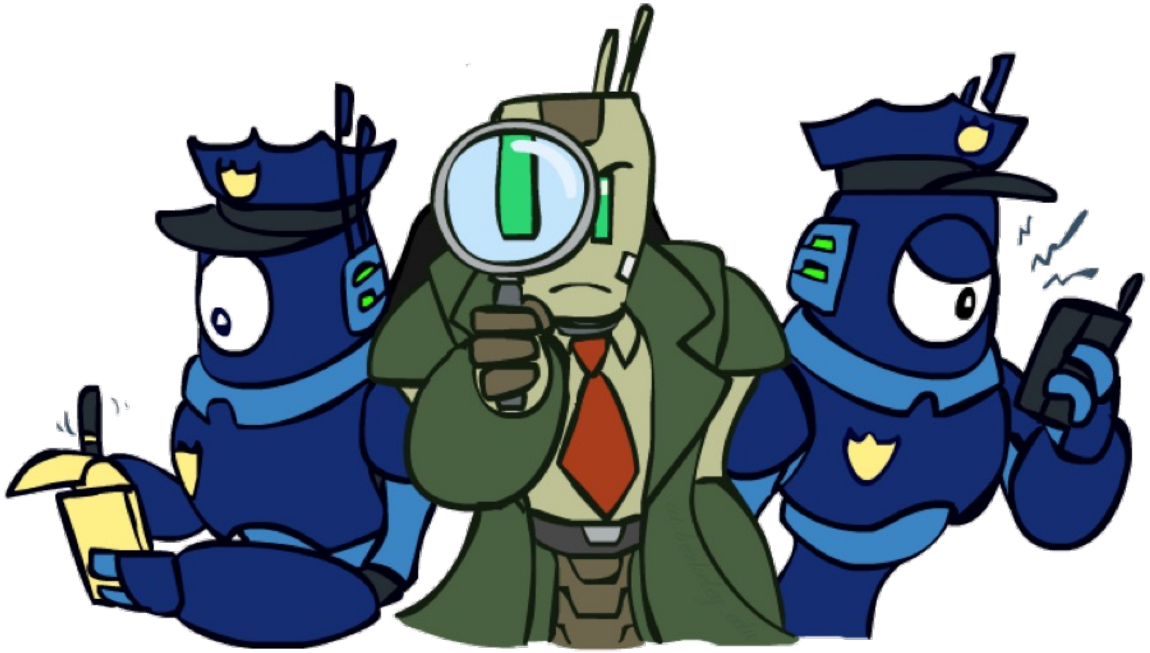
CMSC 471

Constraint Satisfaction Problems III



Instructor: KMA Solaiman

These slides were modified from Dan Klein and Pieter Abbeel at UC Berkeley [ai.berkeley.edu] and Frank Ferraro [ferraro@umbc.edu].

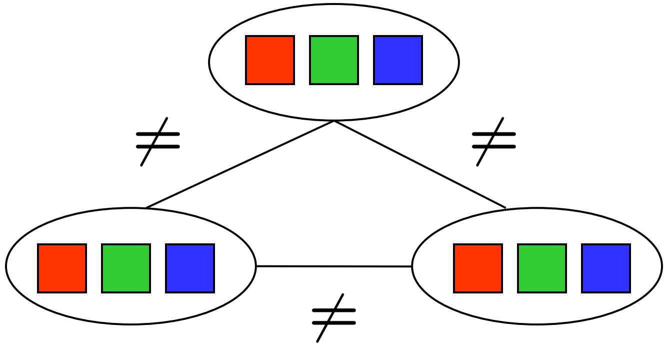
Today

- Efficient Solution of CSPs

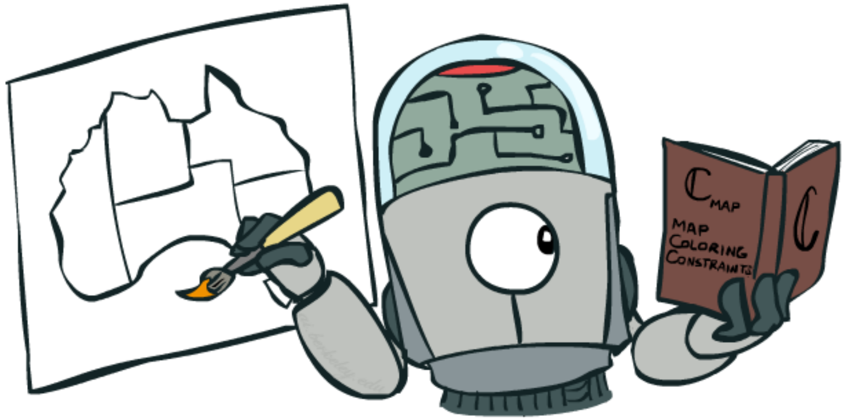
- Local Search

Reminder: CSPs

- CSPs:
  - Variables
  - Domains
  - Constraints
    - Implicit (provide code to compute)
    - Explicit (provide a list of the legal tuples)
    - Unary / Binary / N-ary



- Goals:
  - Here: find any solution
  - Also: find all, find best, etc.



Backtracking Example

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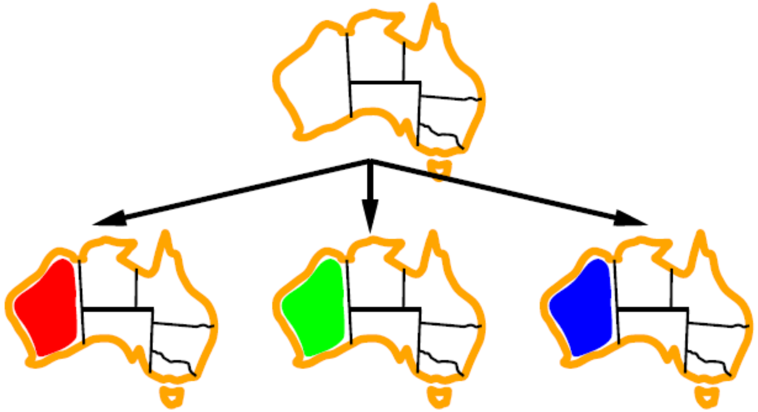

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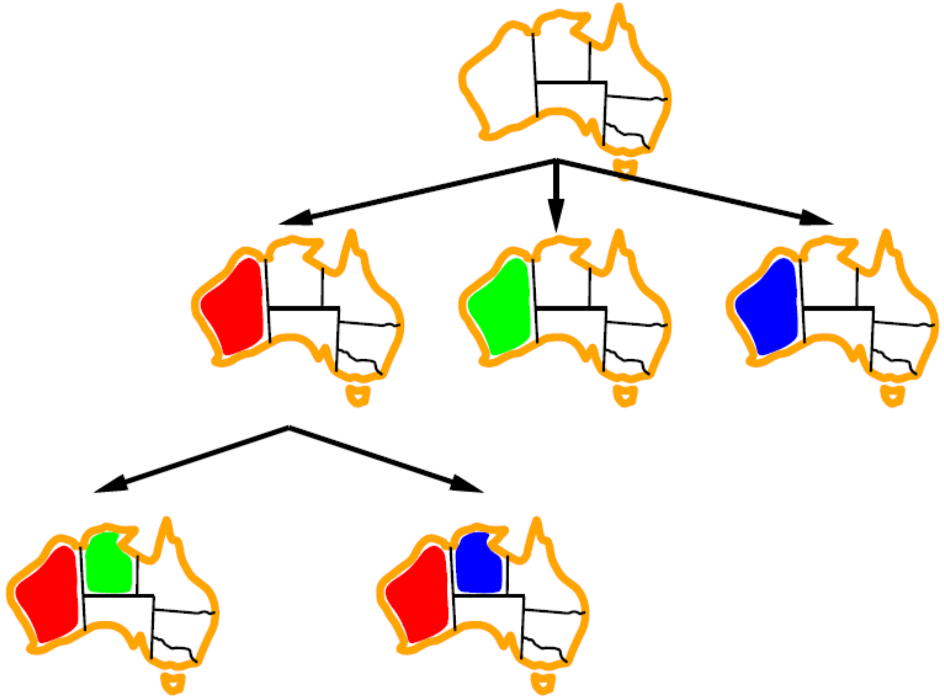

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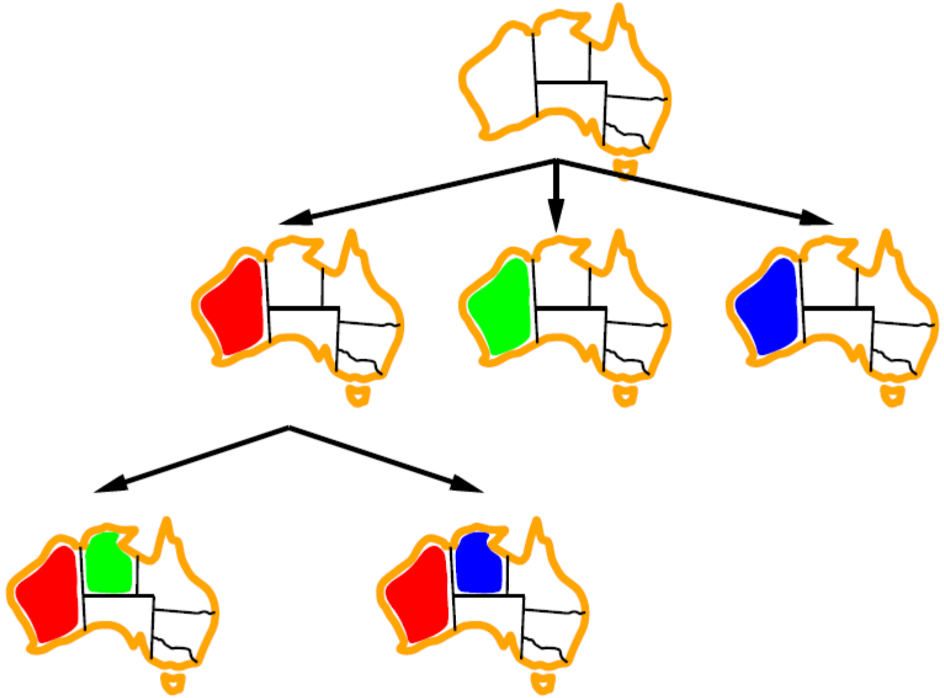

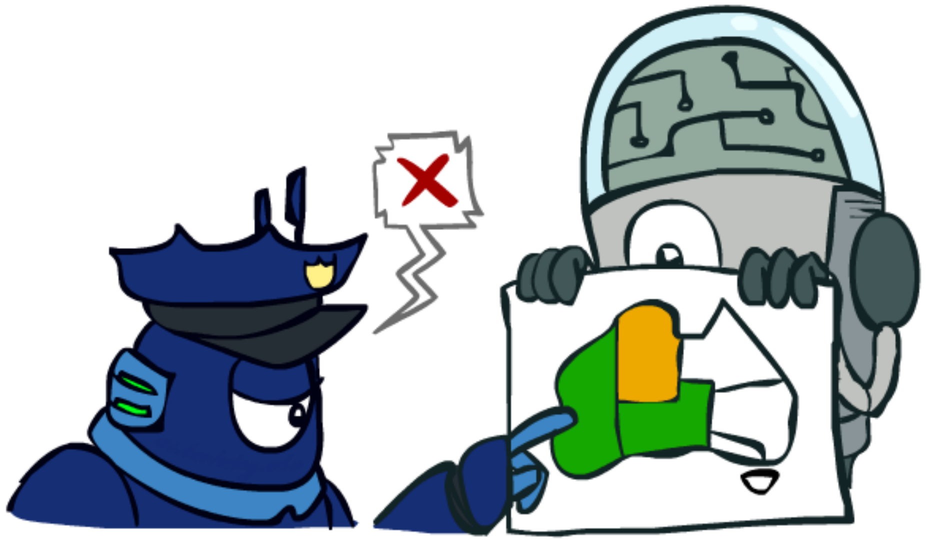

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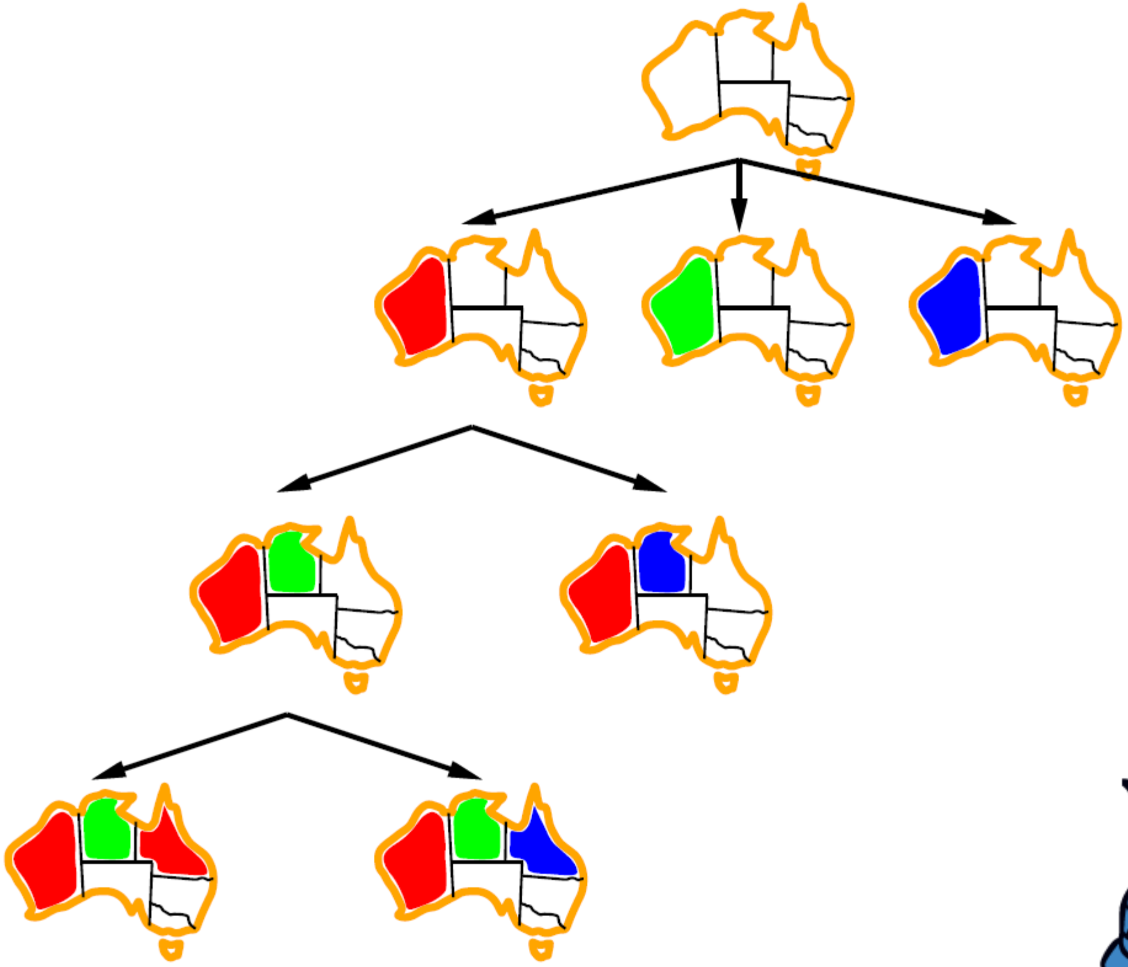

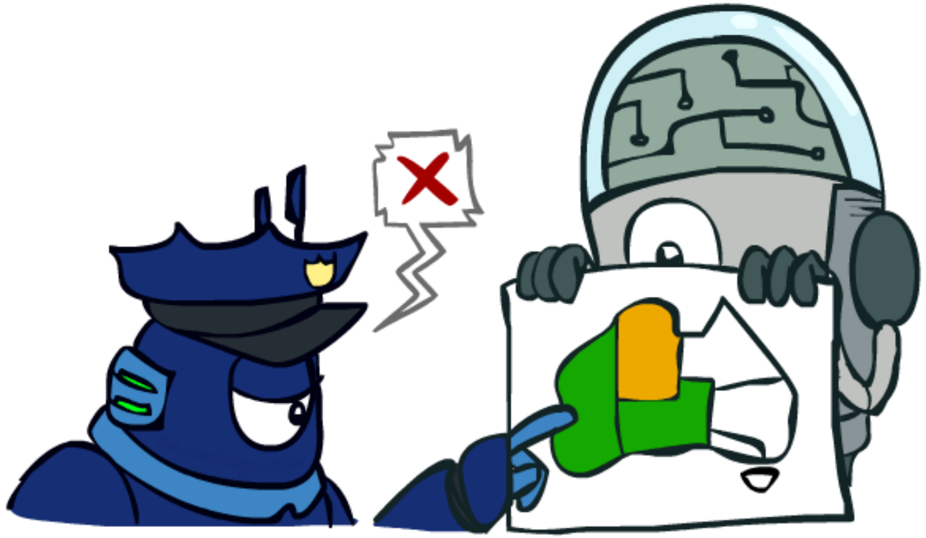

Improving Backtracking

- General-purpose ideas give huge gains in speed
  - … but it's all still NP-hard

- Filtering: Can we detect inevitable failure early?

- Ordering:
  - Which variable should be assigned next? (MRV)
  - In what order should its values be tried? (LCV)

- Structure: Can we exploit the problem structure?

Arc Consistency and Beyond

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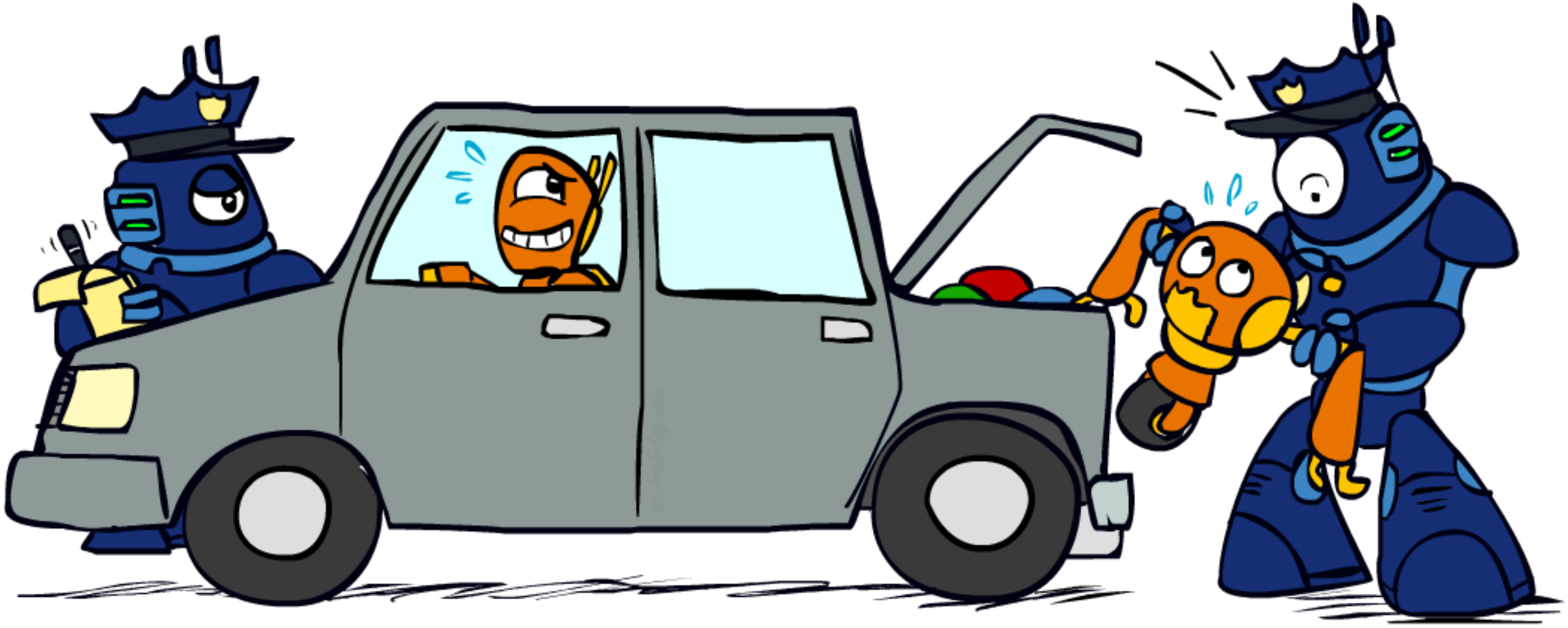
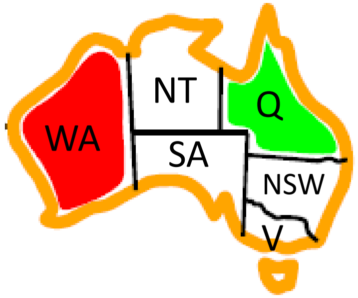
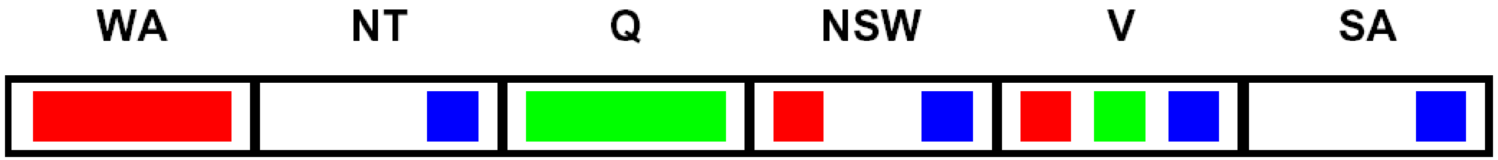


Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:





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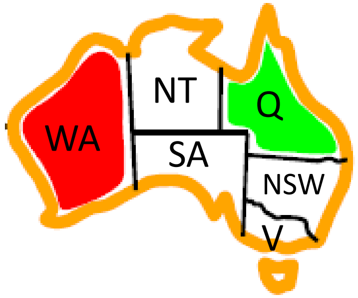

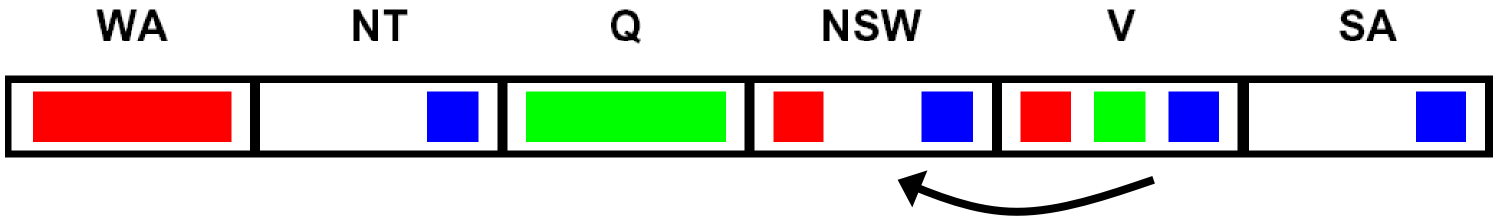

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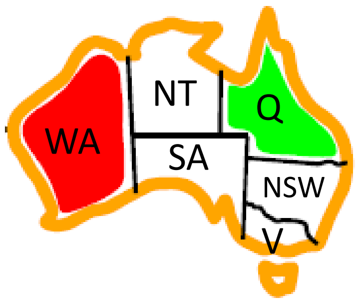

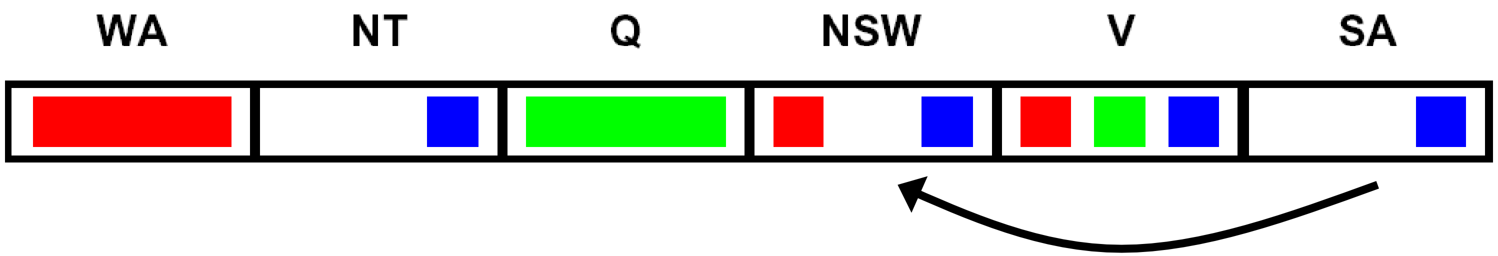

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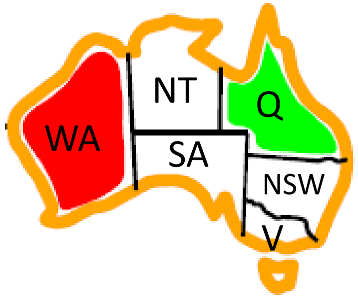

