \text{transitive}(\ r\ ) \rightarrow (\forall xyz)\ r(x,y) \wedge r(y,z) \rightarrow r(x,z))$
- More expressive, but reasoning is undecide-able, in general

Expressing uniqueness

[syntactic sugar]

- Often want to say that there is a single, unique object that satisfies a condition
- There exists a unique x such that king(x) is true
  - $\exists x\ king(x) \land \forall y\ (king(y) \rightarrow x{=}y)$
  - $\exists x\ king(x) \land \neg\exists y\ (king(y) \land x \neq y)$
  - $\exists!\ x\ king(x)$
- Every country has exactly one ruler
  - $\forall c\ country(c) \rightarrow \exists!\ r\ ruler(c,r)$
- Iota operator: ι x P(x) means "the unique x such that p(x) is true"
  - The unique ruler of Freedonia is dead
  - dead(ι x ruler(freedonia,x))

Examples of FOL in use

- Semantics of W3C's Semantic Web stack (RDF, RDFS, OWL) is defined in FOL
- OWL Full is equivalent to FOL
- Other OWL profiles support a subset of FOL and are more efficient
- The semantics of schema.org is only defined in natural language text
- Wikidata's knowledge graph (and Google's) has a richer schema

FOL Summary

- First order logic (FOL) introduces predicates, functions and quantifiers
- More expressive, but reasoning more complex
  - Reasoning in propositional logic is NP hard, FOL is semi-decidable
- Common AI knowledge representation language
  - Other KR languages (e.g., OWL) are often defined by mapping them to FOL
- FOL variables range over objects
  - HOL variables range over functions, predicates or sentences

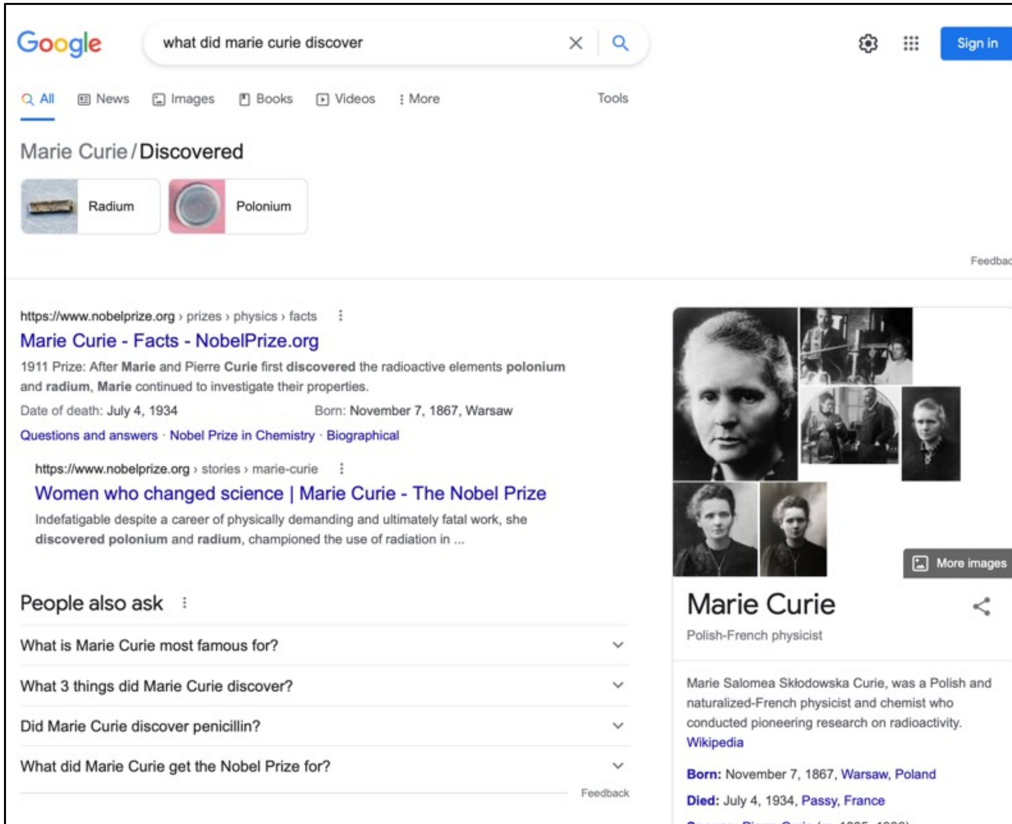
Examples of FOL in use

Many practical approaches embrace the approach that "some data is better than none"