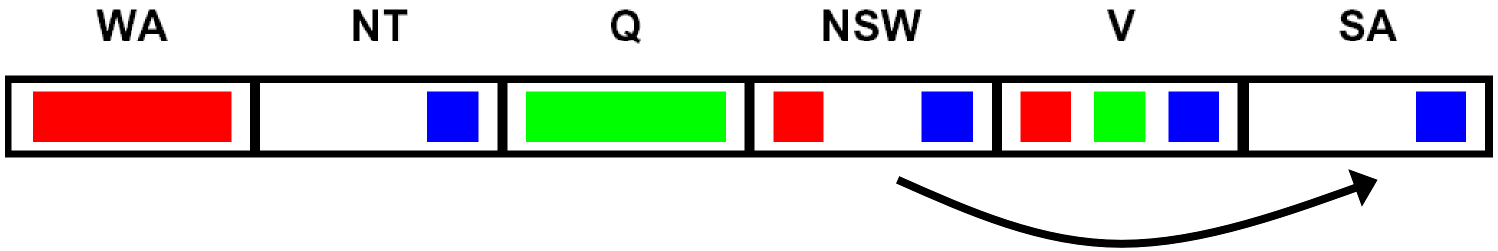

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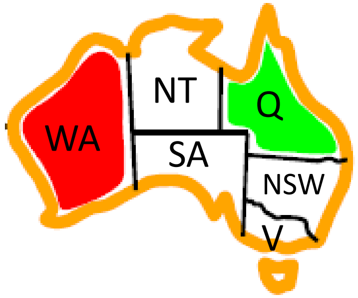

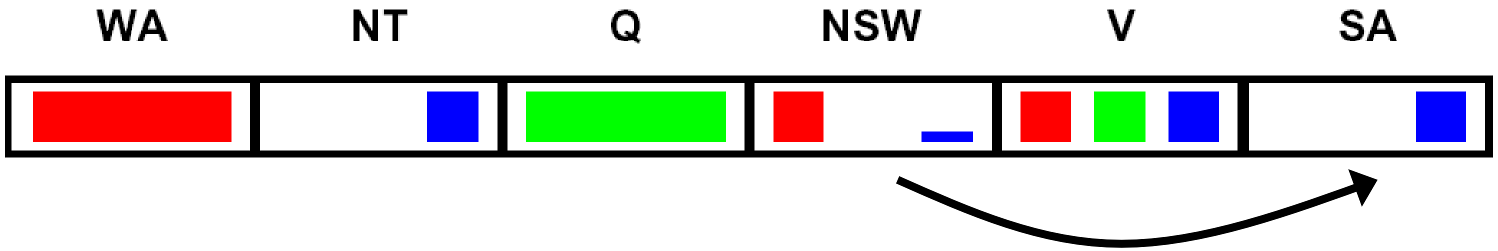

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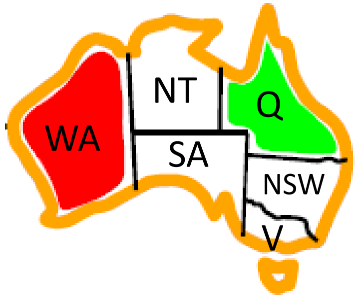

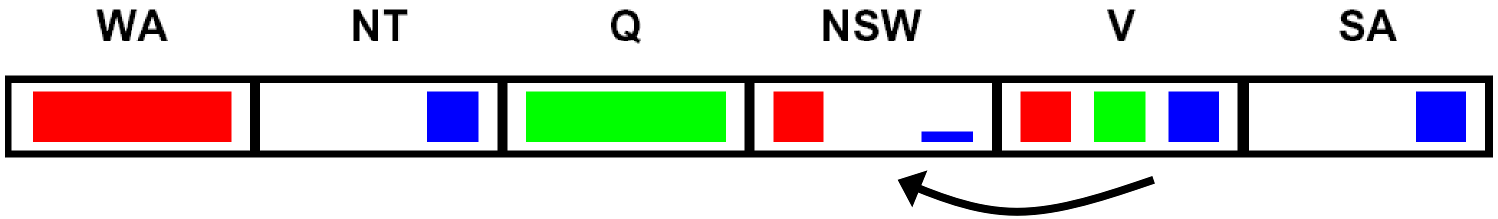

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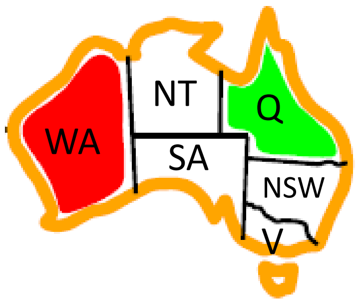

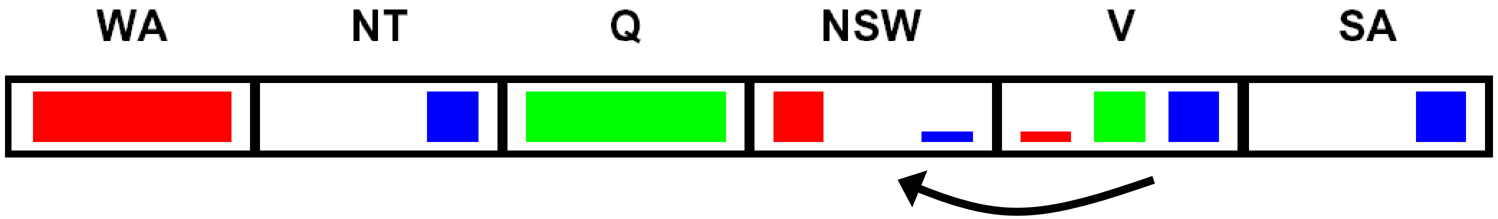

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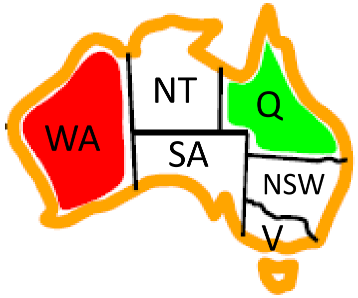

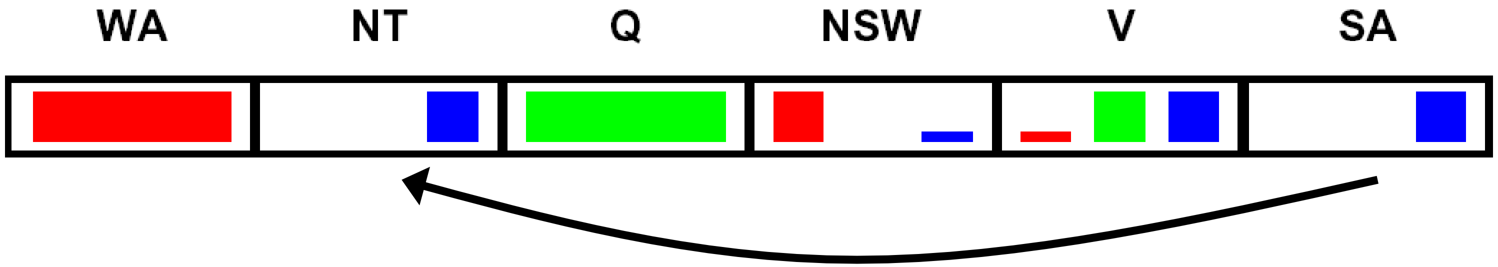

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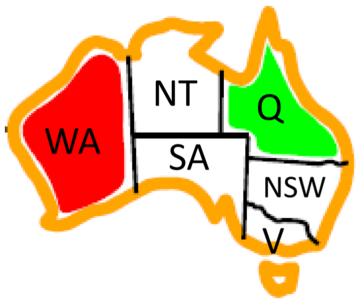

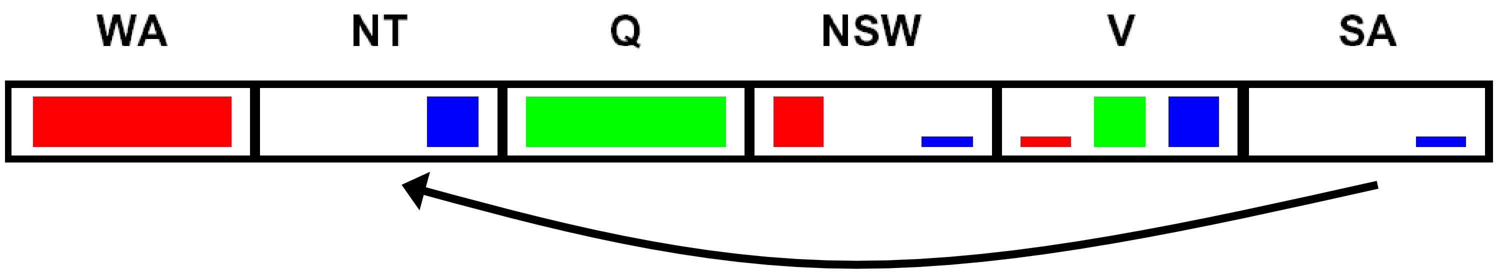

- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

Remember: Delete from the tail!

Enforcing Arc Consistency in a CSP

```
function AC-3( csp) returns the CSP, possibly with reduced domains
   inputs: csp, a binary CSP with variables {X1, X2, ..., Xn}
   local variables: queue, a queue of arcs, initially all the arcs in csp

   while queue is not empty do
      (Xi, Xj) ← REMOVE-FIRST(queue)
      if REMOVE-INCONSISTENT-VALUES(Xi, Xj) then
         for each Xk in NEIGHBORS[Xi] do
            add (Xk, Xi) to queue
```

```
function REMOVE-INCONSISTENT-VALUES( Xi, Xj) returns true iff succeeds
   removed ← false
   for each x in DOMAIN[Xi] do
      if no value y in DOMAIN[Xj] allows (x,y) to satisfy the constraint Xi ↔ Xj
         then delete x from DOMAIN[Xi];  removed ← true
   return removed
```

- Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$
- … but detecting all possible future problems is NP-hard – why?

[Demo: CSP applet (made available by aispace.org) -- n-queens]

Ordering

Ordering: Minimum Remaining Values

- Variable Ordering: Minimum remaining values (MRV):
  - Choose the variable with the fewest legal left values in its domain
  - Aka most constrained variables

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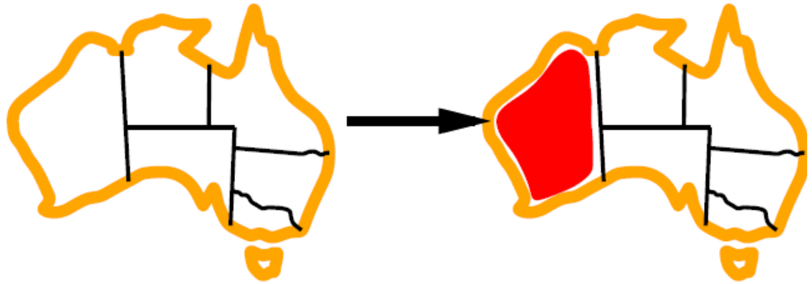

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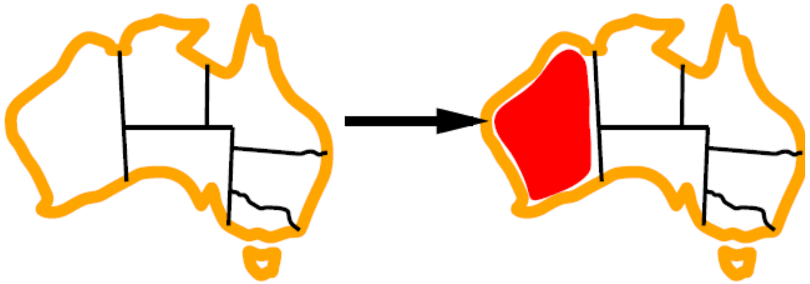

After assigning value to WA, both NT and SA have only two values in their domains
– choose one of them rather than Q, NSW, V or T

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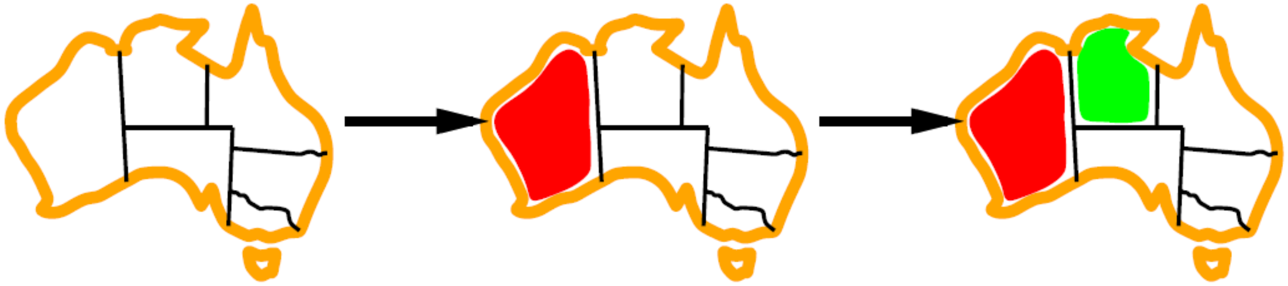

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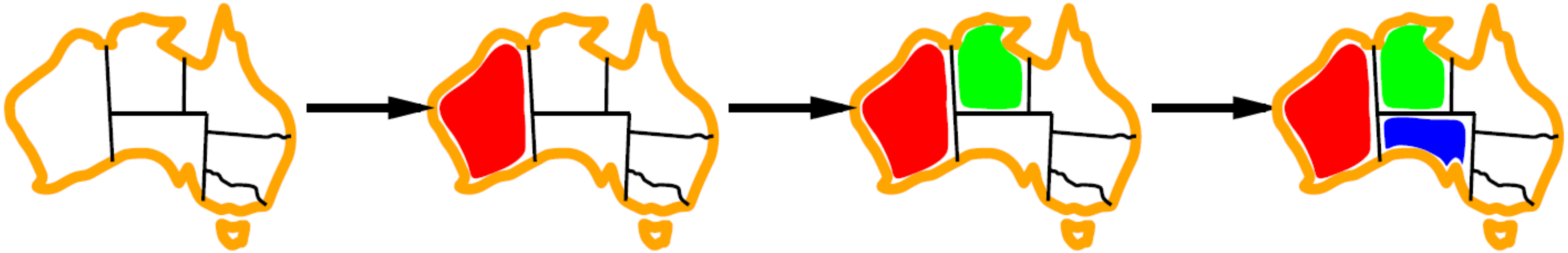

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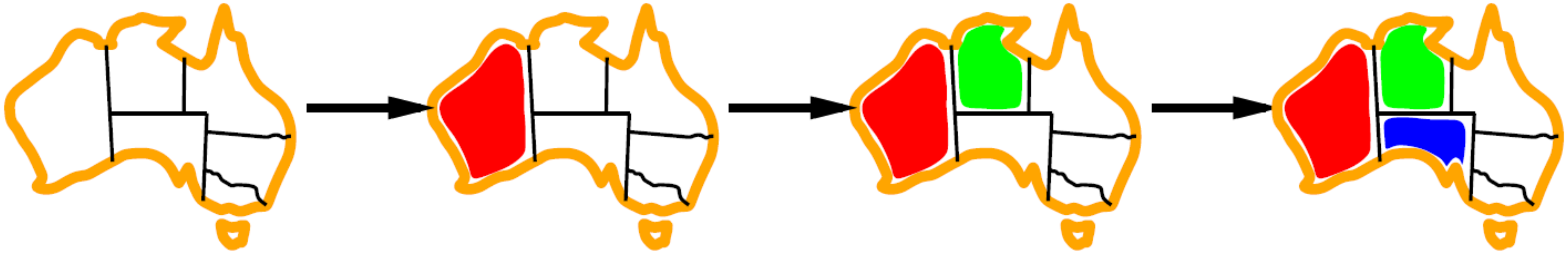

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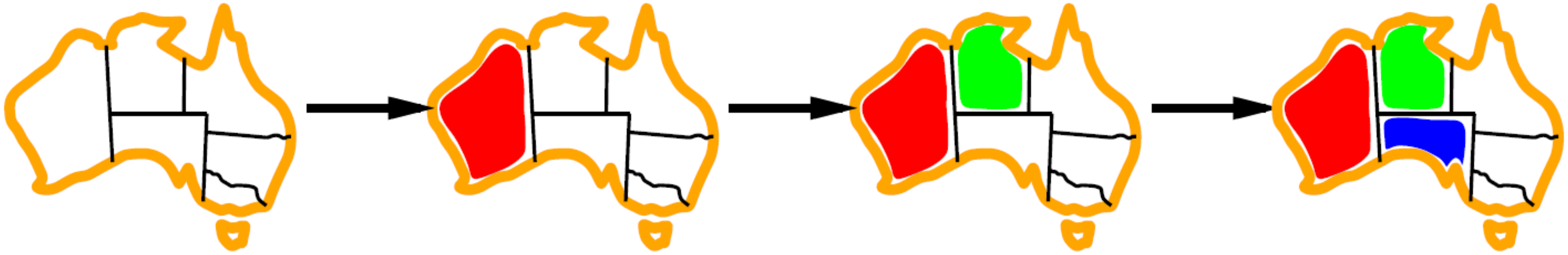

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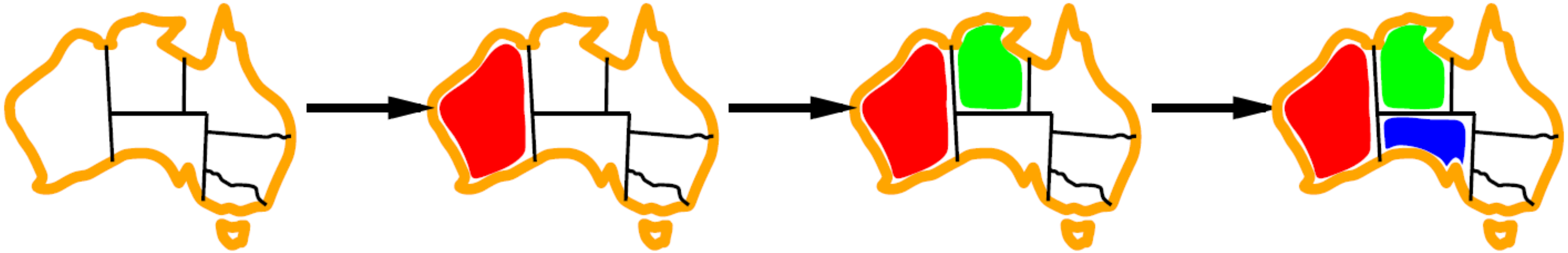

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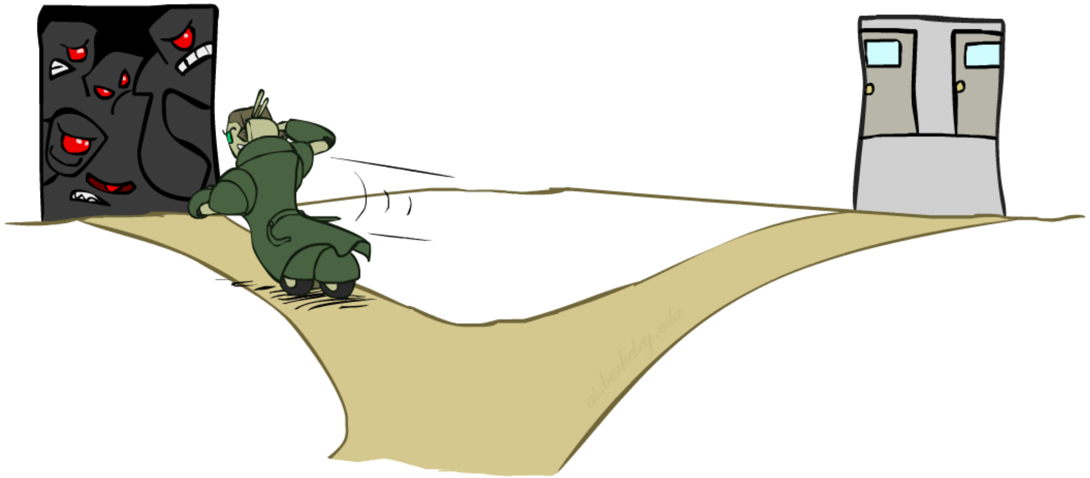

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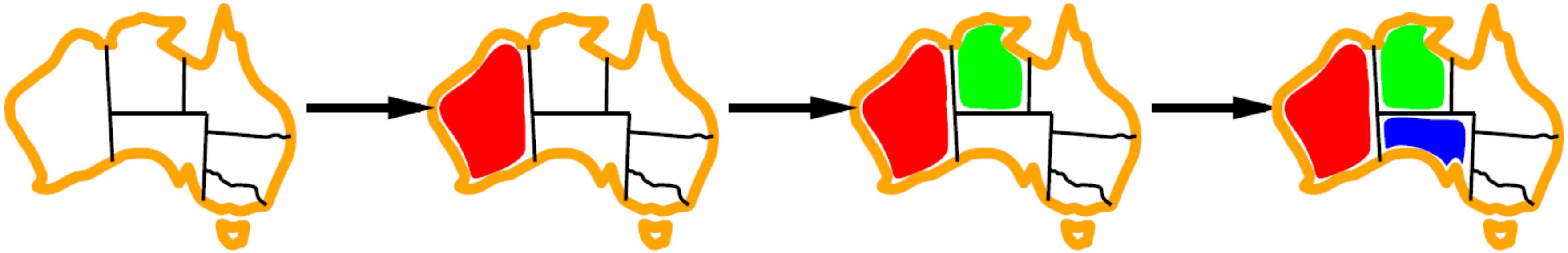
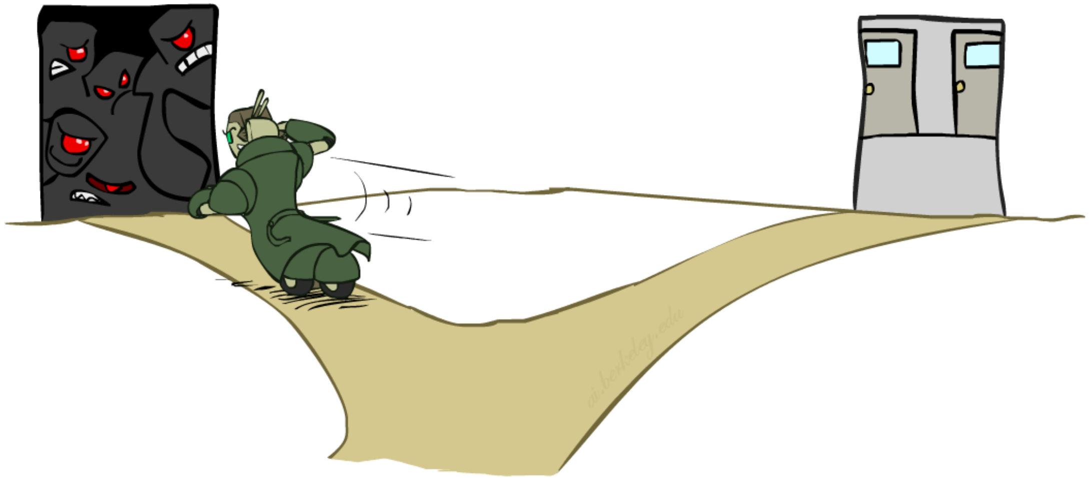
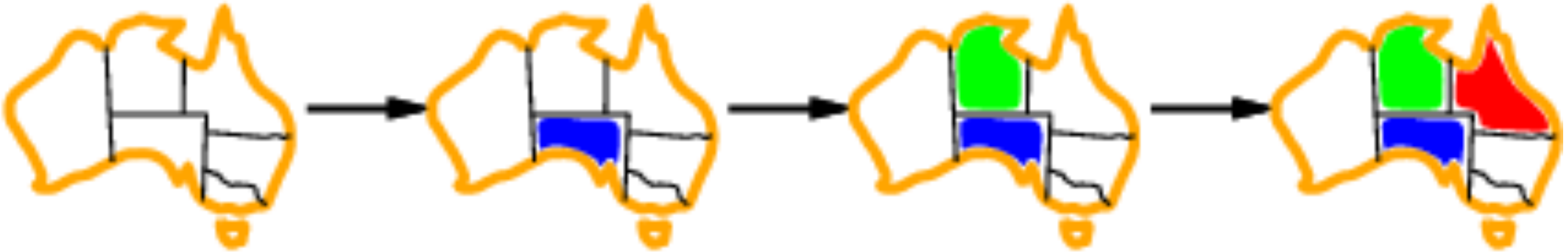