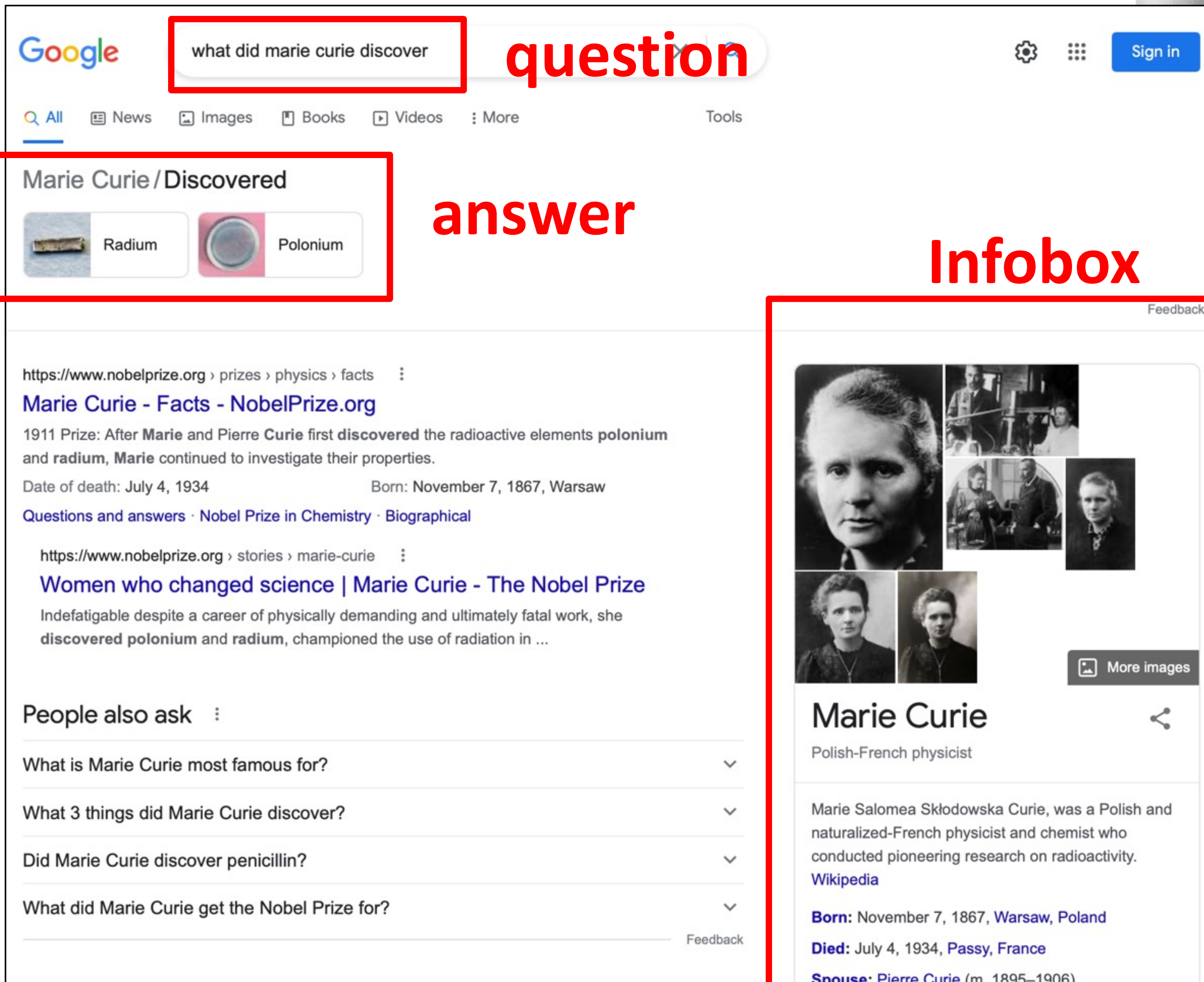
- The semantics of schema.org is only defined in natural language text
- Wikidata's knowledge graph has a rich schema
  - Many constraint/logical violations are flagged with warnings
  - However, not all, see this Wikidata query that finds people who are their own grandfather