- Why min rather than max?
- Also called "most constrained variable"
- "Fail-fast" ordering



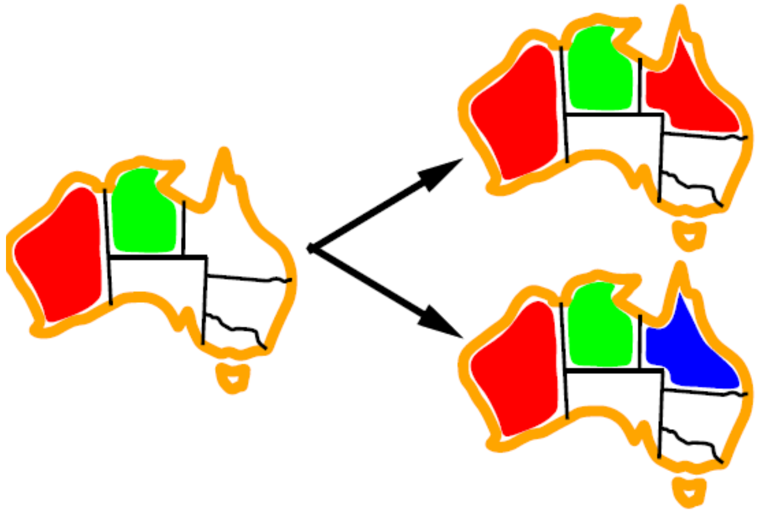
- Tie-breaker among Minimum remaining values
- Choose variable involved in largest # of constraints on remaining variables



- After assigning SA to be blue, WA, NT, Q, NSW and V all have just two values left.
- But WA and V have only one constraint (WA has constraint with NT, and V with NSW) on remaining variables and T none, so choose one of NT, Q & NSW (each of which has 2 cons. left)

Ordering: Least Constraining Value

- Value Ordering: Least Constraining Value
  - Given a choice of variable, choose the *least constraining value*
  - I.e., the one that rules out the fewest values in the remaining variables
  - Note that it may take some computation to determine this! (E.g., rerunning filtering)



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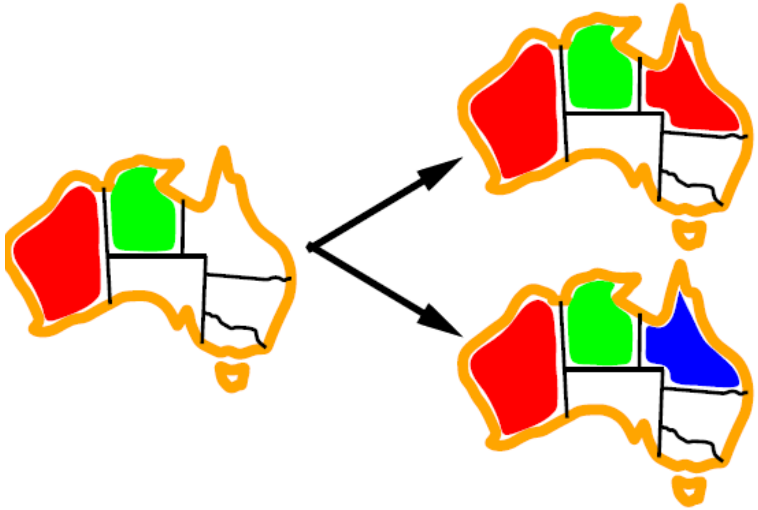

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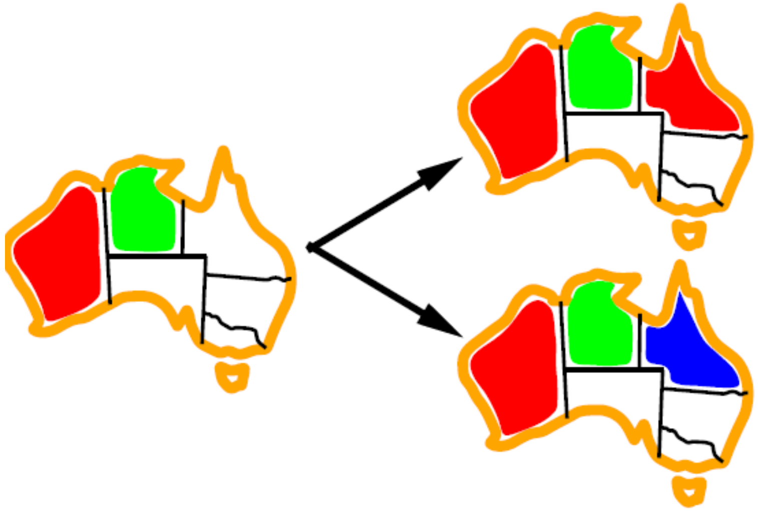

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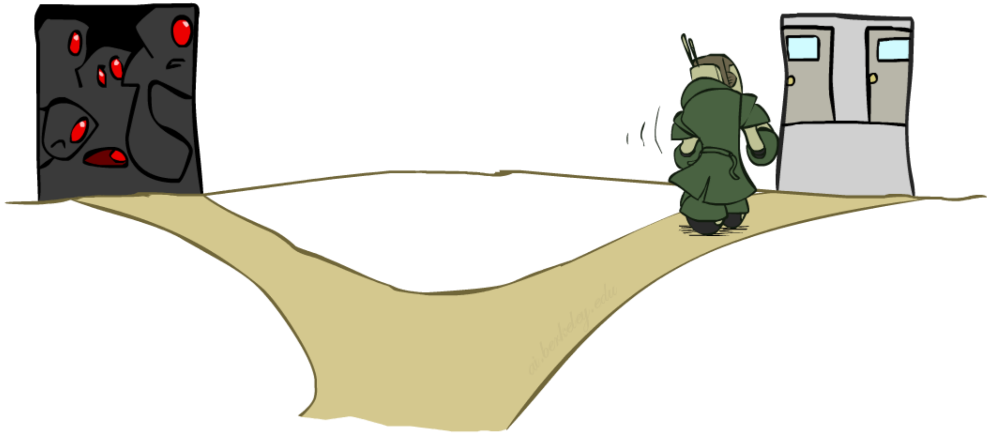

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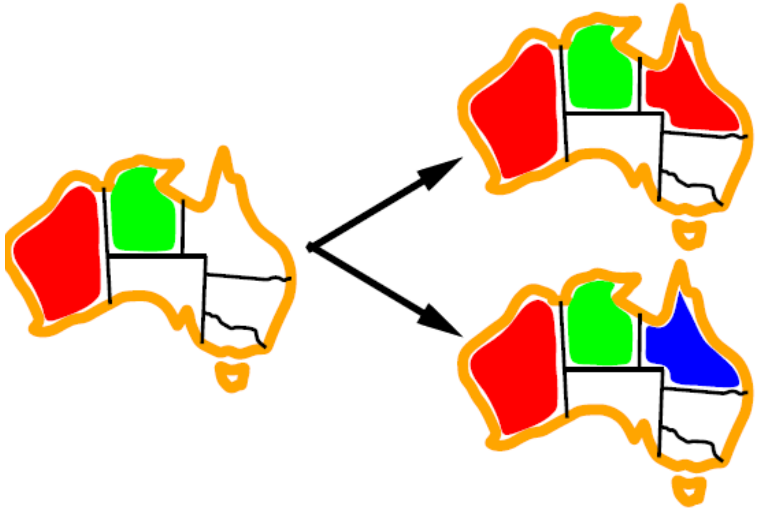
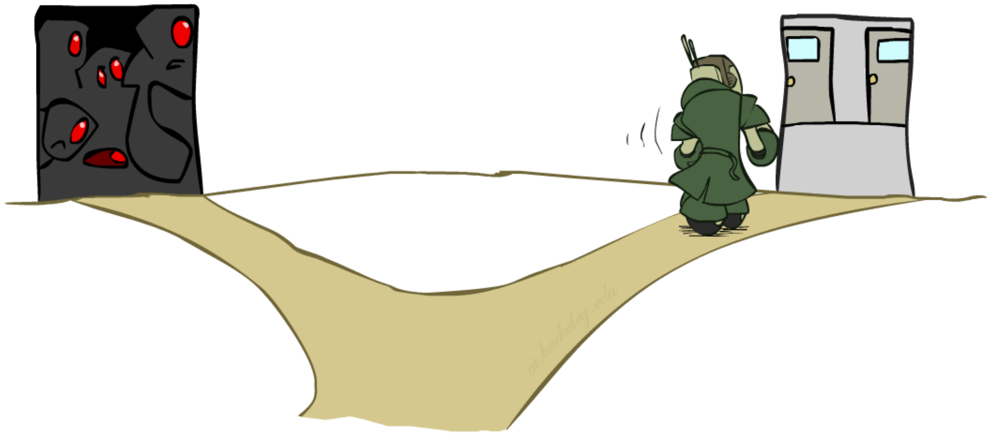


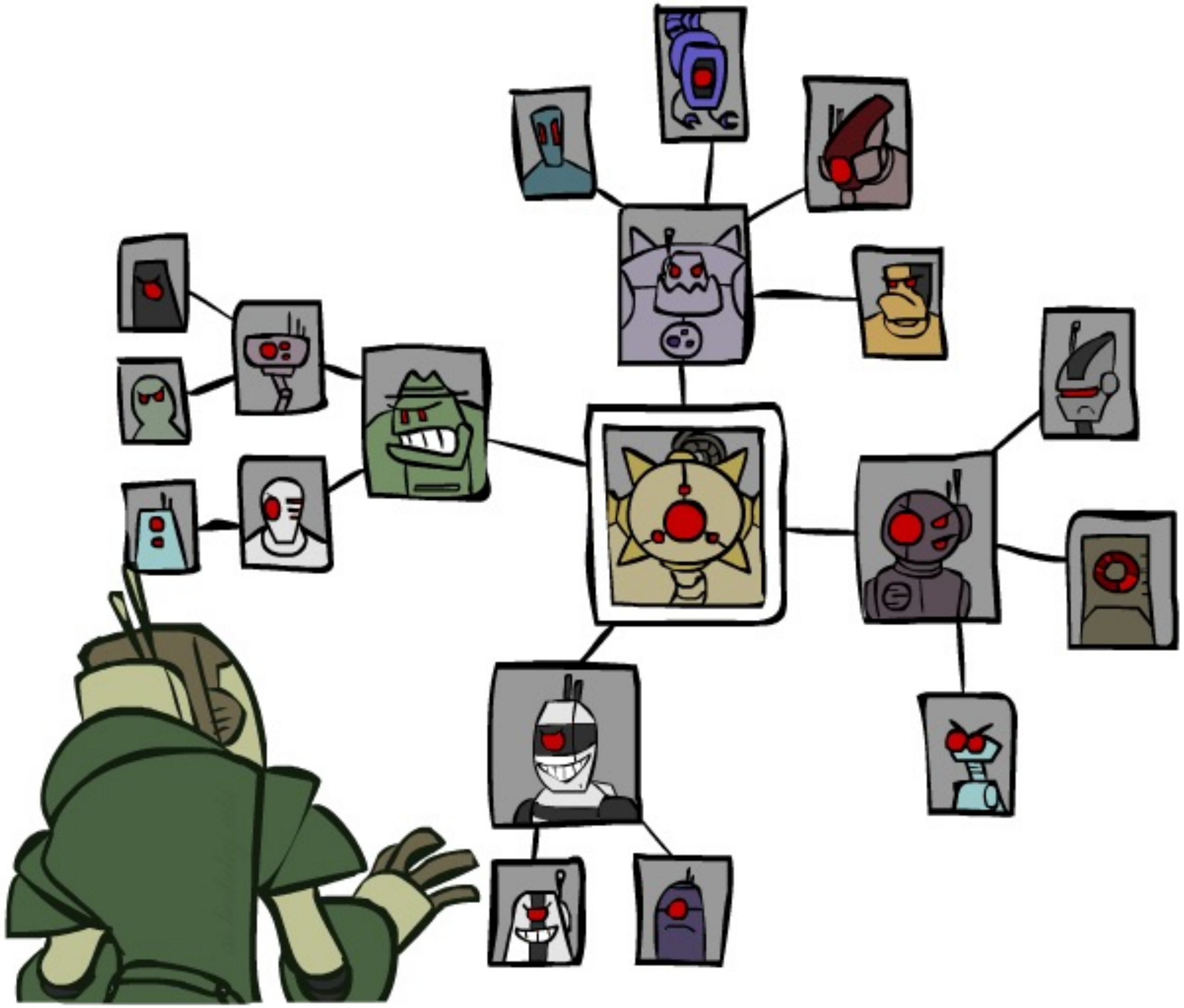
- Why least rather than most?

- Combining these ordering ideas makes 1000 queens feasible



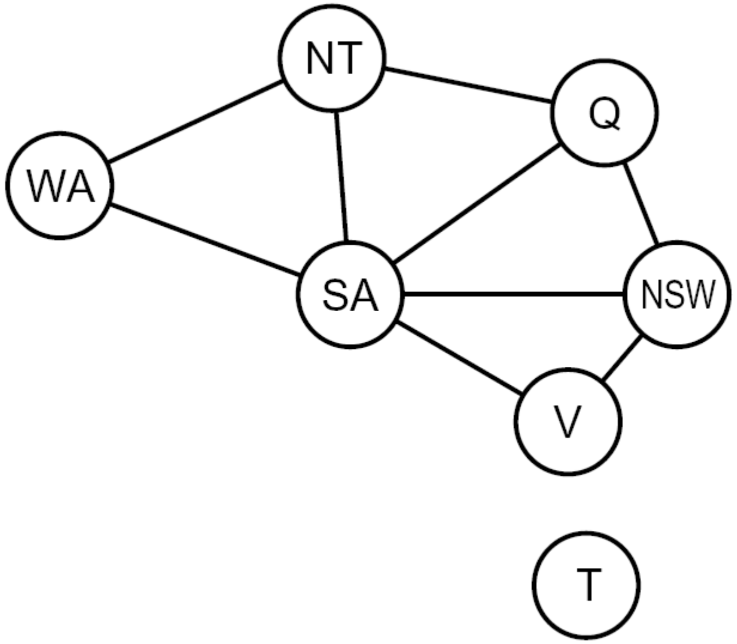
Demo: Coloring -- Backtracking + Forward Checking + Ordering

Structure

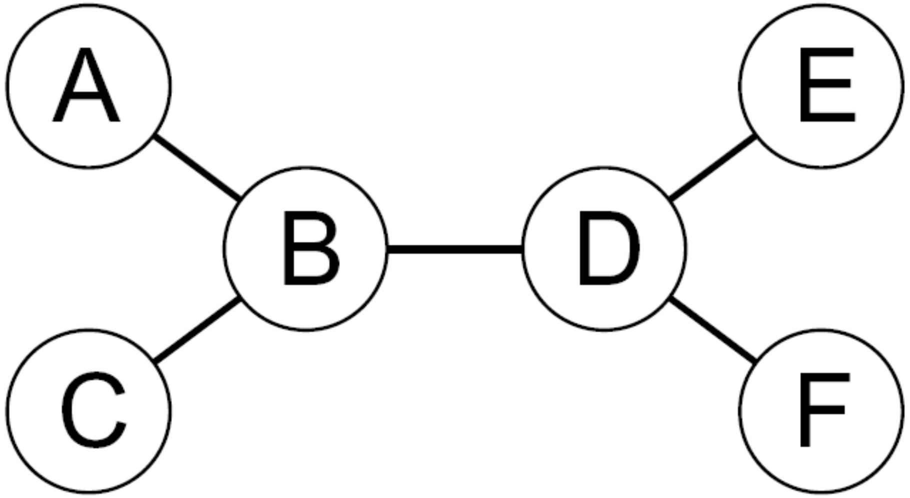


Problem Structure

- Extreme case: independent subproblems
  - Example: Tasmania and mainland do not interact
- Independent subproblems are identifiable as connected components of constraint graph
- Suppose a graph of n variables can be broken into subproblems of only c variables:
  - Worst-case solution cost is $O((n/c)(d^c))$, linear in n
  - E.g., n = 80, d = 2, c =20
  - 2^{80} = 4 billion years at 10 million nodes/sec
  - $(4)(2^{20})$ = 0.4 seconds at 10 million nodes/sec



Tree-Structured CSPs

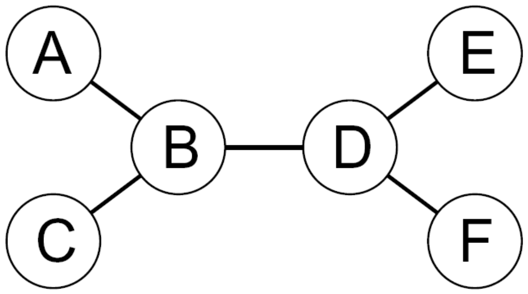


- Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\ d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$

- This property also applies to probabilistic reasoning (later): an example of the relation between syntactic restrictions and the complexity of reasoning

Tree-Structured CSPs

- Algorithm for tree-structured CSPs:
  - Order: Choose a root variable, order variables so that parents precede children



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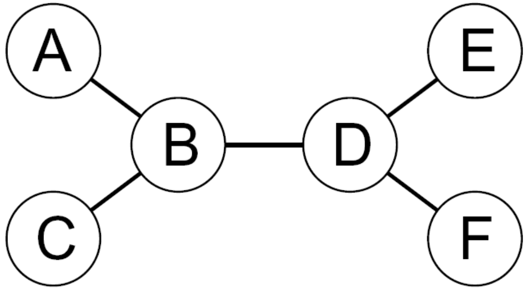

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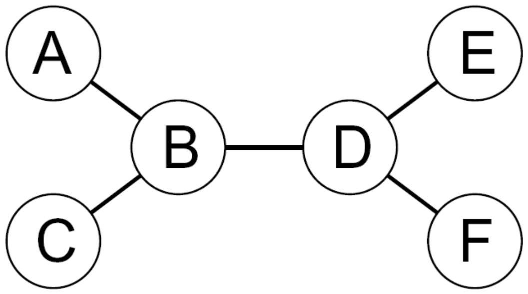

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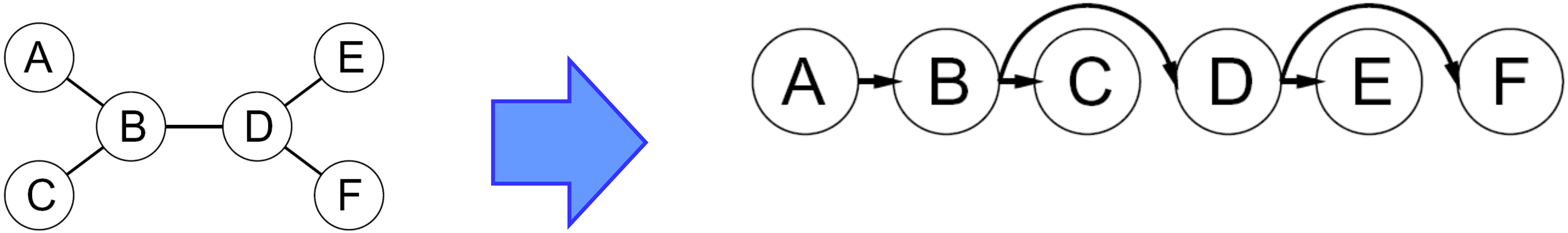

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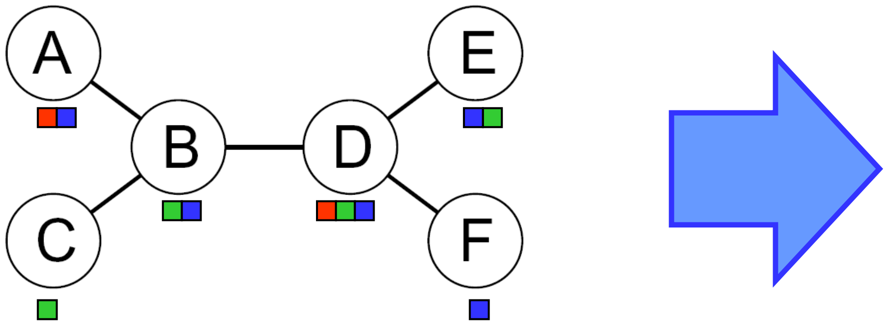

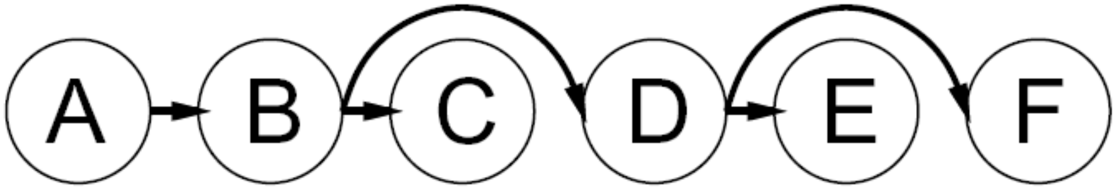

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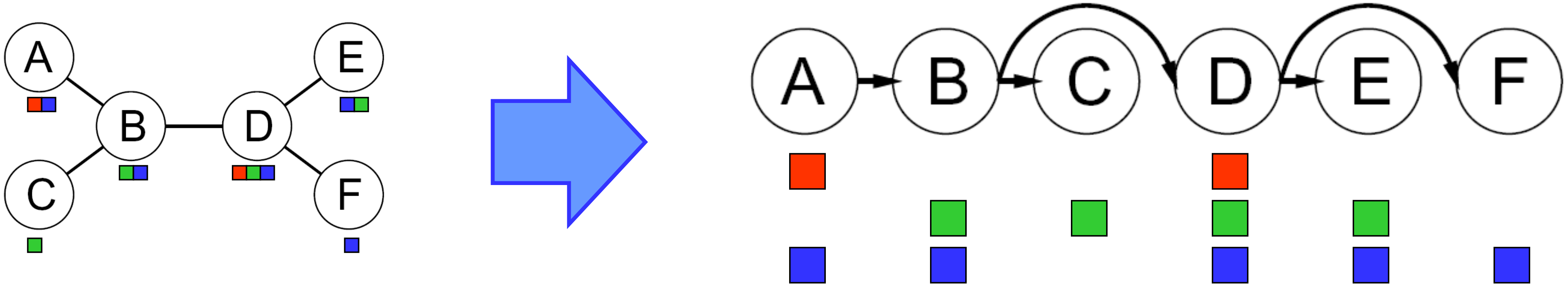
  - Order: Choose a root variable, order variables so that parents precede children



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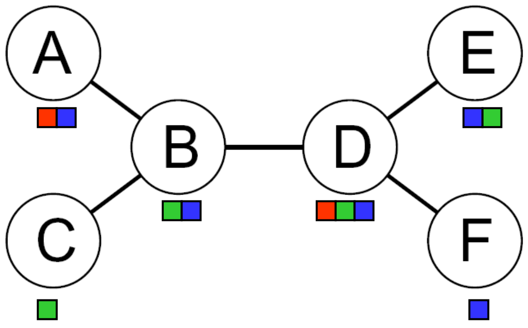