Virtual assistants and Infoboxes

- Web search engines and virtual assistants like Alexa use custom **knowledge graphs** to
  - help understand queries and content of web pages & documents
  - Answer questions
  - Show infoboxes
- Wikidata shares roots with these
- All draw on the similar knowledge, like the ~300 Wikipedia & Wikimedia sites



Virtual assistants & search engines



CNF (Conjunctive Normal Form)

All of the following formulas in the variables $A, B, C, D, E,$ and F are in conjunctive normal form:

- $(A \vee \neg B \vee \neg C) \wedge (\neg D \vee E \vee F)$
- $(A \vee B) \wedge (C)$
- $(A \vee B)$
- (A)

Each sentence is a disjunction of one or more literals (positive or negative atoms)

The following formulas are **not** in conjunctive normal form:

- $\neg(B \vee C)$, since an OR is nested within a NOT
- $(A \wedge B) \vee C$
- $A \wedge (B \vee (D \wedge E))$, since an AND is nested within an OR

Every formula can be equivalently written as a formula in conjunctive normal form. The three non-examples in CNF are:

- $(\neg B) \wedge (\neg C)$
- $(A \vee C) \wedge (B \vee C)$
- $(A) \wedge (B \vee D) \wedge (B \vee E)$.

Logical Inference: Overview

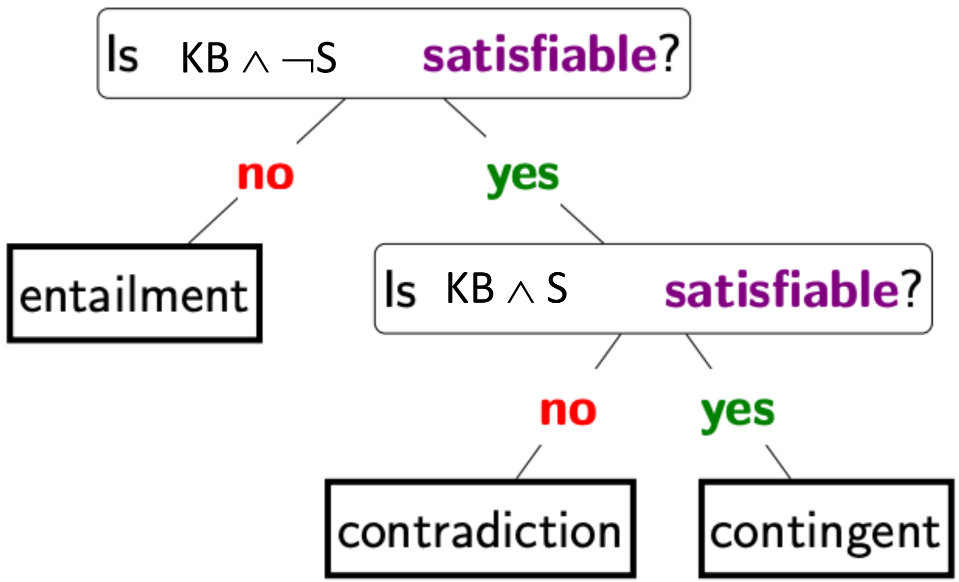
- Model checking for propositional logic
- Rule based reasoning in first-order logic
  - Inference rules and generalized modes ponens
  - Forward chaining
  - Backward chaining
- Resolution-based reasoning in first-order logic
  - Clausal form
  - Unification
  - Resolution as search

Satisfiability

Definition: satisfiability

A knowledge base KB is **satisfiable** if $\mathcal{M}(\text{KB}) \neq \emptyset$.

Reduce Ask[S] and Tell[S] to satisfiability:



"A knowledge base KB is satisfiable if there exists at least one model for KB."

- We can say that a sentence S by itself is satisfiable if there is some model that satisfies S.
- Finally, a knowledge base (which is no more than just the conjunction of its formulas) is satisfiable if there is some model that satisfies all the formulas $S \in KB$.