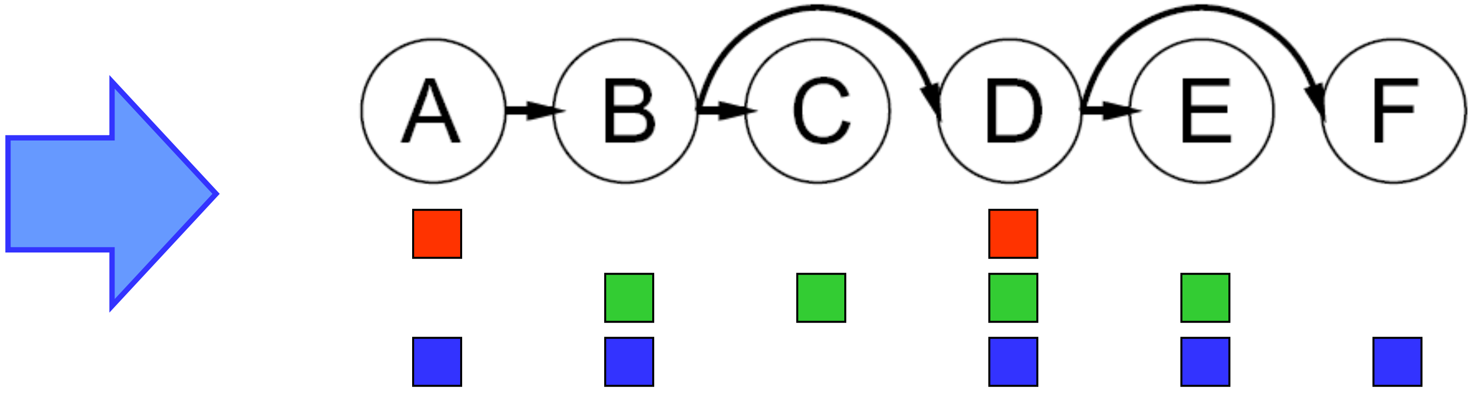

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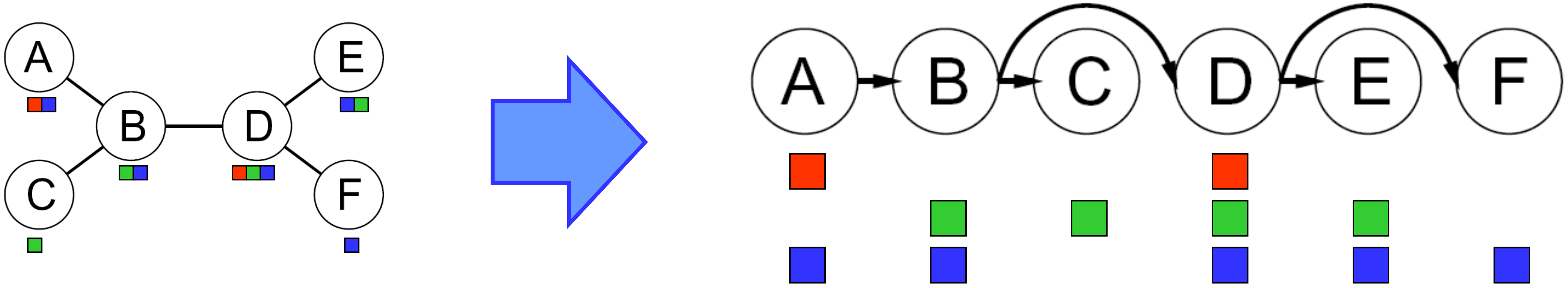

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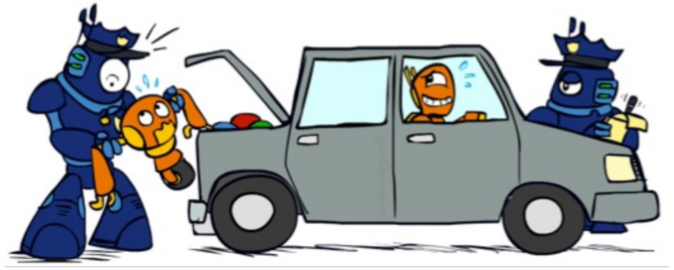

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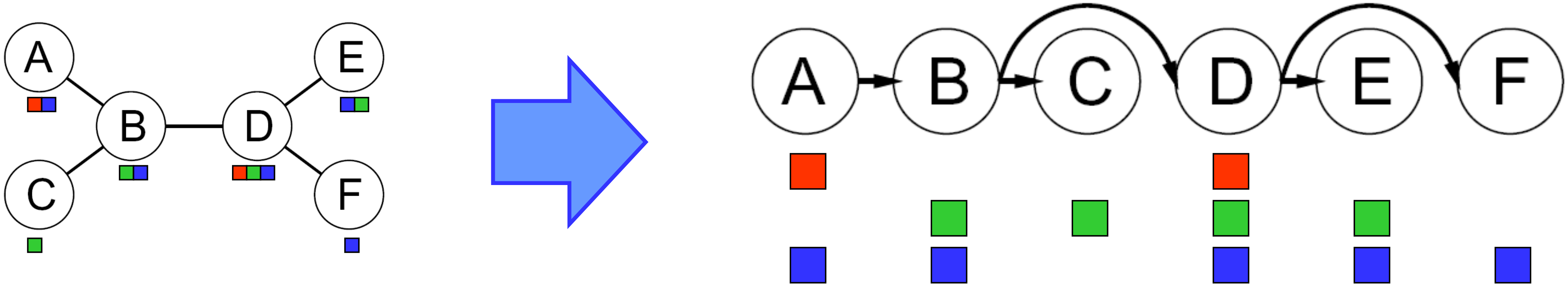

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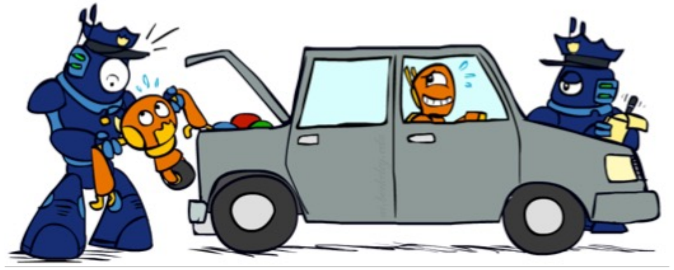

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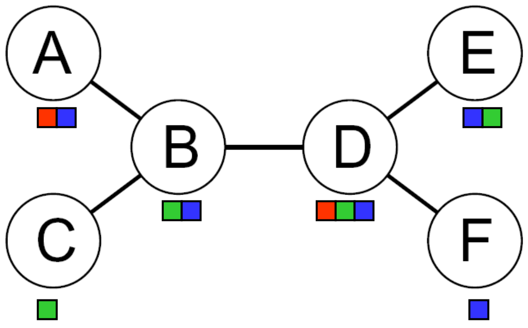

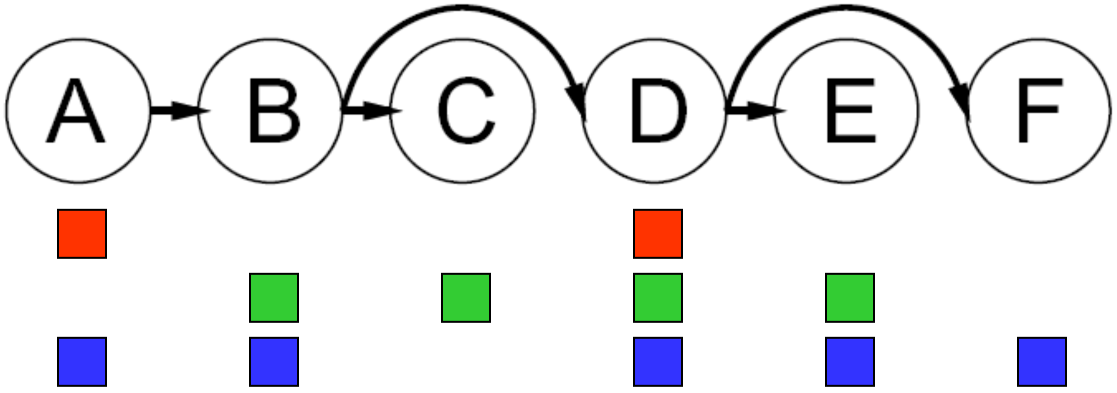

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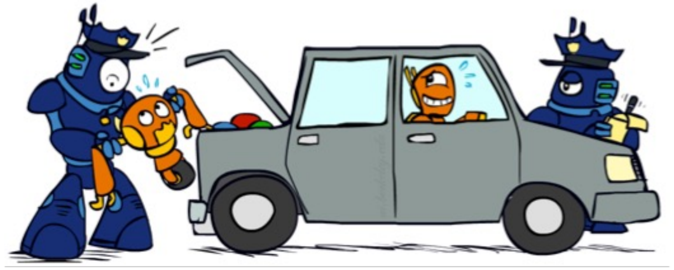
- Runtime: $O(n\ d^2)$ (why?)



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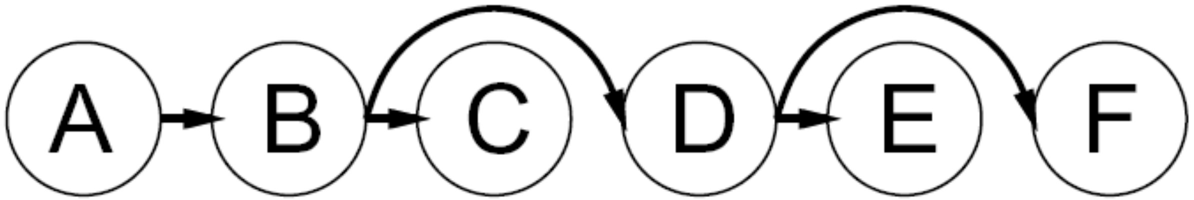
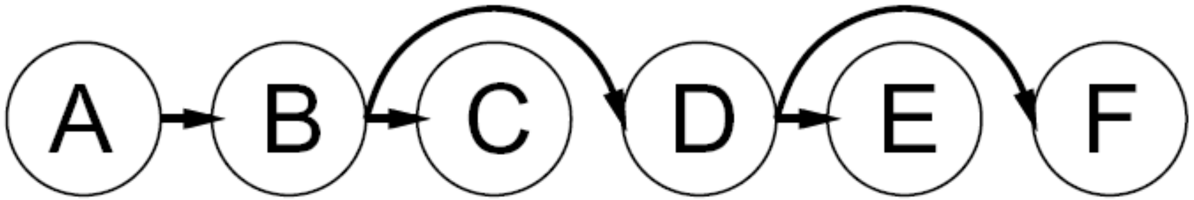


Tree-Structured CSPs

- Claim 1: After backward pass, all root-to-leaf arcs are consistent



Tree-Structured CSPs

- Claim 1: After backward pass, all root-to-leaf arcs are consistent
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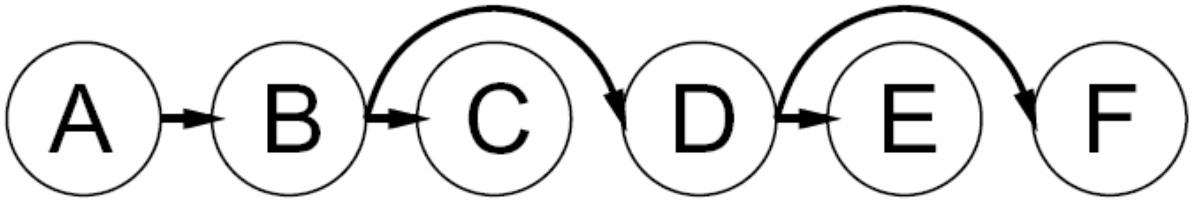

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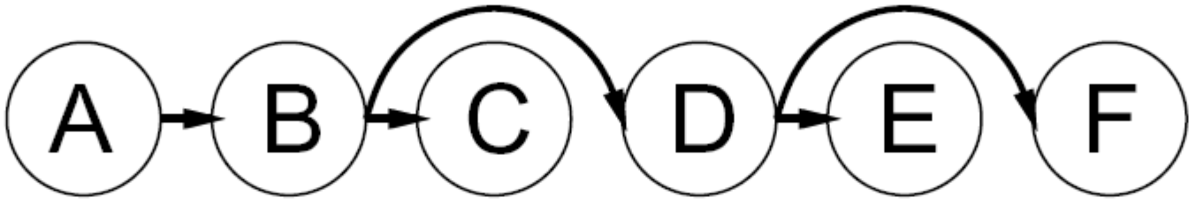

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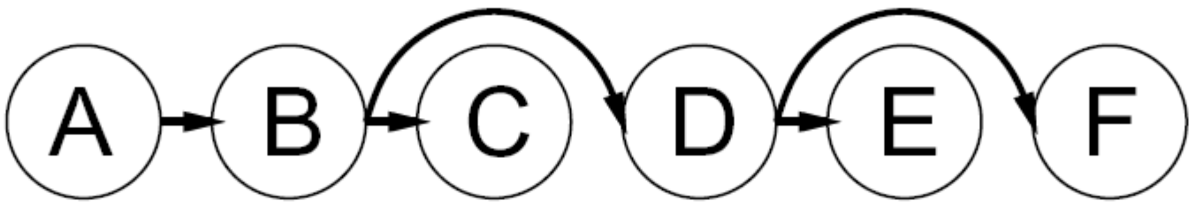

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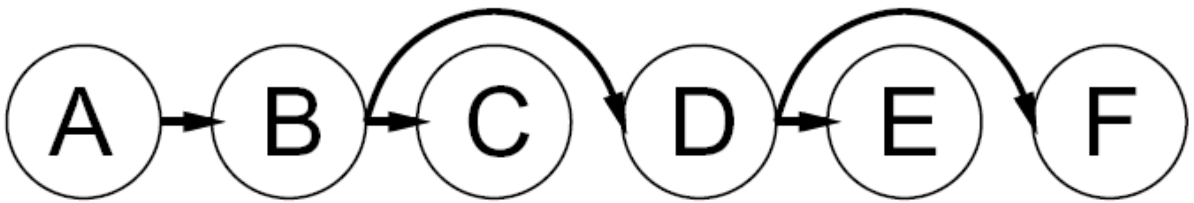

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- Why doesn't this algorithm work with cycles in the constraint graph?

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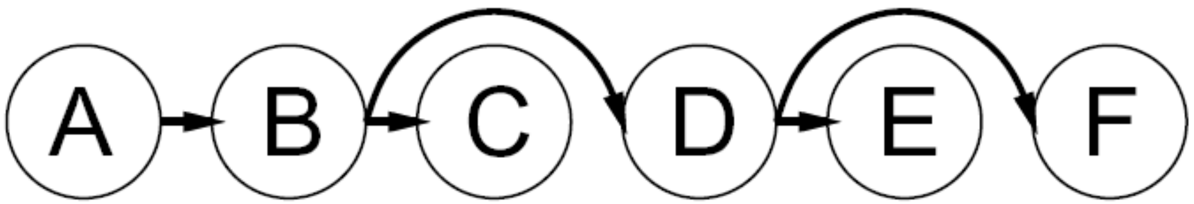

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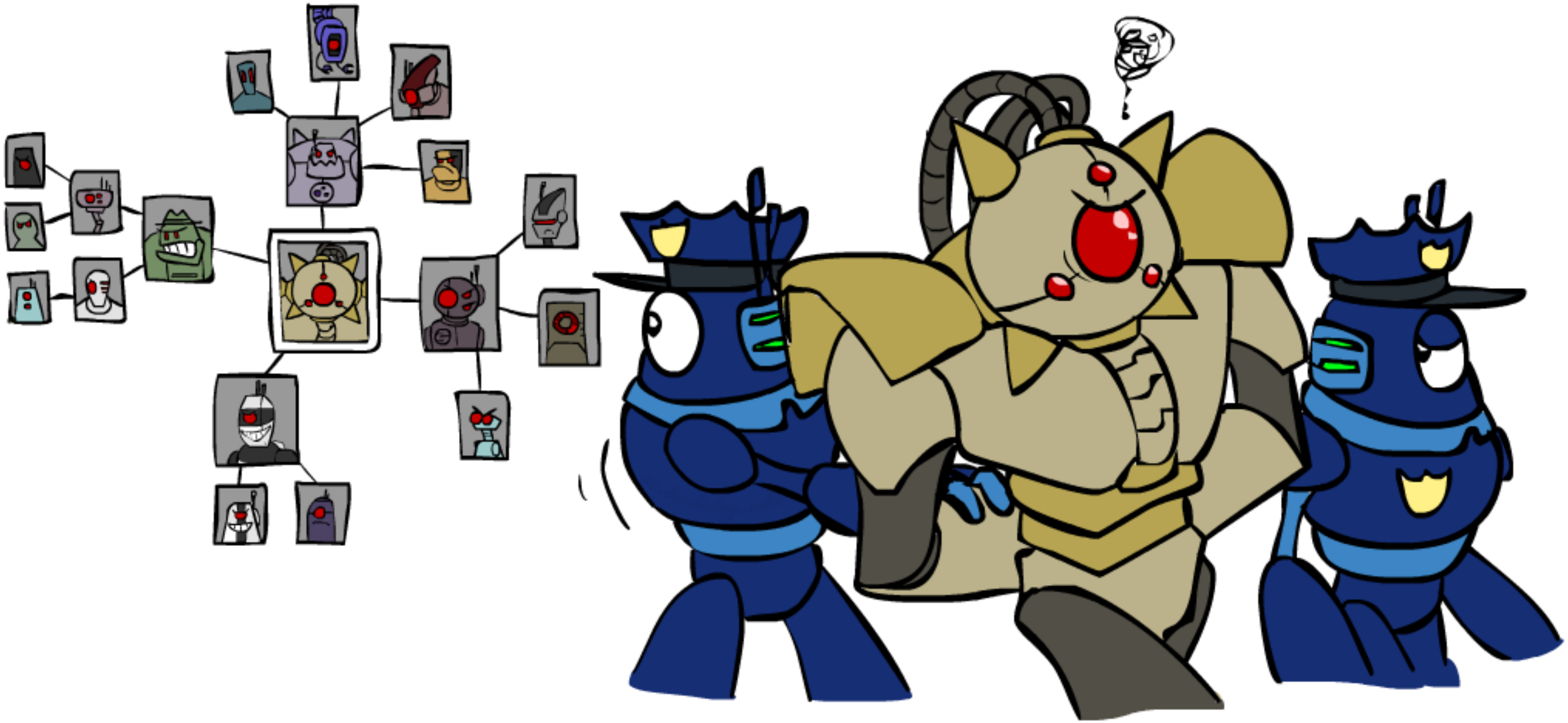
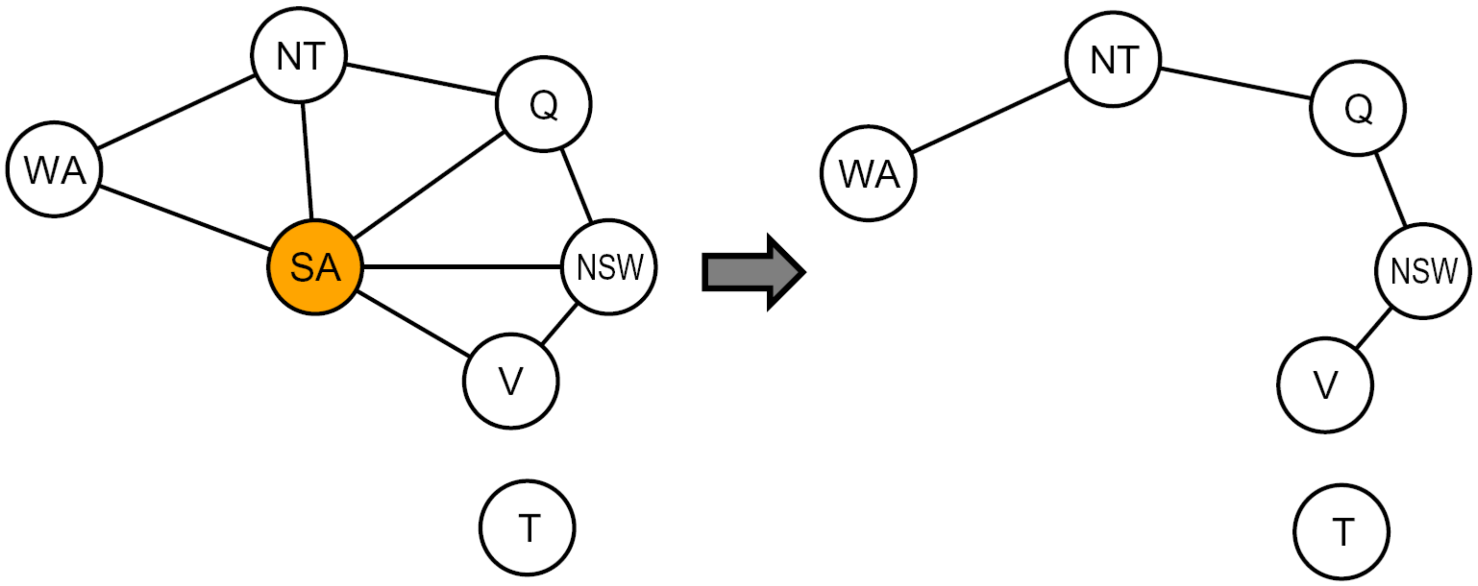
- Note: we'll see this basic idea again with Bayes' nets

Improving Structure



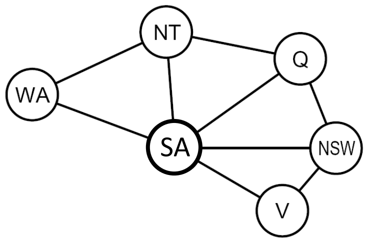
Nearly Tree-Structured CSPs



- Conditioning: instantiate a variable, prune its neighbors' domains
- Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree
- Cutset size c gives runtime $O((d^c) (n-c) d^2)$, very fast for small c

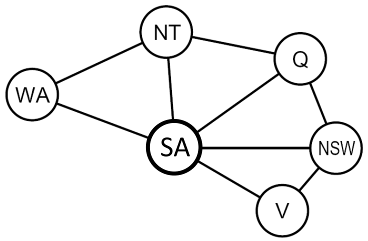
Cutset Conditioning

Cutset Conditioning



Cutset Conditioning

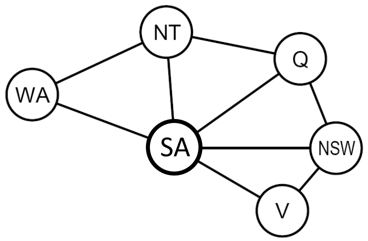
Choose a cutset



Cutset Conditioning

Choose a cutset

Instantiate the cutset
(all possible ways)



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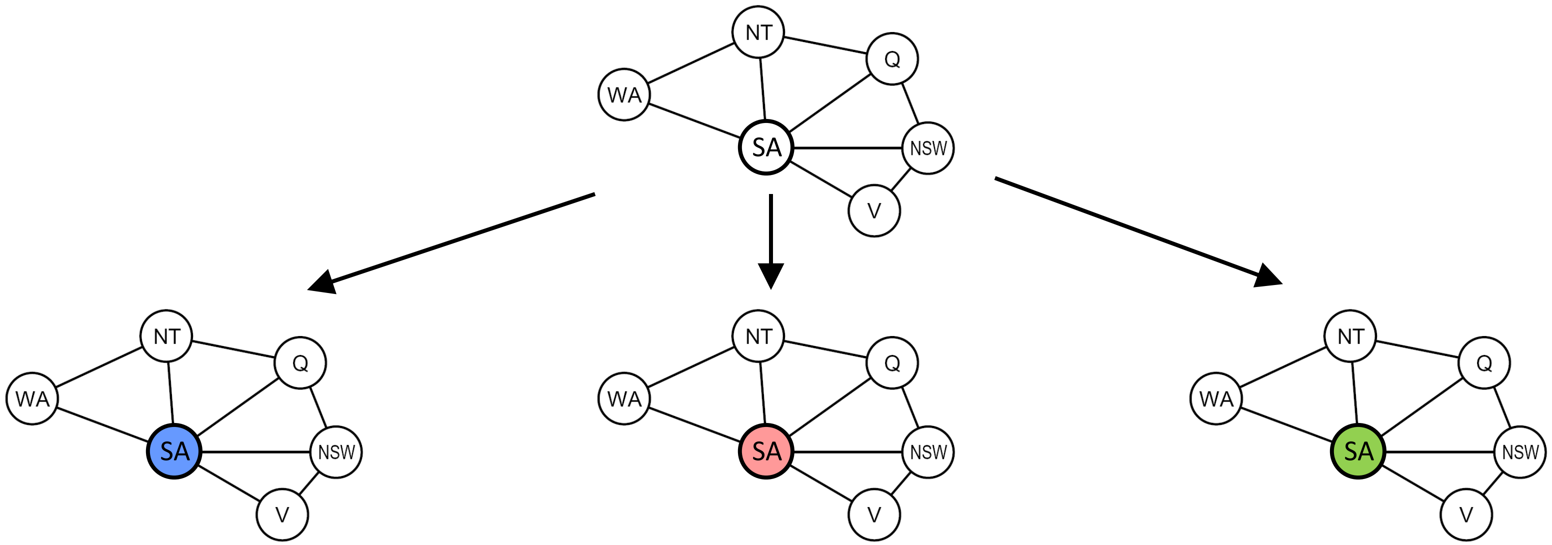
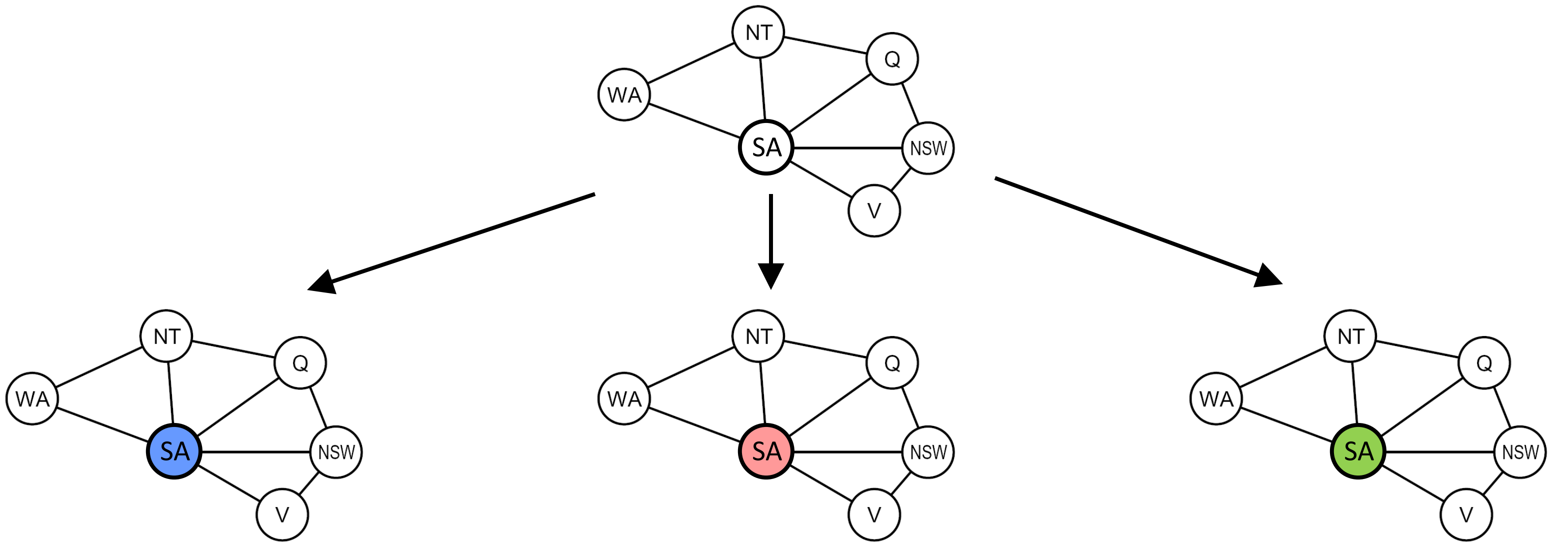