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The KB

$P \land Q$

$R \land \neg P$

Models for the KB

P	Q	R

The KB has no models. There is no assignment of True or False to every variable that makes every sentence in the KB true

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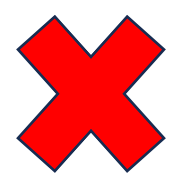
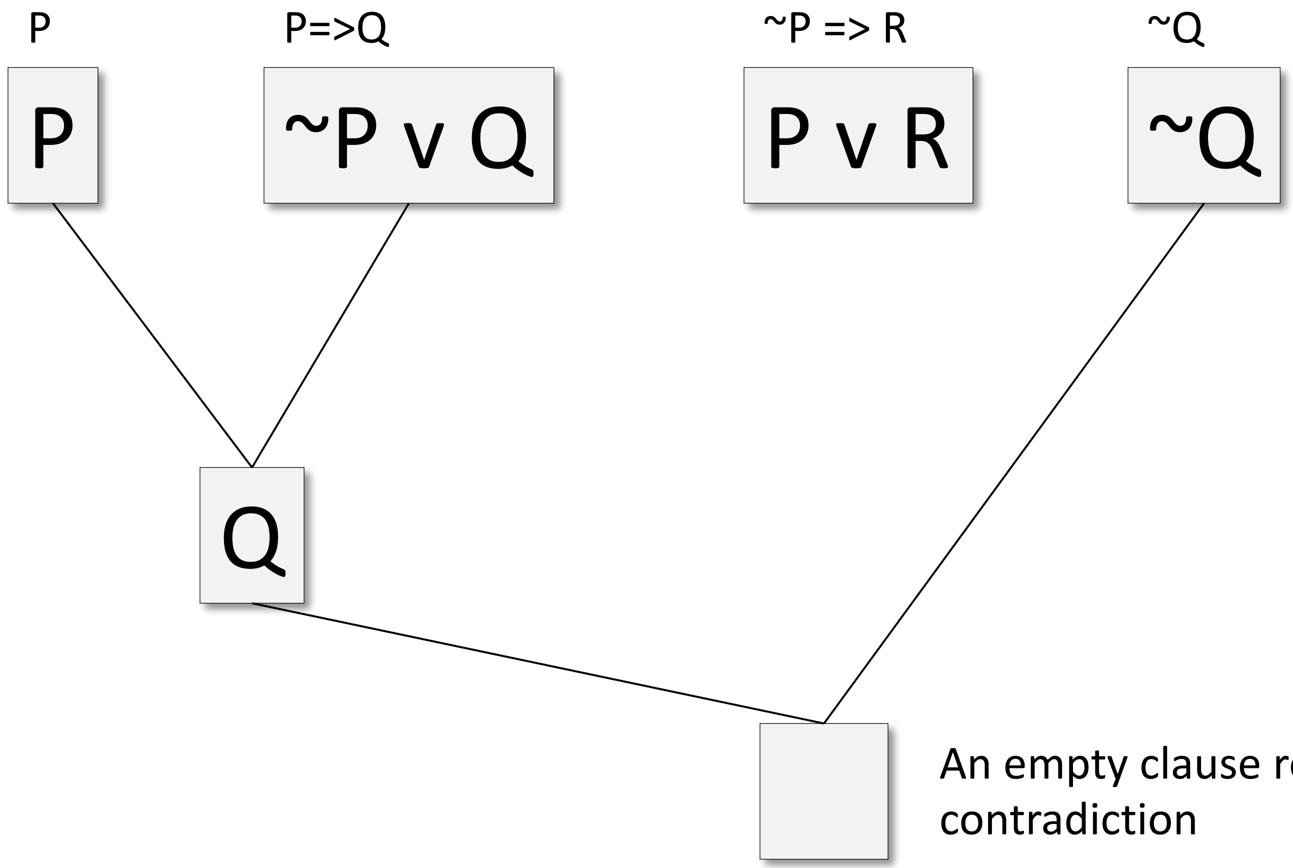


From Satisfiability to Proof

- To see if a satisfiable KB entails sentence S, see if $\underline{KB \wedge \neg S}$ is satisfiable
  - If it is not, then the KB entails S
  - If it is, then the KB does not entail S
  - This is a refutation proof
- Consider the KB with (P, P=>Q, ~P=>R)
  - Does the KB it entail Q? R?

Does the KB entail Q?