Cutset Conditioning

Choose a cutset

Instantiate the cutset (all possible ways)

Compute residual CSP for each assignment



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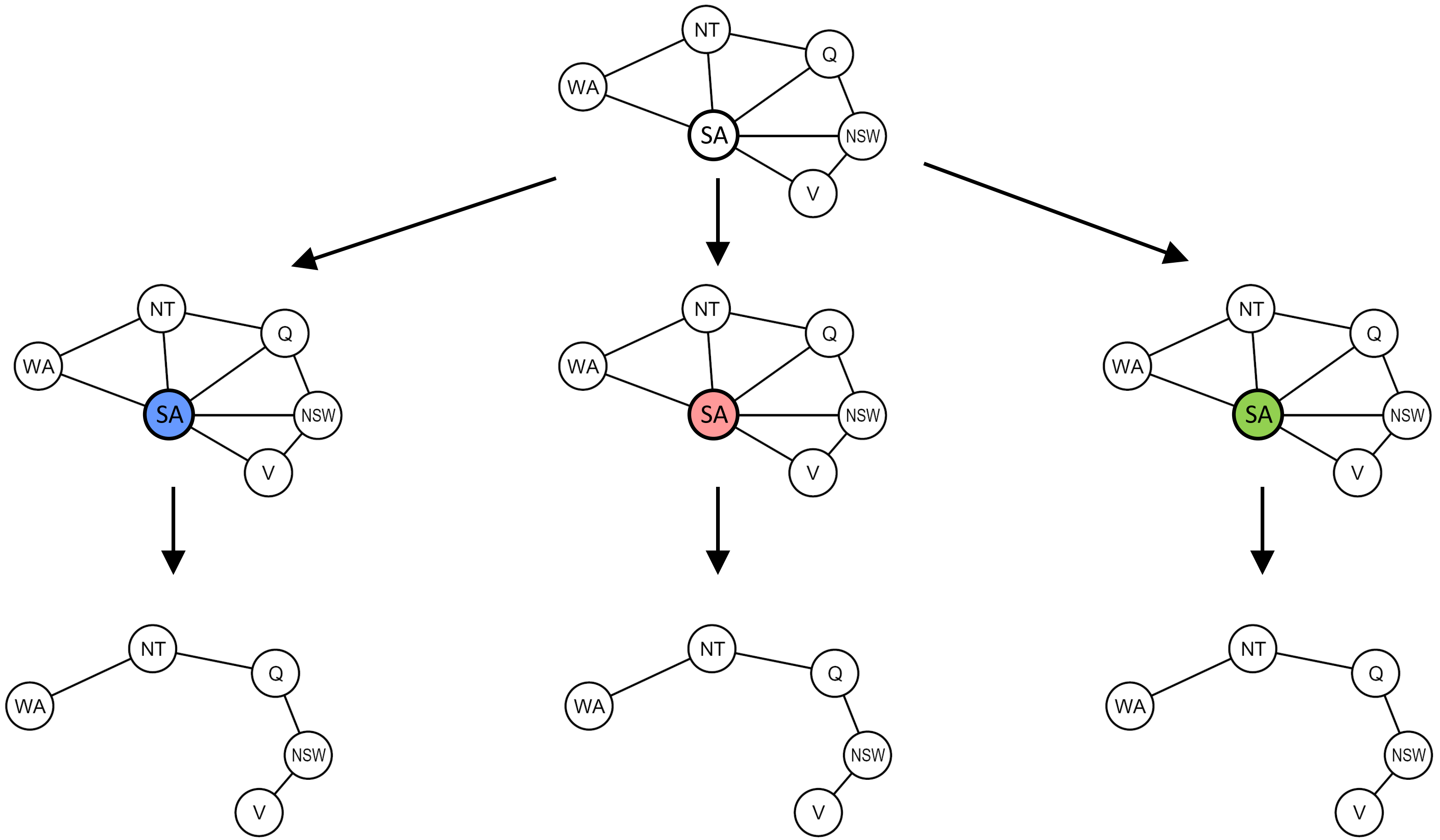

Cutset Conditioning

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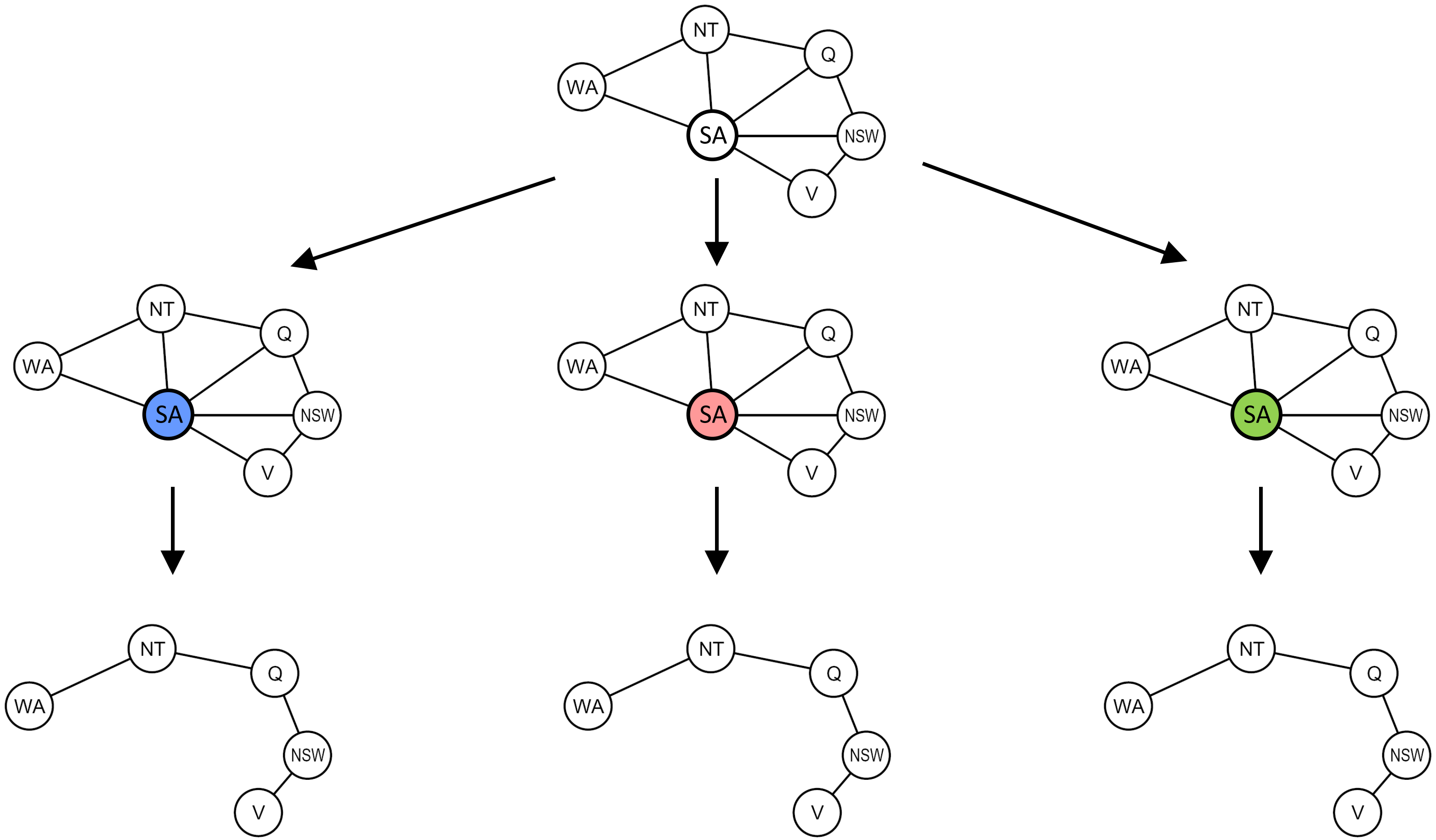
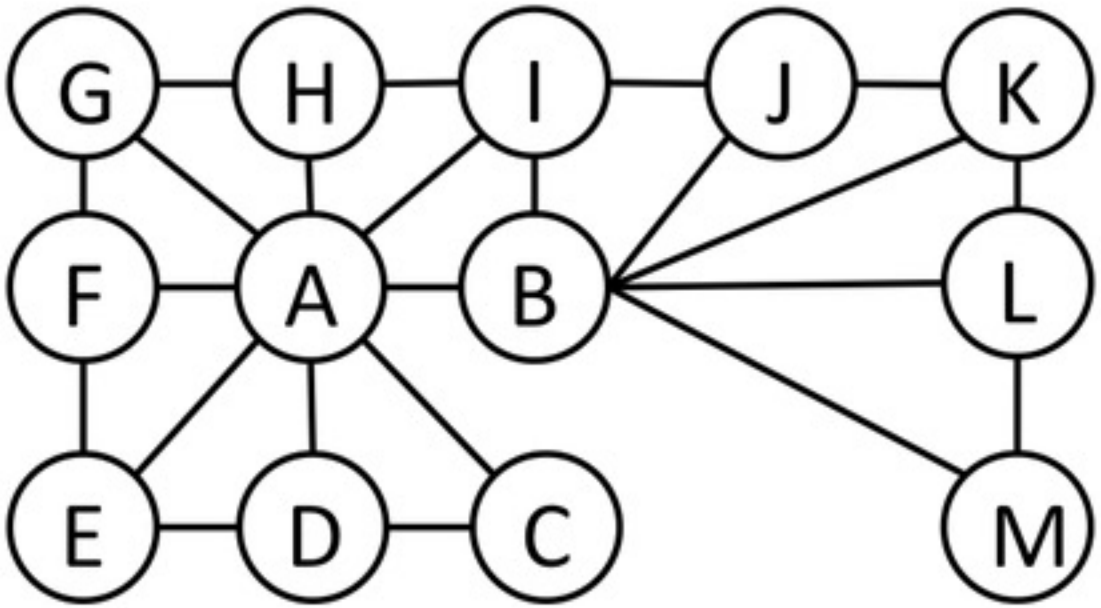
Compute residual CSP for each assignment

Solve the residual CSPs (tree structured)

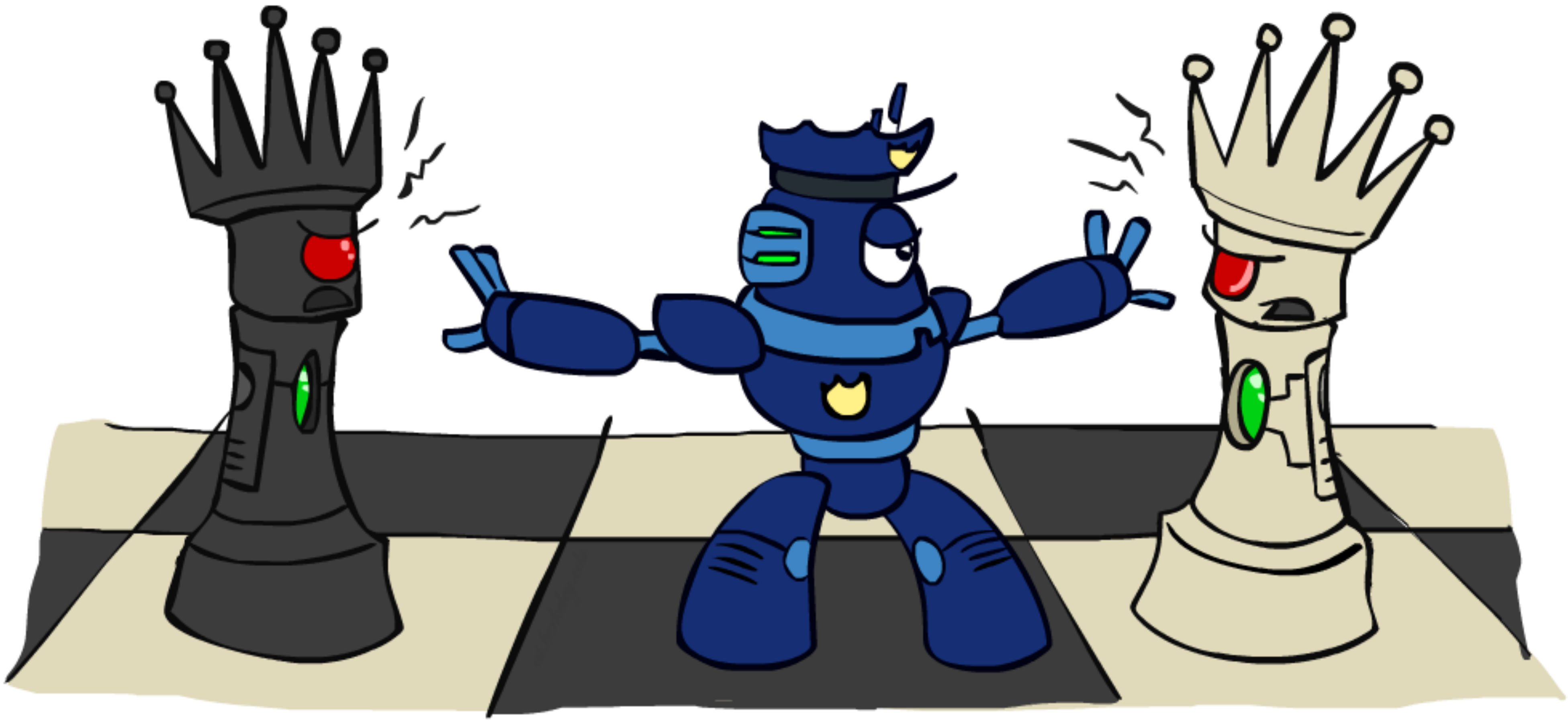


Cutset Quiz

- Find the smallest cutset for the graph below.



Iterative Improvement



Iterative Algorithms for CSPs

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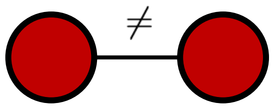

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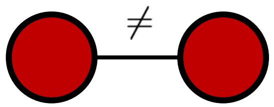
  - No fringe! Live on the edge.



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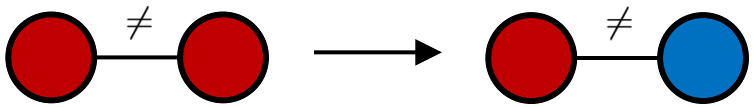

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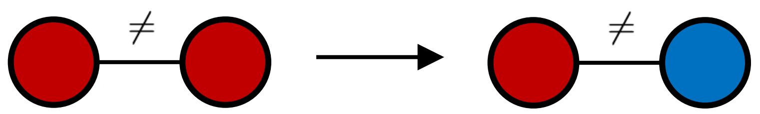
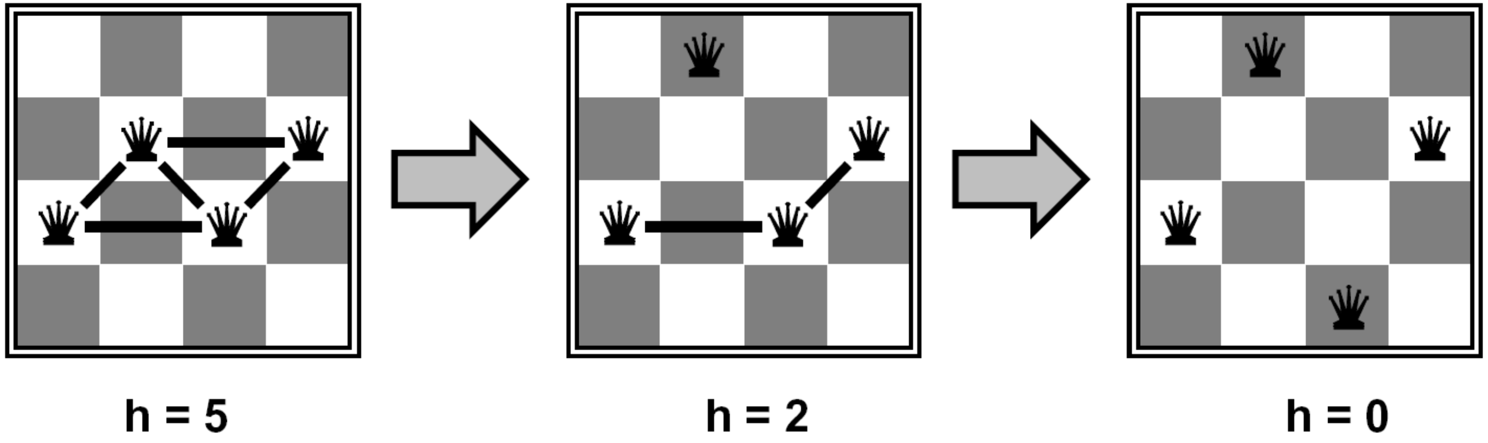


- Algorithm: While not solved,
  - Variable selection: randomly select any conflicted variable
  - Value selection: min-conflicts heuristic:
    - Choose a value that violates the fewest constraints
    - I.e., hill climb with h(n) = total number of violated constraints

Example: 4-Queens



- States: 4 queens in 4 columns (4^4 = 256 states)
- Operators: move queen in column
- Goal test: no attacks
- Evaluation: c(n) = number of attacks

[Demo: n-queens – iterative improvement (L5D1)]
[Demo: coloring – iterative improvement]

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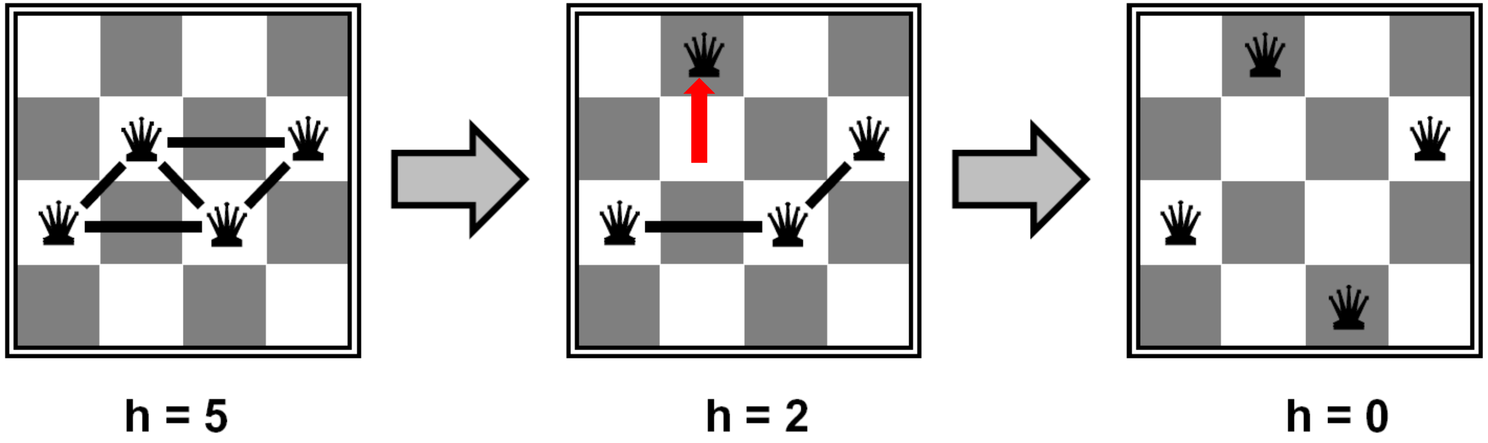

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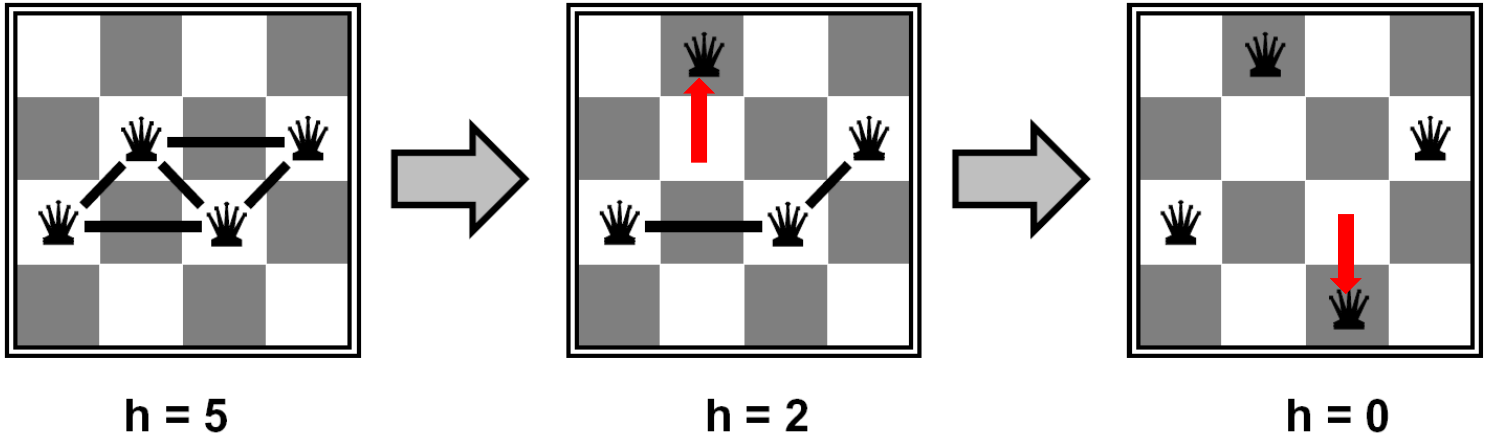

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Basic Local Search Algorithm

Assign one domain value d_i to each variable v_i

while no solution & not stuck & not timed out:

  bestCost $\leftarrow \infty$;   bestList $\leftarrow$ [ ];

  for each variable v_i where Cost(Value(v_i)) > 0

    for each domain value d_i of v_i

      if Cost(d_i) < bestCost

        bestCost $\leftarrow$ Cost(d_i)

        bestList $\leftarrow$ [d_i]

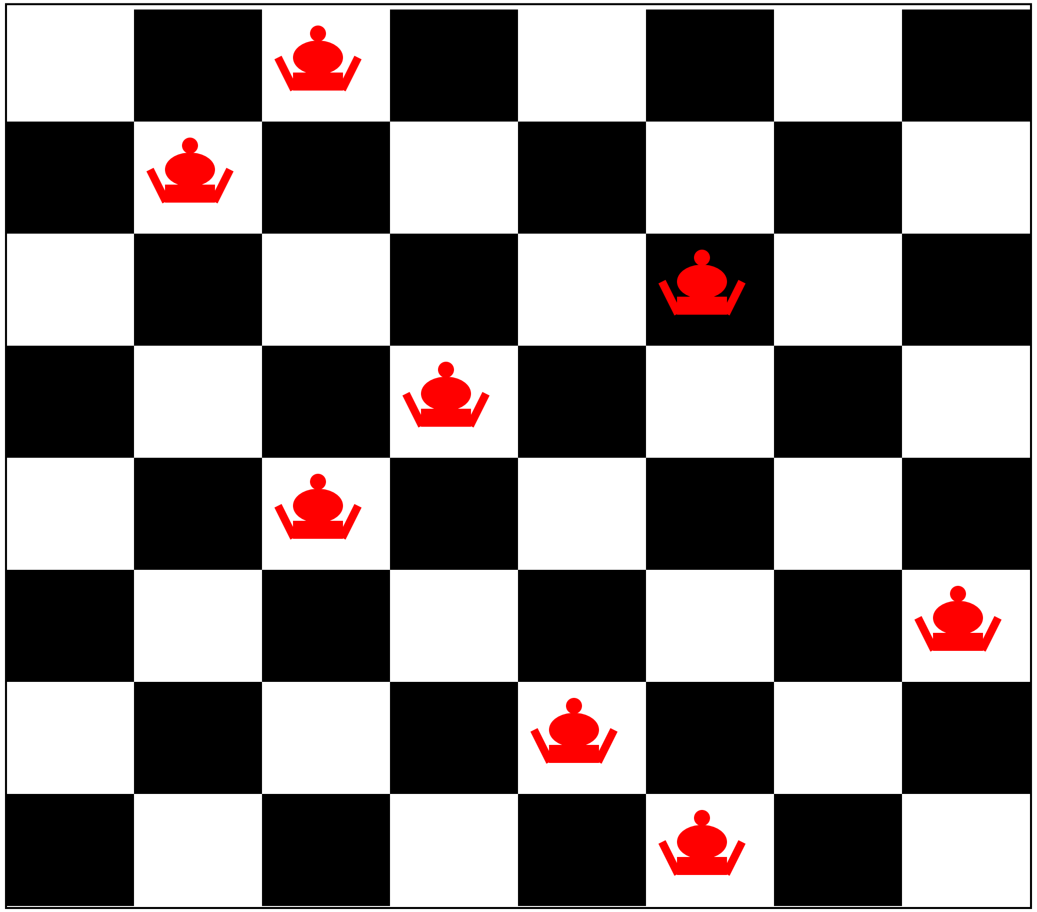
      else if Cost(d_i) = bestCost

        bestList $\leftarrow$ bestList $\cup$ d_i

  Take a randomly selected move from bestList

Eight Queens using Local Search

Place 8 Queens randomly on the board

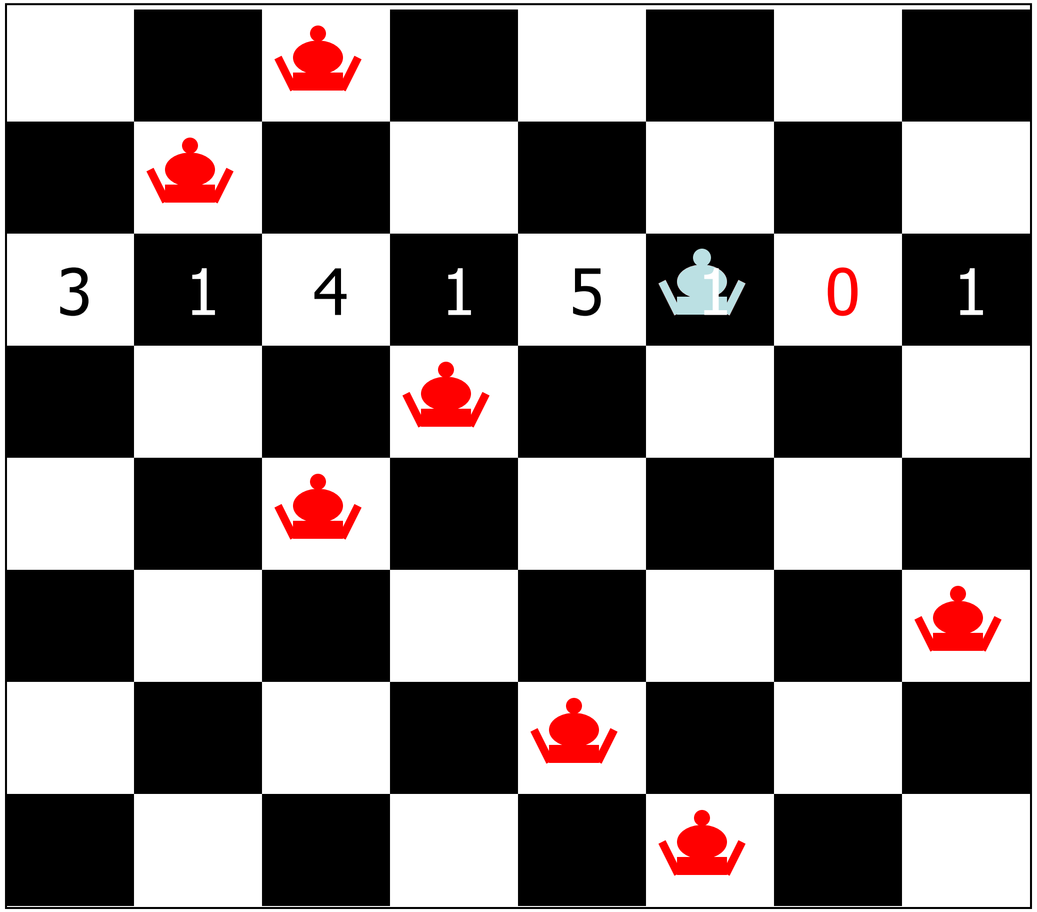


Eight Queens using Local Search

Pick a Queen:

Calculate cost

of each move



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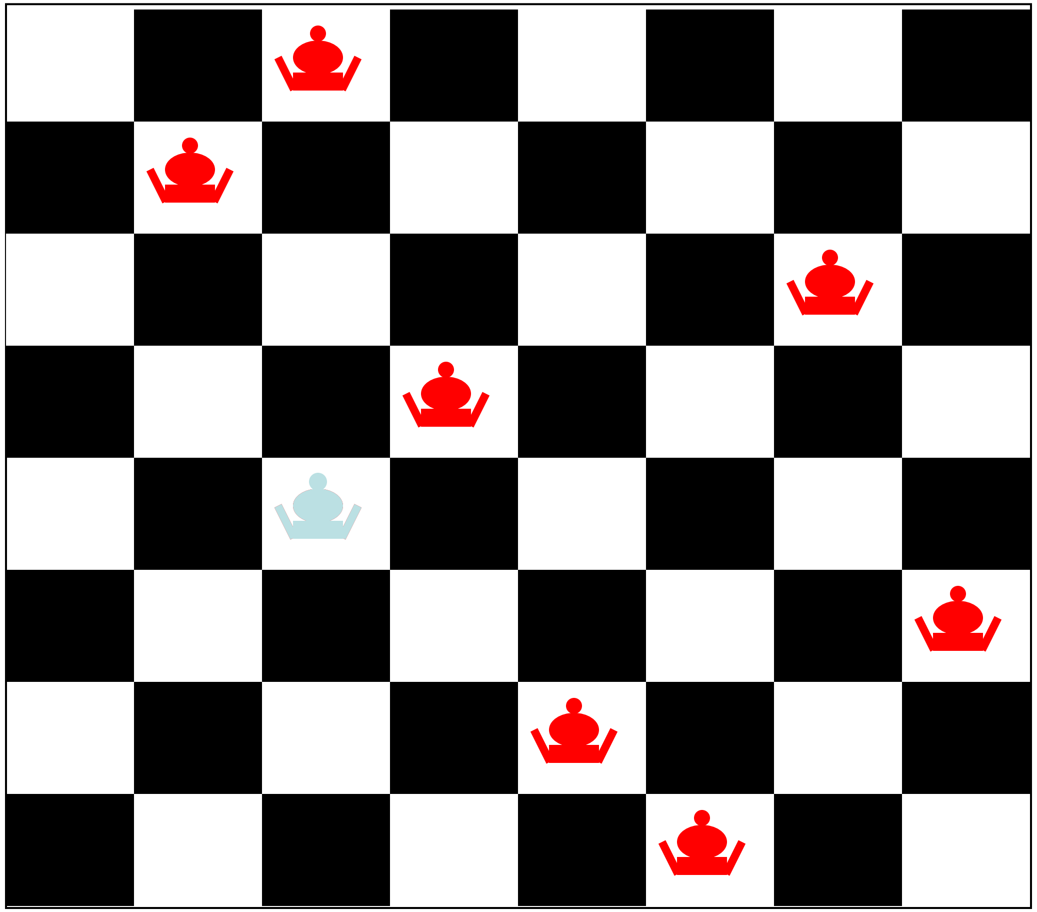
Eight Queens using Local Search

Take least cost move then try another Queen



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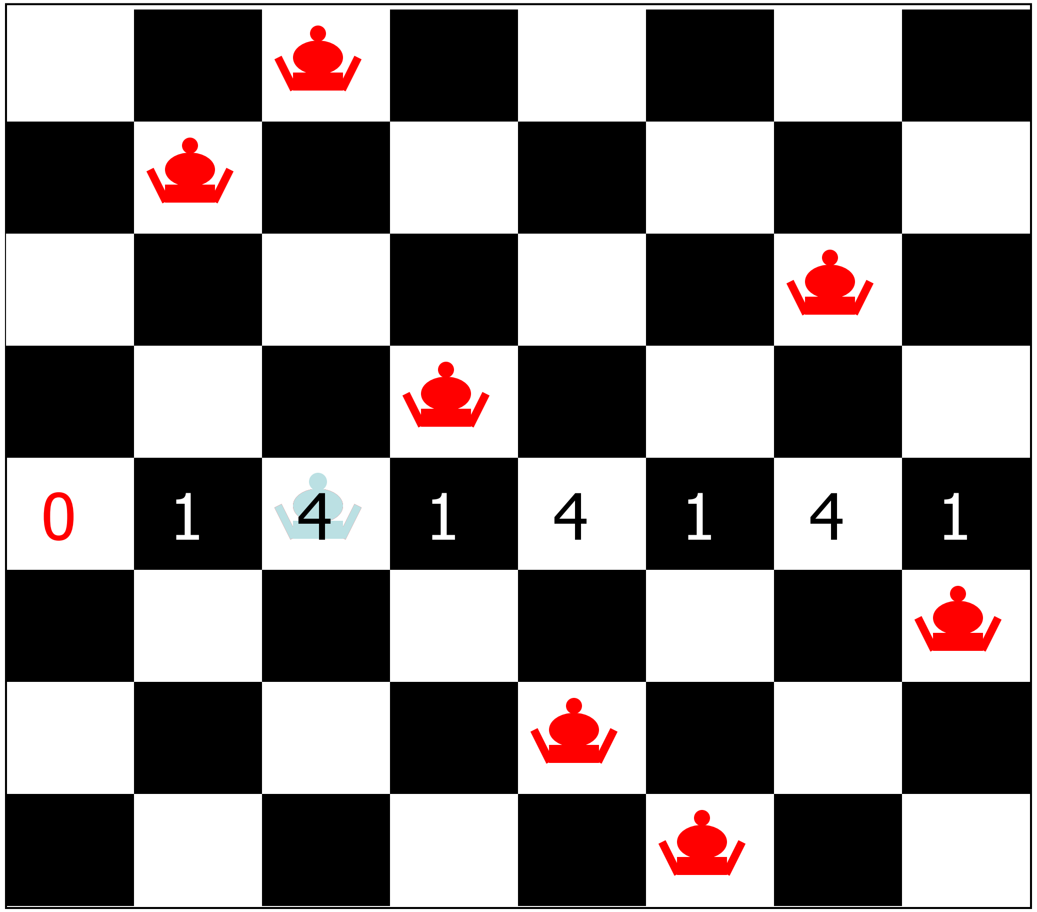

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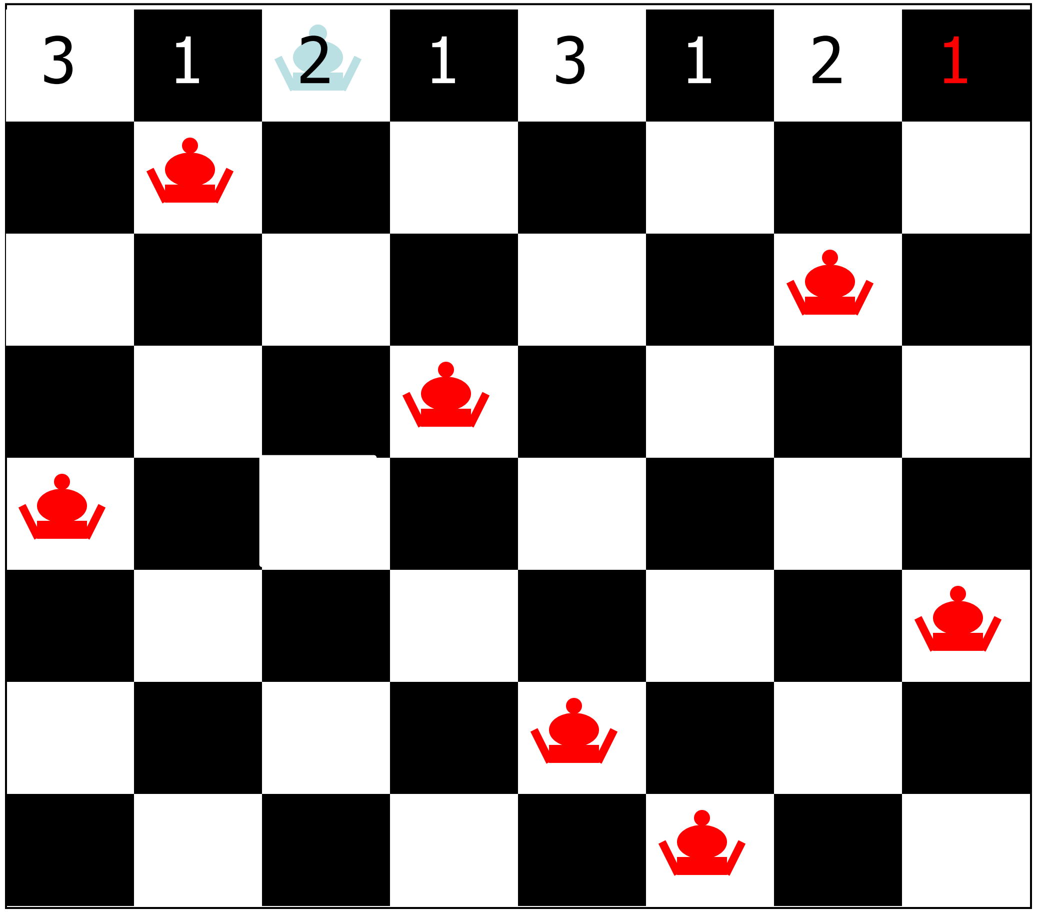

…and so on, until….

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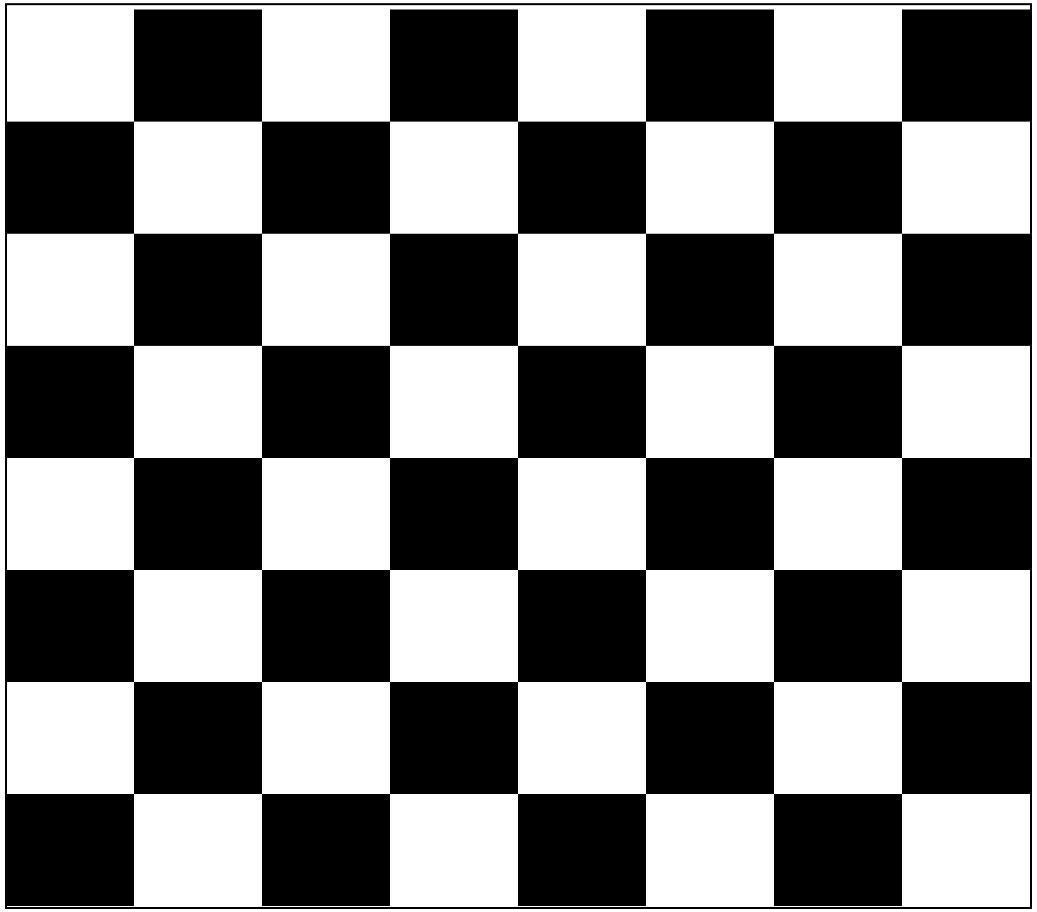
Eight Queens using Local Search



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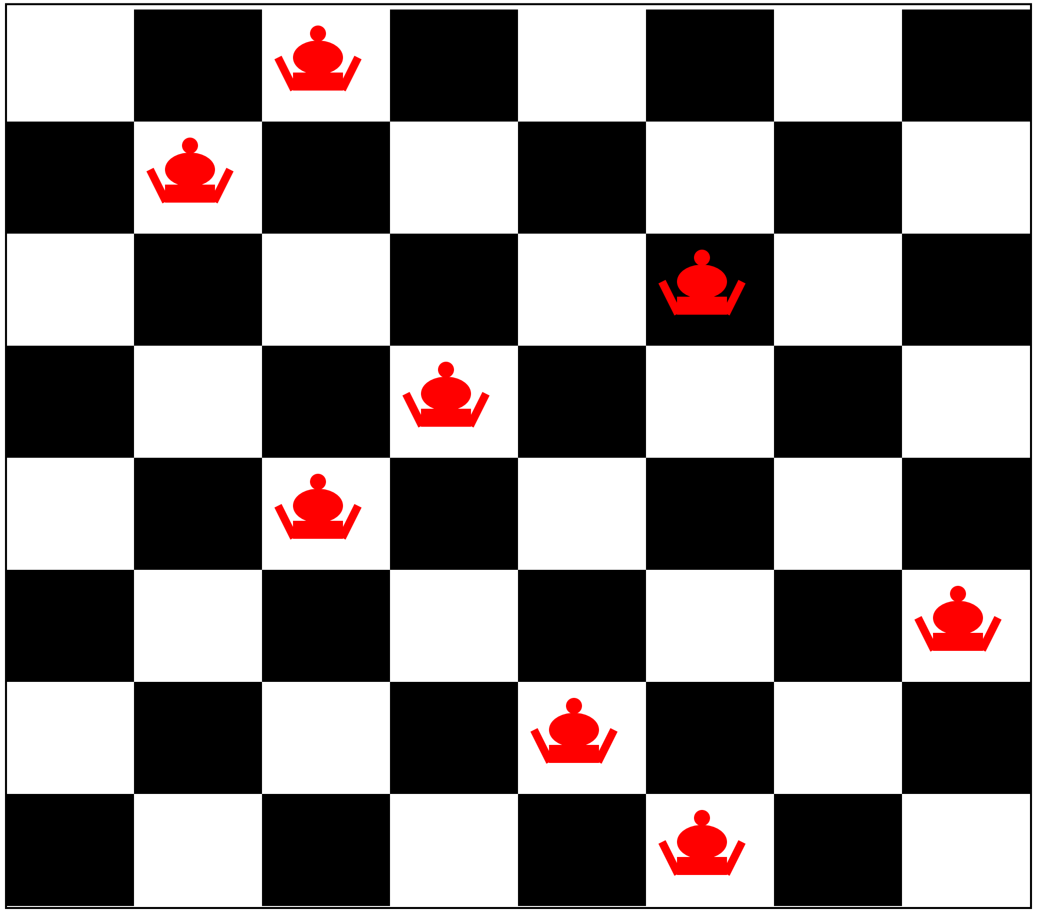

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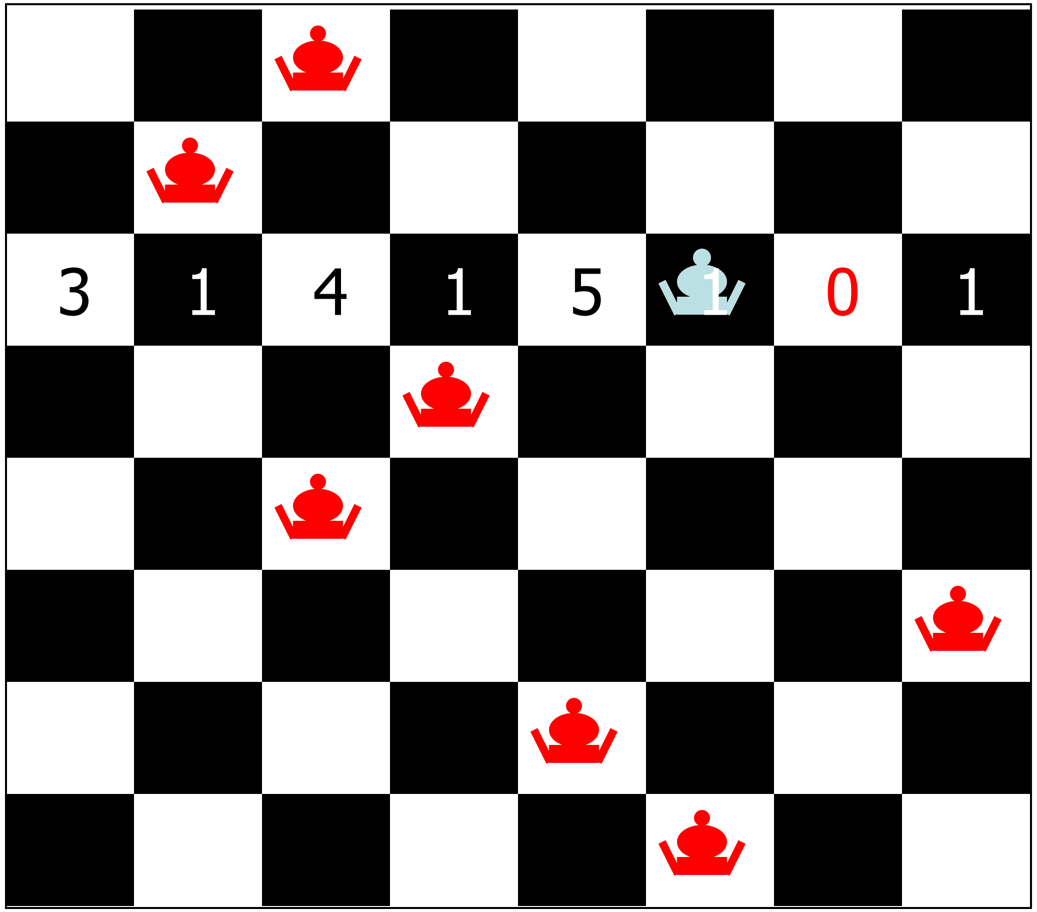