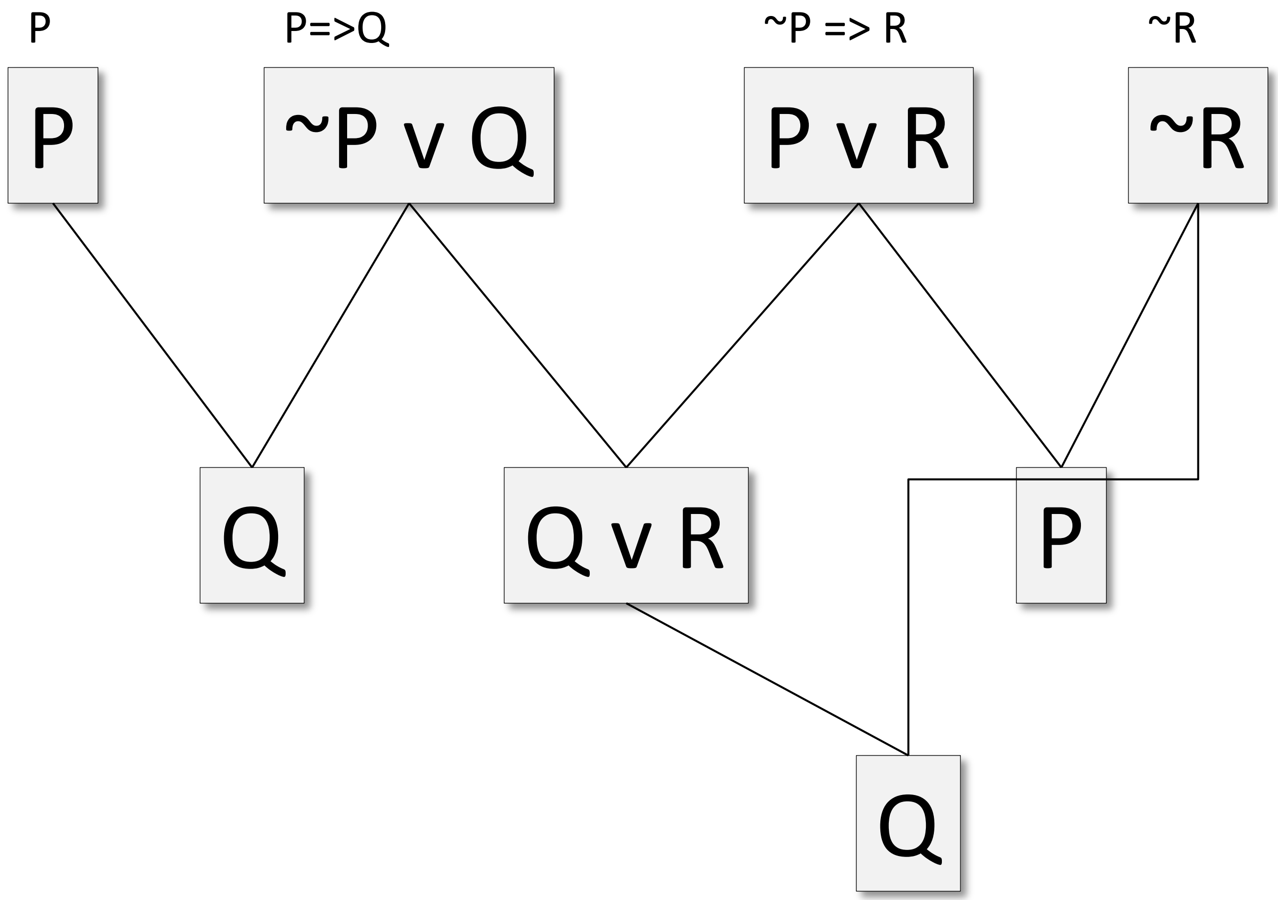
KB
P
P=>Q
~P=>R



An empty clause represents a contradiction

We assume that every sentence in the KB is true. Adding ~Q to the KB yields a contradiction, so ~Q must be false, so **Q must be true**.

Does the KB entail R?



Adding ~R to KB does not produce a contradiction after drawing all possible conclusions, so it could be False, so **KB doesn't entail R**. (but we also can't say KB entails not R).

Model checking

Checking satisfiability (SAT) in propositional logic is special case of solving CSPs!

Mapping:

propositional symbol	$\Rightarrow$	variable
formula	$\Rightarrow$	constraint
model	$\Leftarrow$	assignment

Model checking

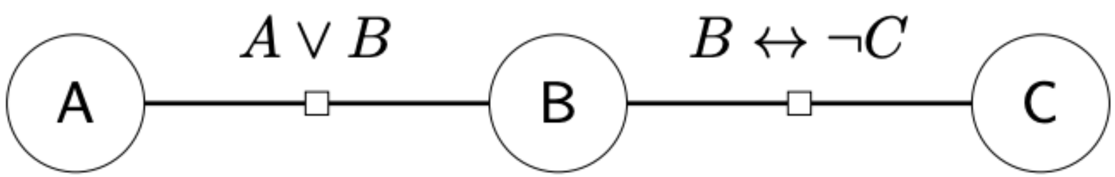
Example: model checking

KB $= \{A \vee B, B \leftrightarrow \neg C\}$

Propositional symbols (CSP variables):

$$\{A, B, C\}$$

CSP:



Consistent assignment (satisfying model):

$$\{A : 1, B : 0, C : 1\}$$

Model checking

Definition: model checking

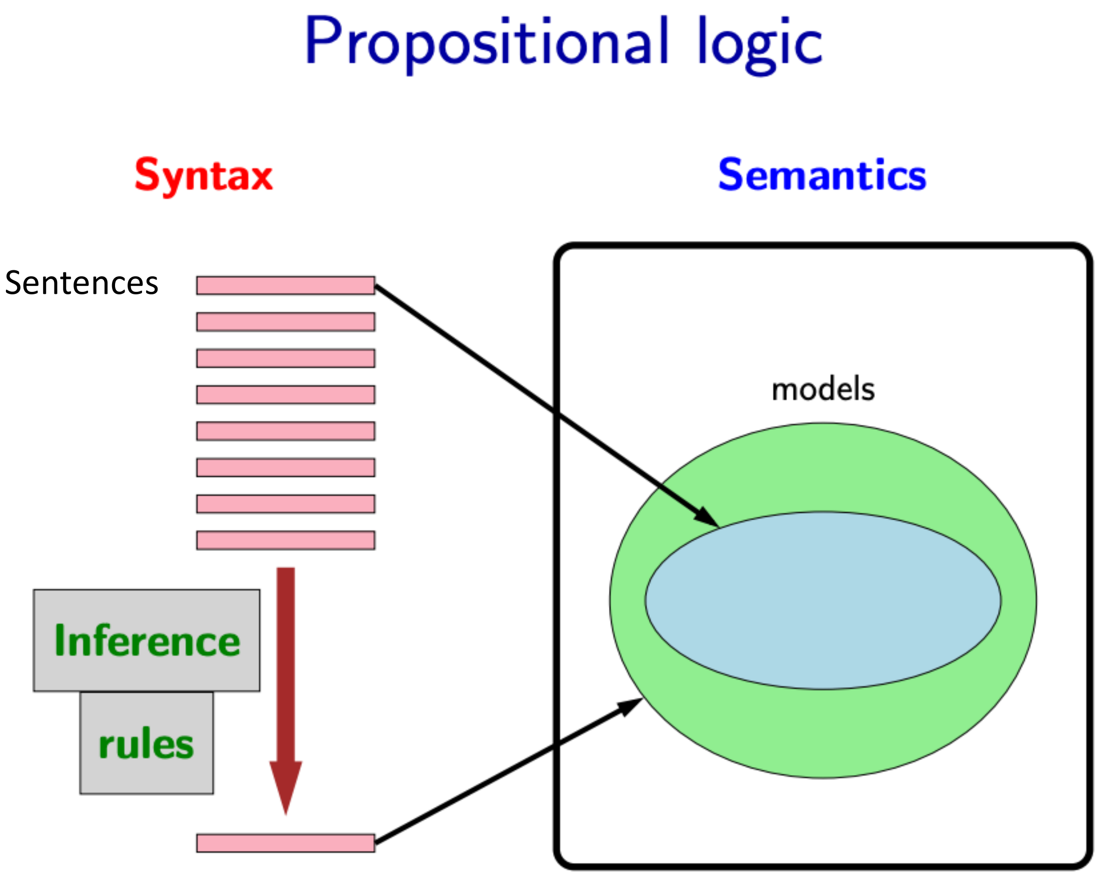
Input: knowledge base KB

Output: exists satisfying model ($\mathcal{M}(\text{KB}) \neq \emptyset$)?

Popular algorithms:

- DPLL (backtracking search + pruning)
- WalkSat (randomized local search)

Difference with Inference



- Used sentences to define sets of models.
- Reasoning on sentences has been through these models (e.g., reduction to satisfiability).
- Inference rules allow us to do reasoning on the sentences themselves without ever instantiating the models.