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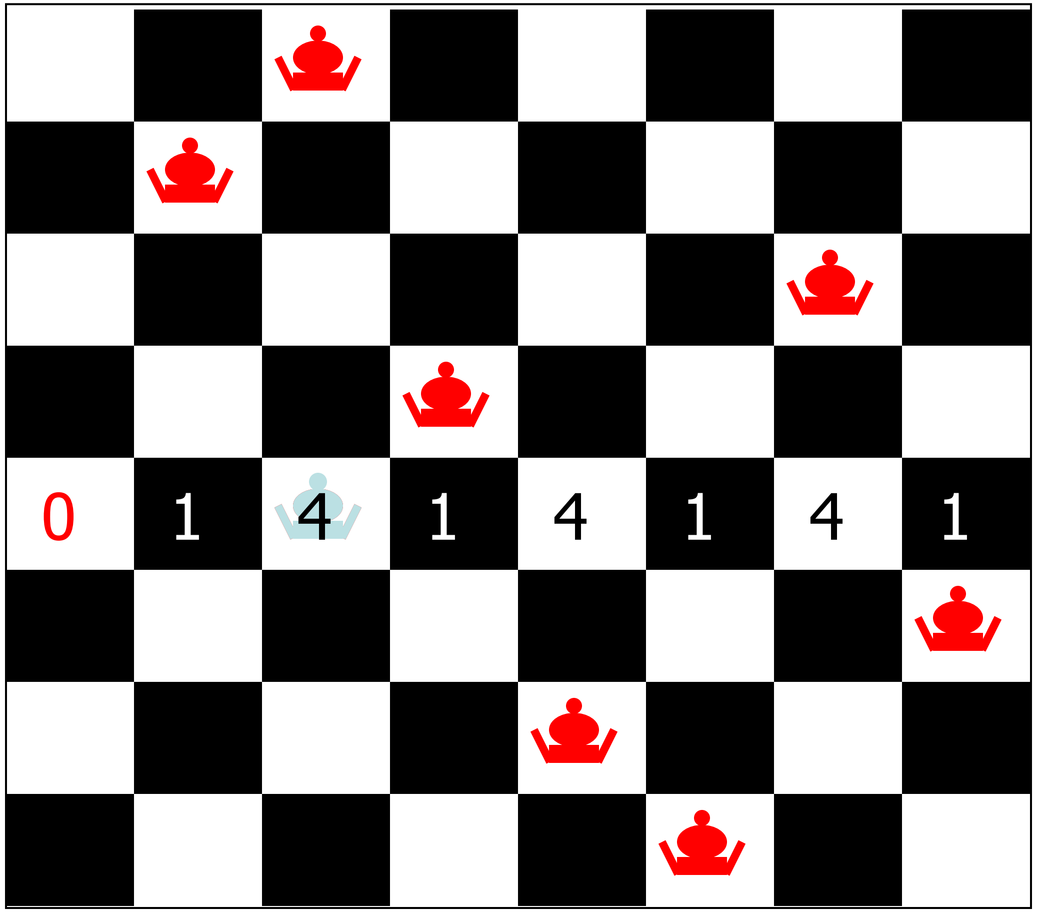

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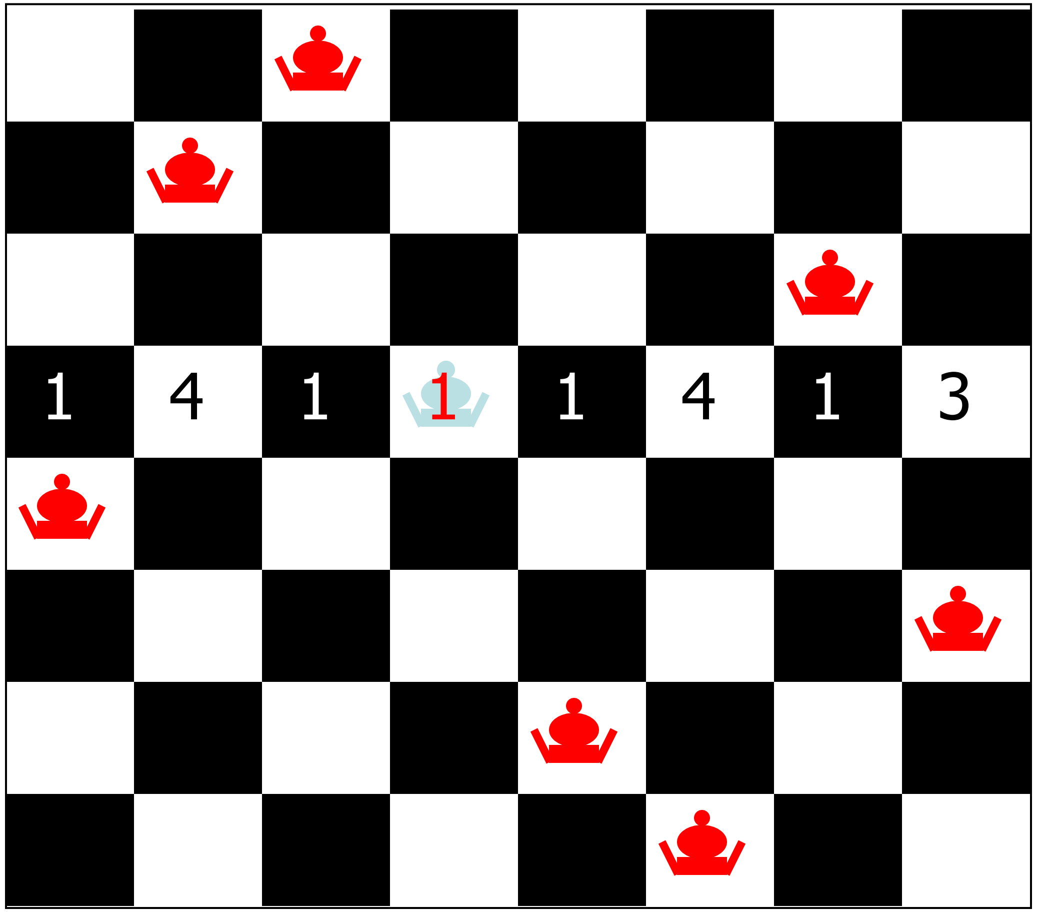

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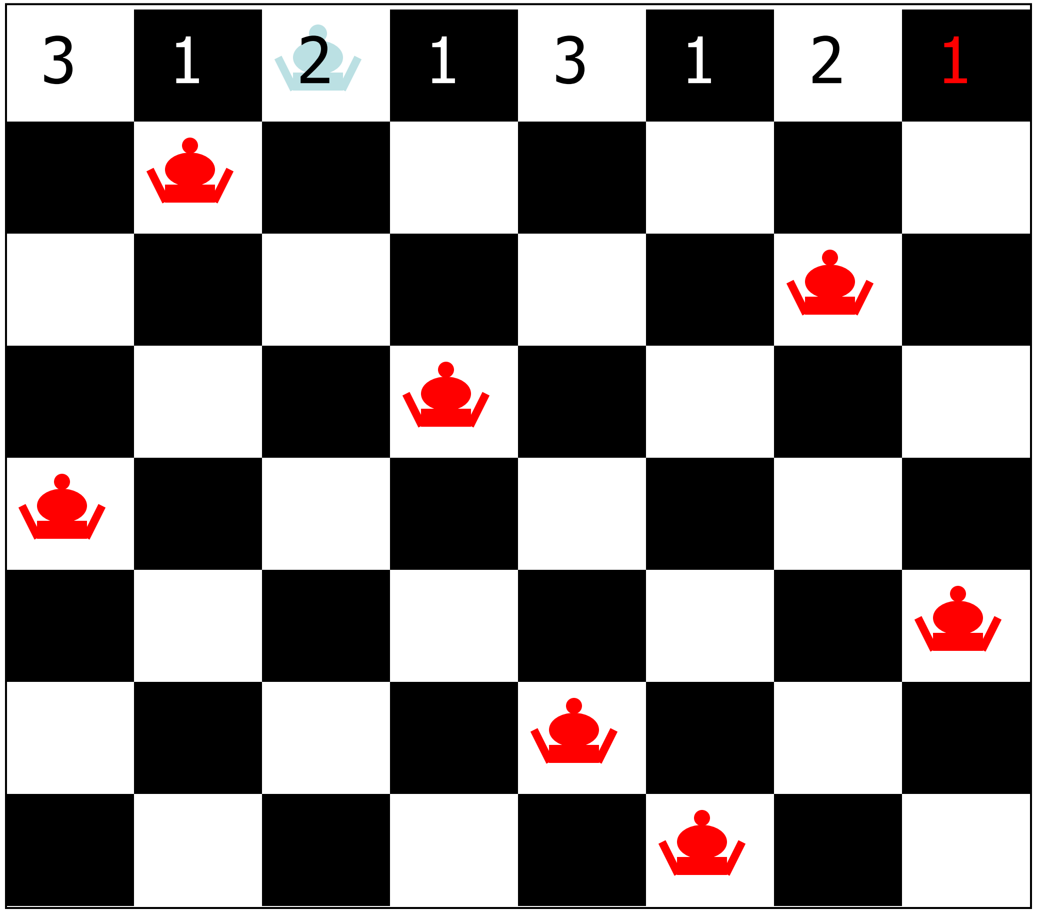

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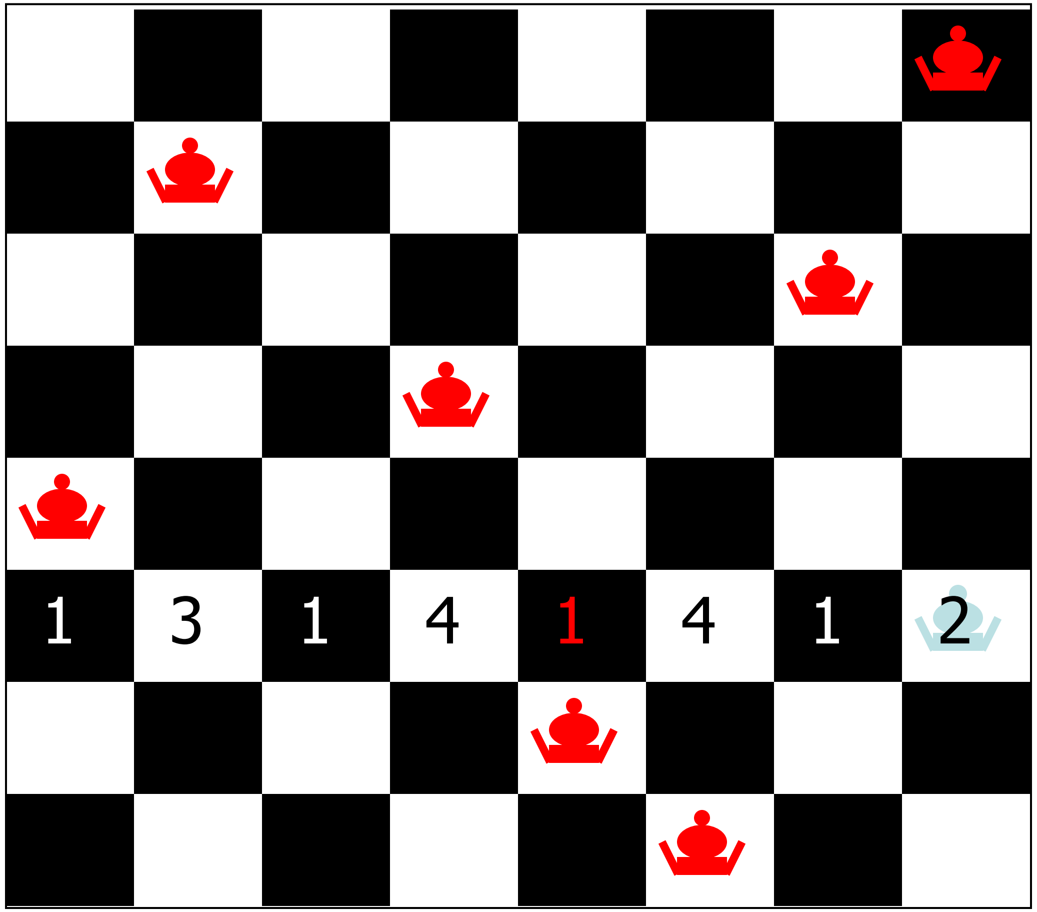

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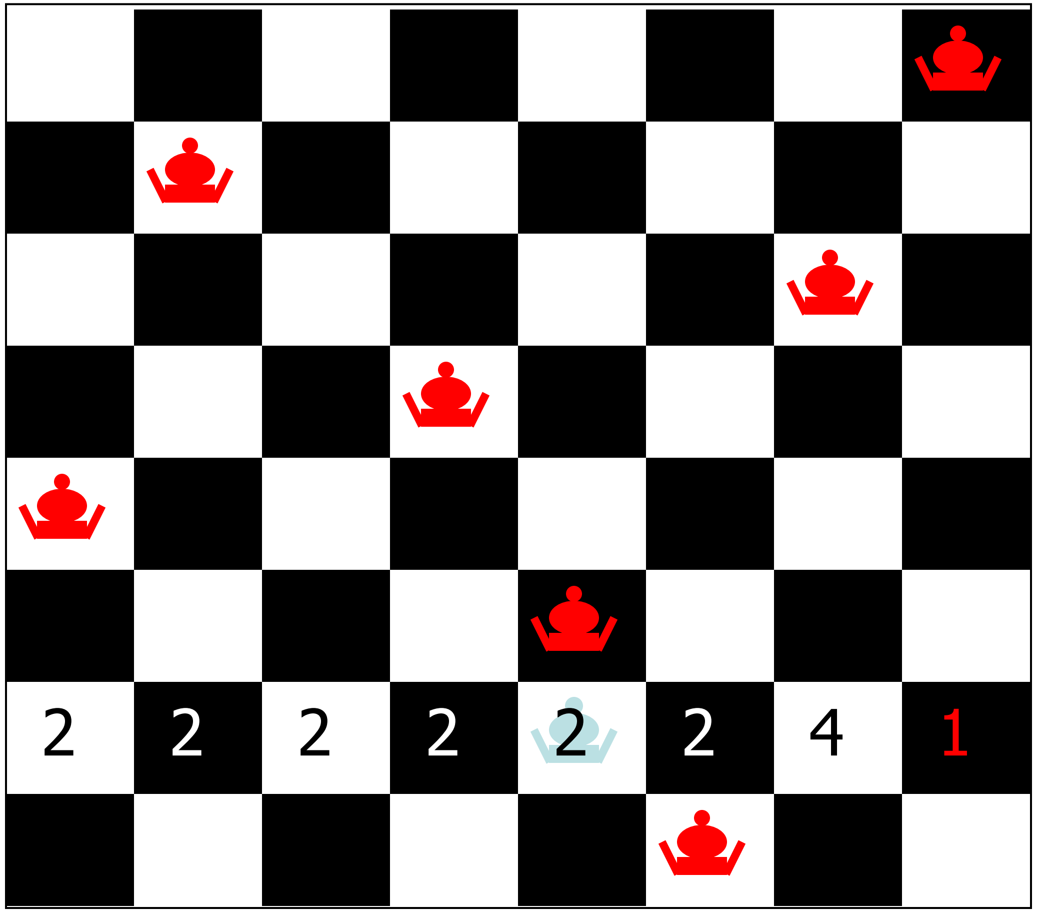

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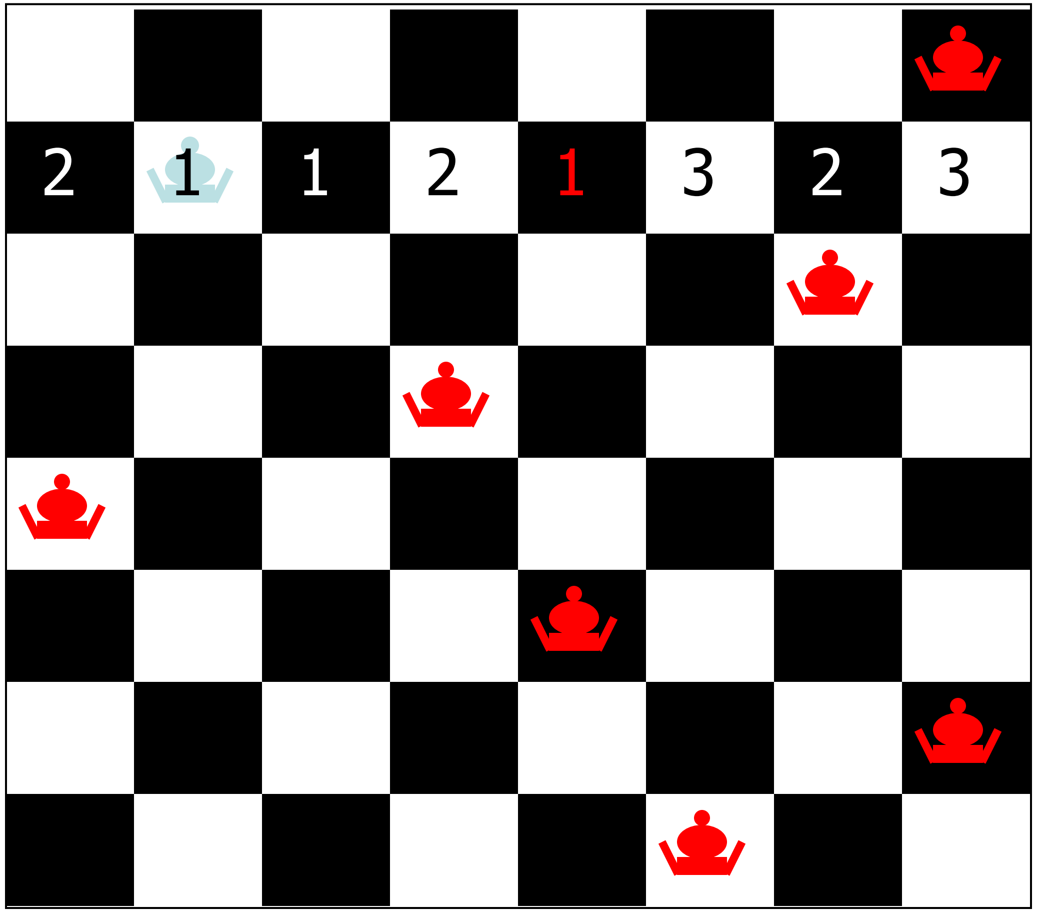

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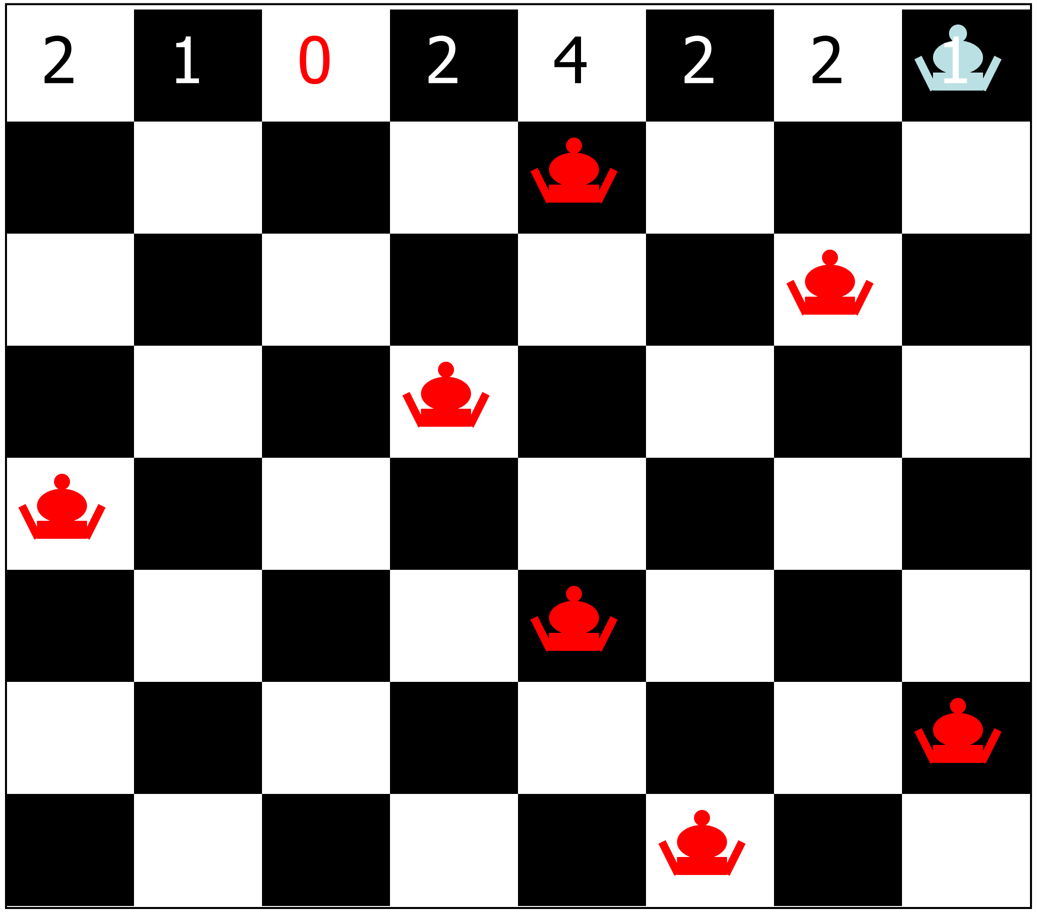

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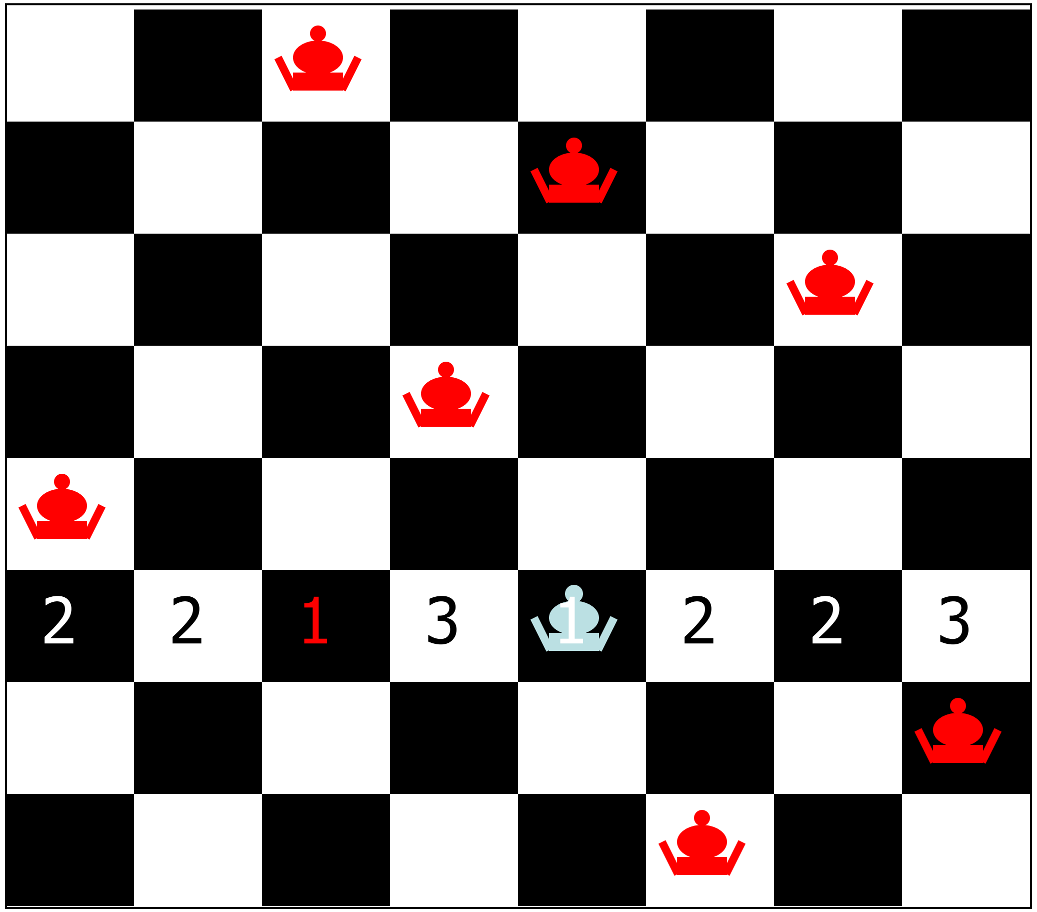