Model Checking using the AIMA Code

```
python> python
Python ...
>>> from logic import *
>>> expr('P & P==>Q & ~P==>R')
((P & (P >> Q)) & (~P >> R))

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R'))
{R: True, P: True, Q: True}

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~R'))
{R: False, P: True, Q: True}

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~Q'))
False

>>>
```

Model Checking using the AIMA Code

expr parses a string, and returns a logical expression

```
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>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~R'))
{R: False, P: True, Q: True}

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~Q'))
False

>>>
```

Model Checking using the AIMA Code

expr parses a string, and returns a logical expression

dpll_satisfiable returns a model if satisfiable else False

```
python> python
Python ...
>>> from logic import *
>>> expr('P & P==>Q & ~P==>R')
((P & (P >> Q)) & (~P >> R))

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R'))
{R: True, P: True, Q: True}

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~R'))
{R: False, P: True, Q: True}

>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~Q'))
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Model Checking using the AIMA Code

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False

>>>
```

expr parses a string, and returns a logical expression

dpll_satisfiable returns a model if satisfiable else False

The KB entails Q but does not entail R

Checking Validity

- Use the functions in aima's logic.py to see which of the following are valid, i.e., true in every model.
- convert these sentences to the appropriate string form that the python code uses
- use the `expr()` function in logic.py to turn each into an Expr object
- use the `tt_true()` function to check for validity.
- `tt_true()` checks an expression object to see if it is valid, i.e., true in all possible models.
- A valid sentence is true for all possible assignments of true and false to its variables, i.e., $P \vee \neg P$

AIMA KB Class

```
>>> kb1 = PropKB()
>>> kb1.clauses
[]
>>> kb1.tell(expr('P==>Q & ~P==>R'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
>>> kb1.tell(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P), P]
>>> kb1.ask(expr('Q'))
{}
>>> kb1.retract(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
```

AIMA KB Class

```
>>> kb1 = PropKB()
>>> kb1.clauses
[]
>>> kb1.tell(expr('P==>Q & ~P==>R'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
>>> kb1.tell(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P), P]
>>> kb1.ask(expr('Q'))
{}
>>> kb1.retract(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
```

PropKB is a subclass

AIMA KB Class

```
>>> kb1 = PropKB()
>>> kb1.clauses
[]
>>> kb1.tell(expr('P==>Q & ~P==>R'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
>>> kb1.tell(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P), P]
>>> kb1.ask(expr('Q'))
{}
>>> kb1.retract(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
```

PropKB is a subclass

A sentence is converted to CNF and the clauses added

AIMA KB Class

```
>>> kb1 = PropKB()
>>> kb1.clauses
[]
>>> kb1.tell(expr('P==>Q & ~P==>R'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
>>> kb1.tell(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P), P]
>>> kb1.ask(expr('Q'))
{}
>>> kb1.retract(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
```

PropKB is a subclass

A sentence is converted to CNF and the clauses added

The KB does not entail Q

AIMA KB Class

```
>>> kb1 = PropKB()
>>> kb1.clauses
[]
>>> kb1.tell(expr('P==>Q & ~P==>R'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
>>> kb1.tell(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P), P]
>>> kb1.ask(expr('Q'))
{}
>>> kb1.retract(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
```

PropKB is a subclass

A sentence is converted to CNF and the clauses added

The KB does not entail Q

After adding P the KB does entail Q

AIMA KB Class

```
>>> kb1 = PropKB()
>>> kb1.clauses
[]
>>> kb1.tell(expr('P==>Q & ~P==>R'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
>>> kb1.tell(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P), P]
>>> kb1.ask(expr('Q'))
{}
>>> kb1.retract(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
```

PropKB is a subclass

A sentence is converted to CNF and the clauses added

The KB does not entail Q

After adding P the KB does entail Q

Retracting P removes it and the KB no longer entails Q

Logic Summary

- **Propositional logic**
  - Problems with propositional logic
- **First-order logic**
  - Properties, relations, functions, quantifiers, …
  - Terms, sentences, wffs, axioms, theories, proofs, …
  - Variations and extensions to first-order logic
- **Logical agents**
  - Reflex agents
  - Representing change: situation calculus, frame problem
  - Preferences on actions
  - Goal-based agents