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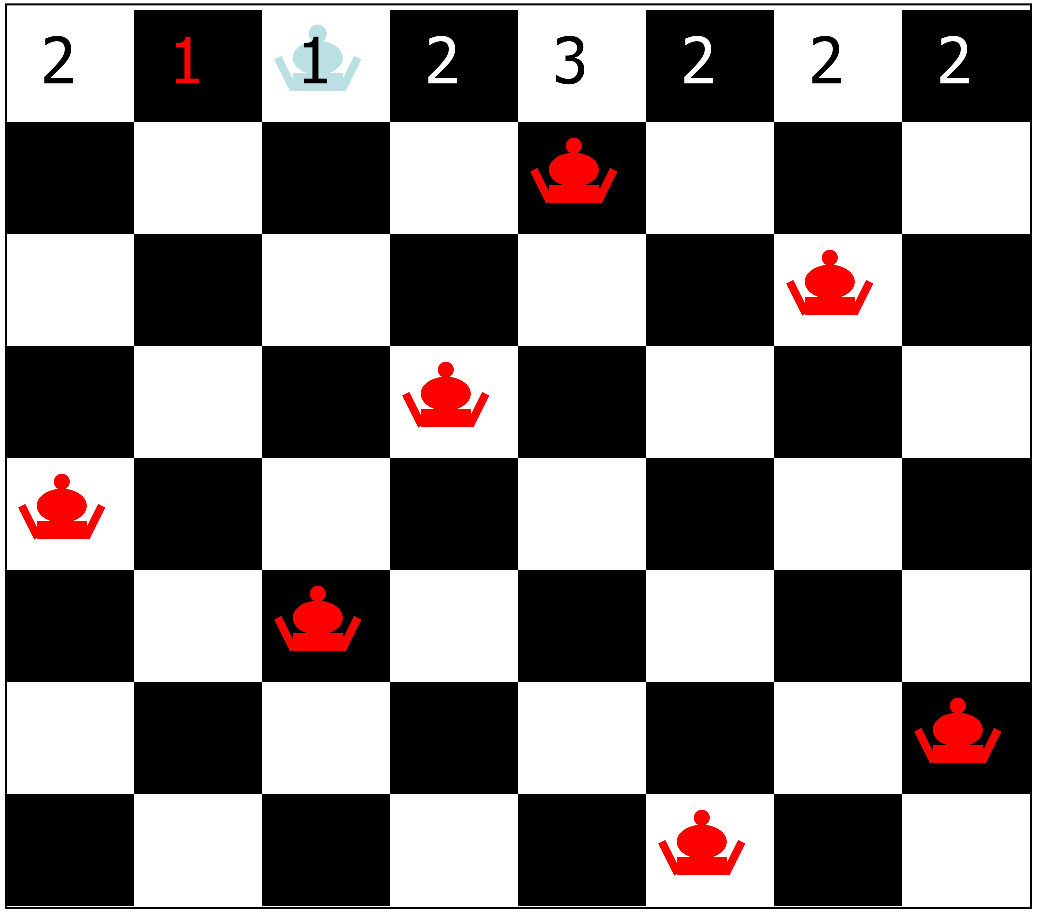

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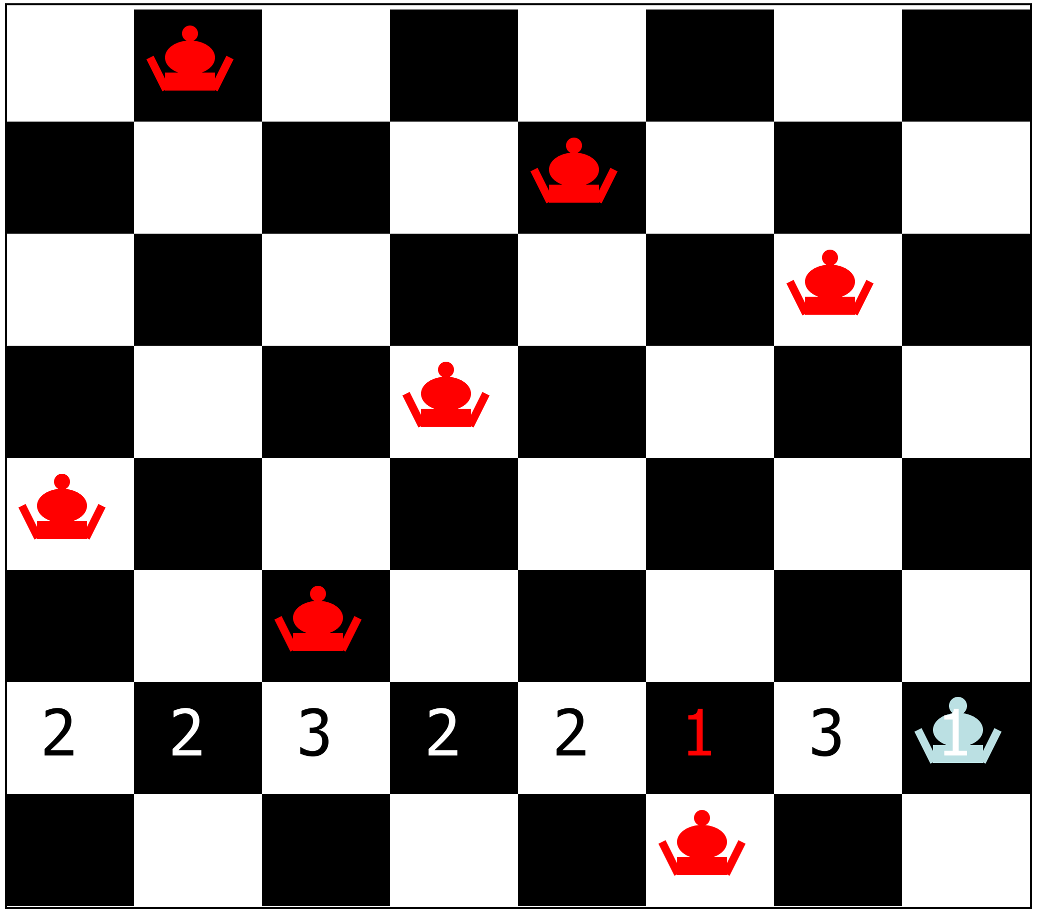

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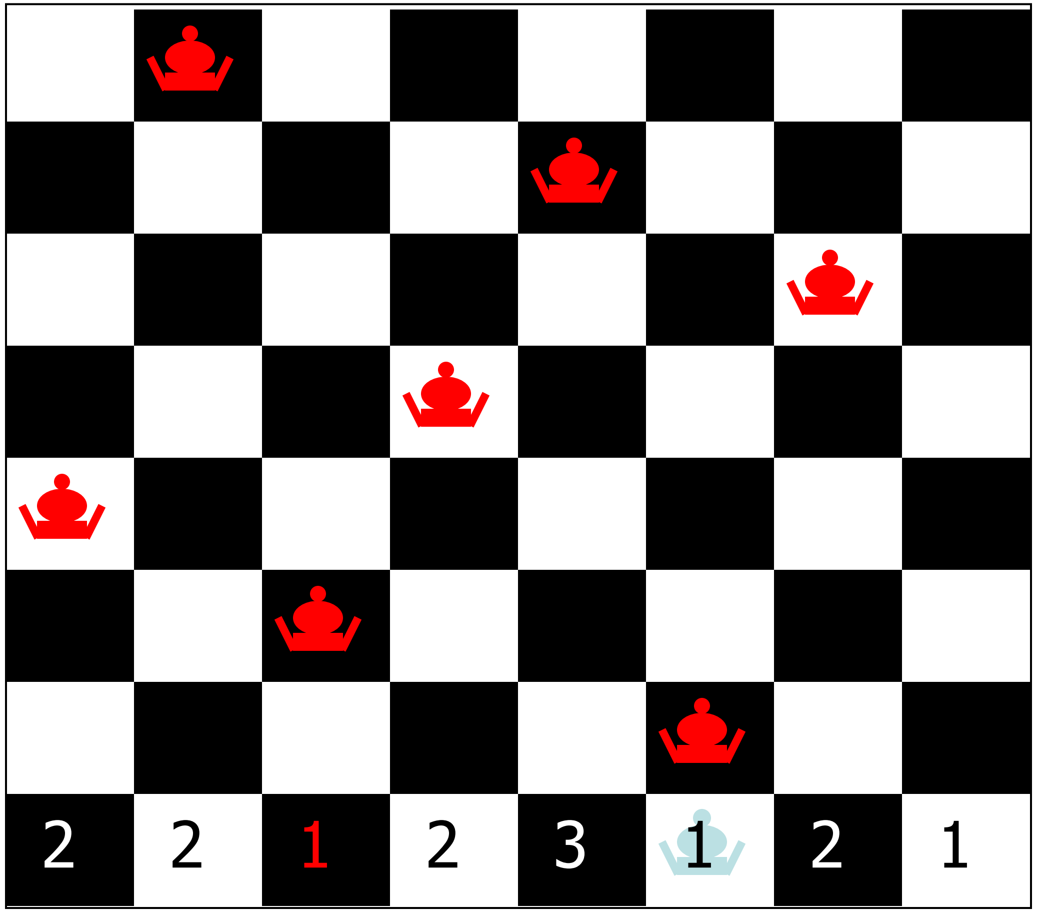

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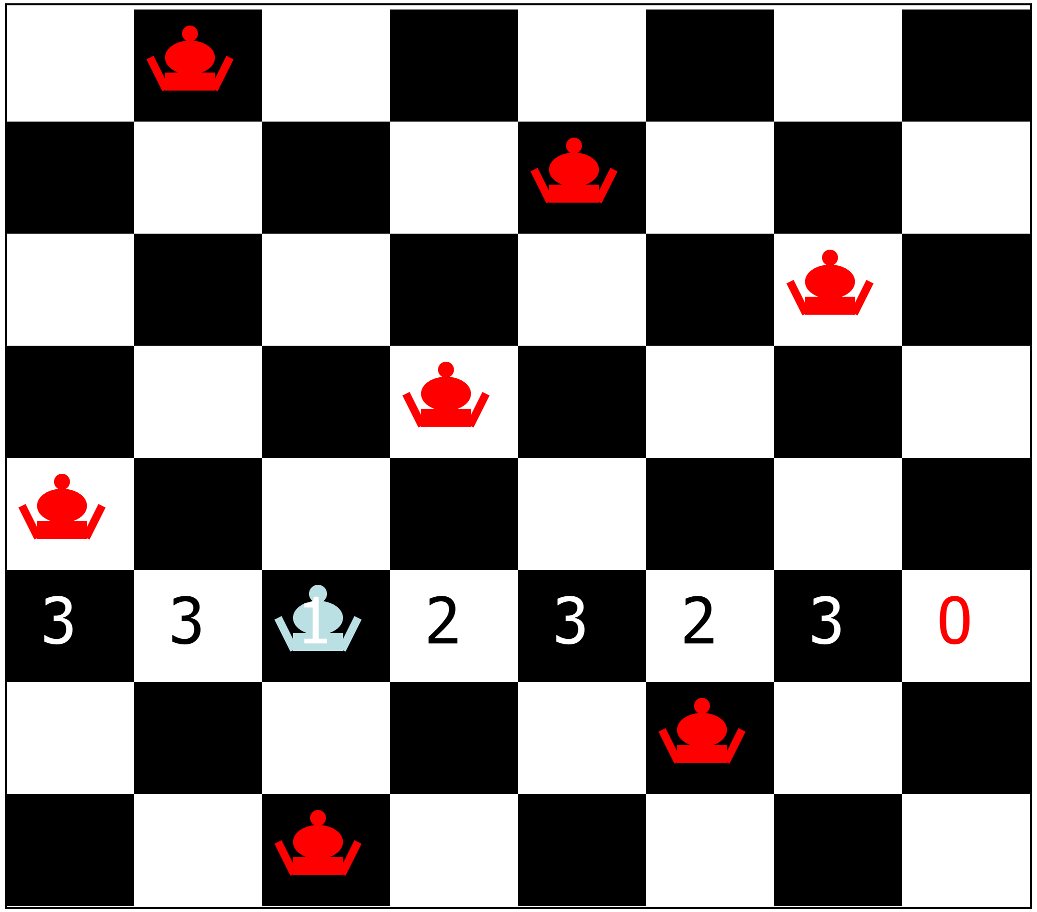

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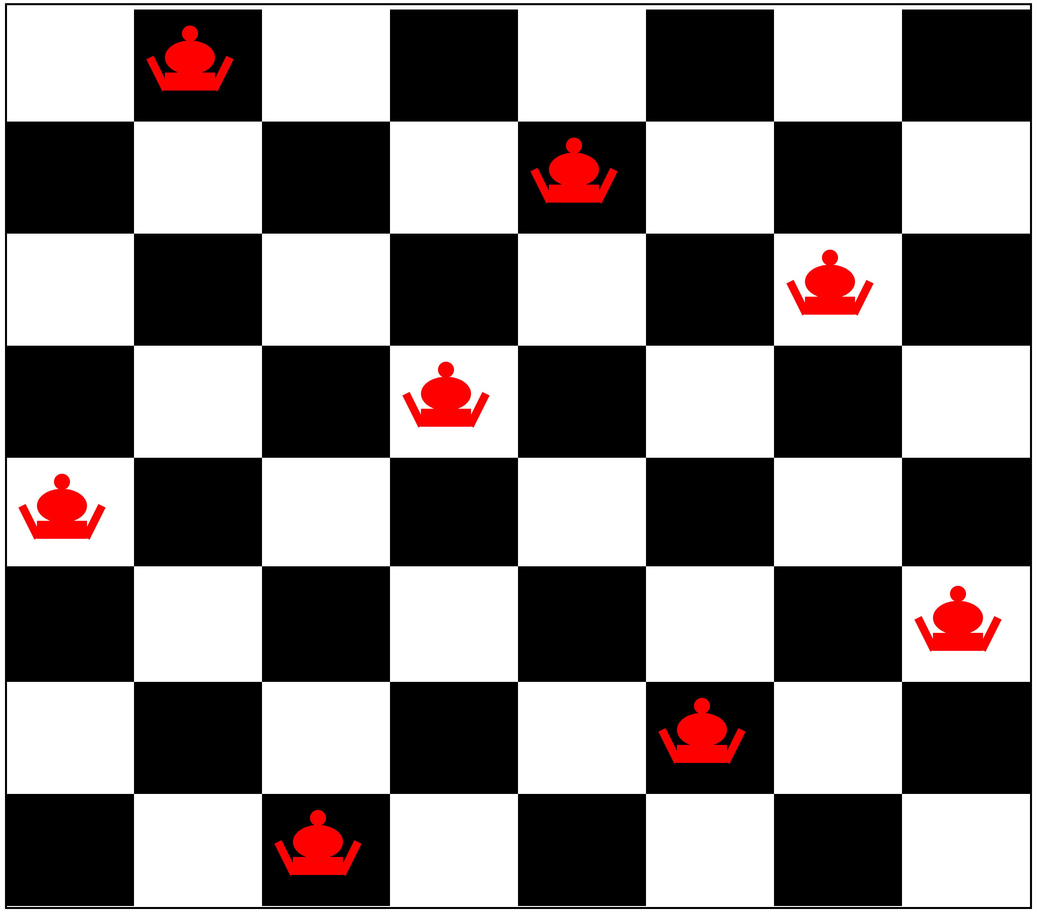
Eight Queens using Local Search

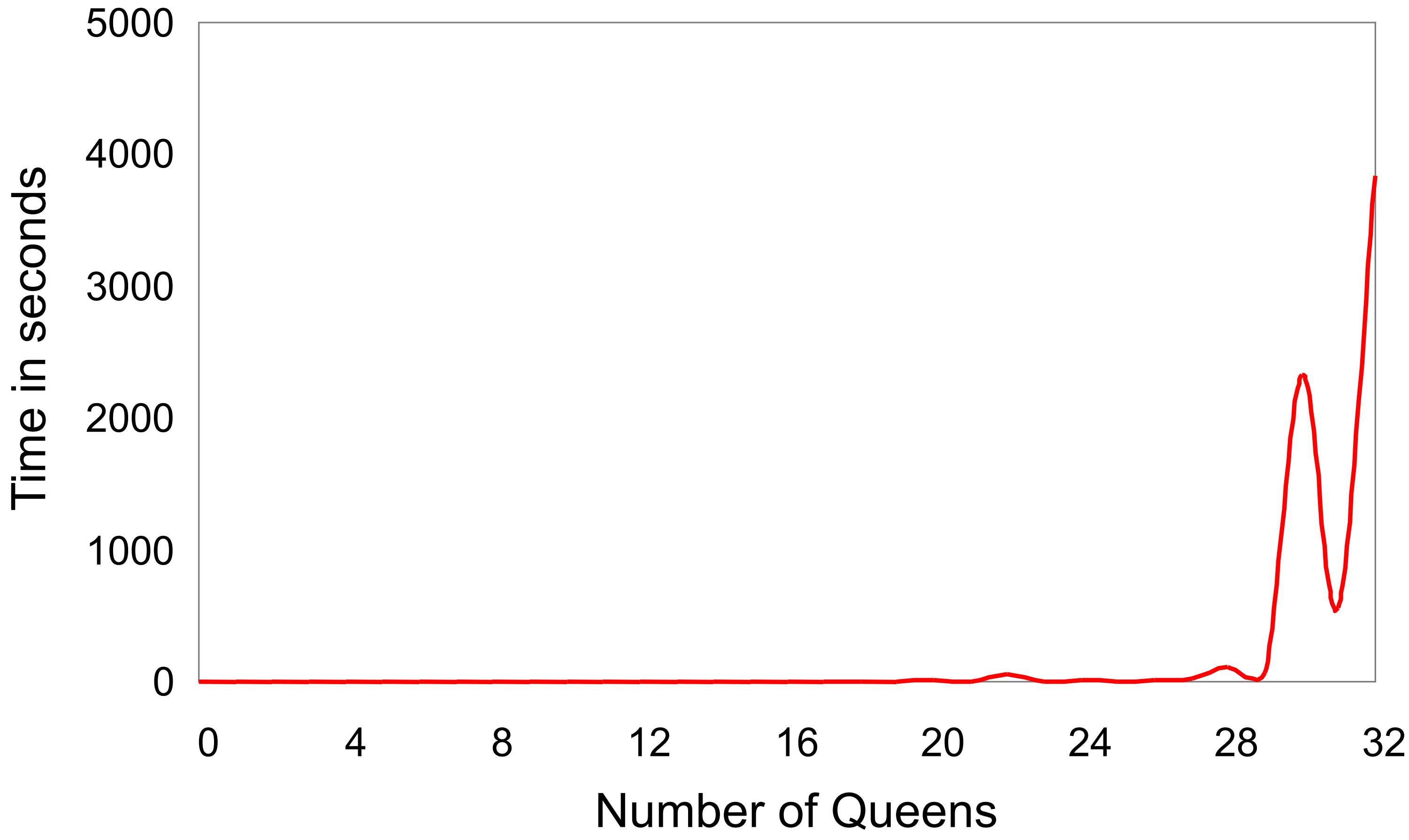
Answer Found



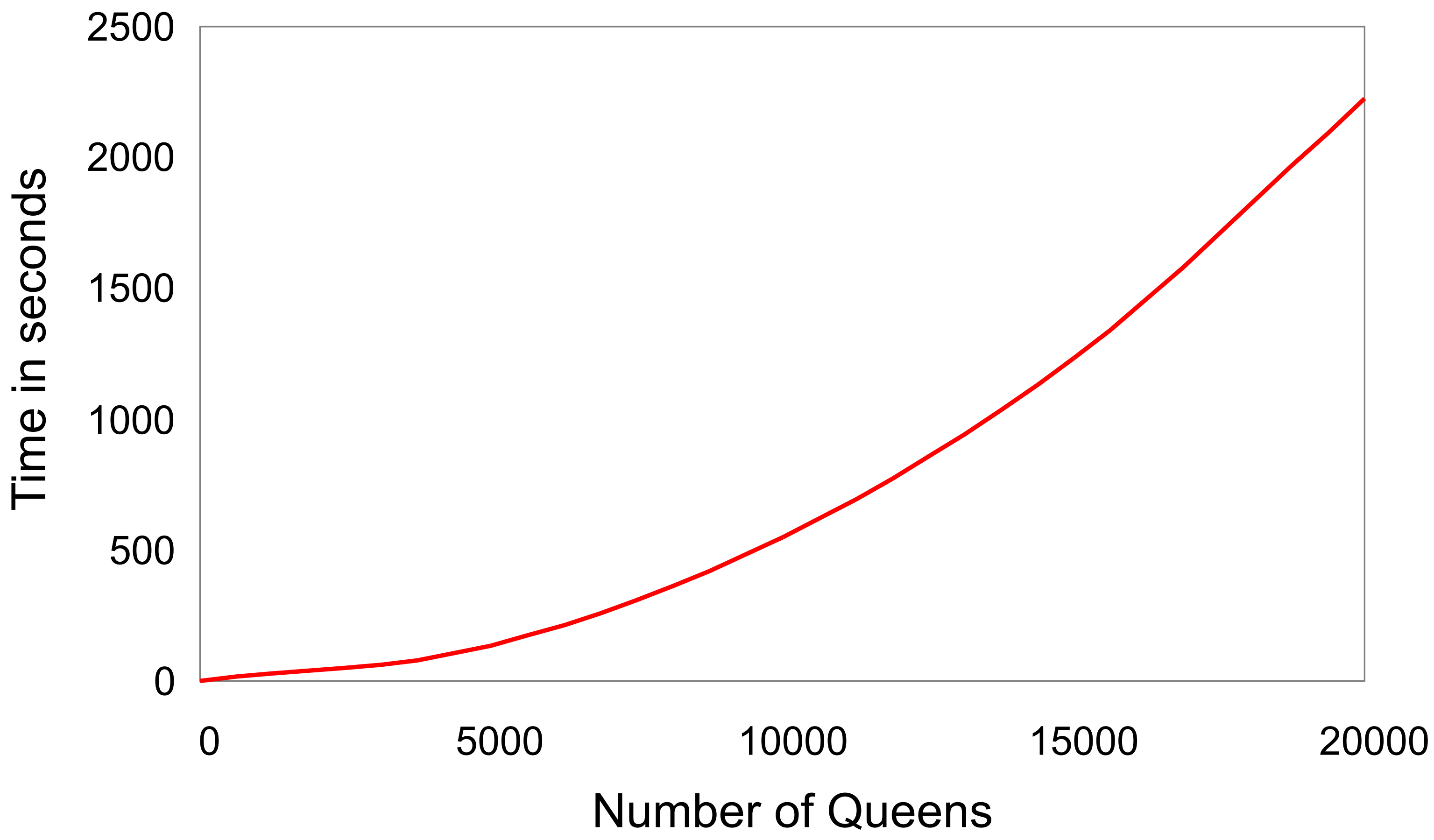
Slide

Video of Demo Iterative Improvement – Coloring

Backtracking Performance



Local Search Performance



Performance of Min-Conflicts

- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000)

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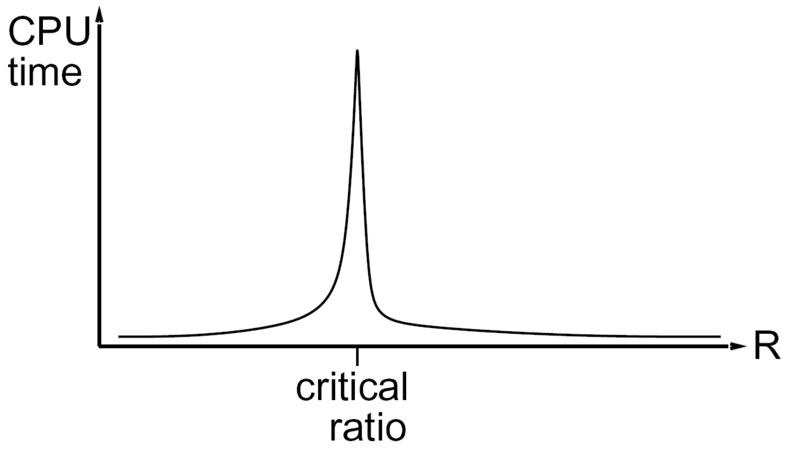

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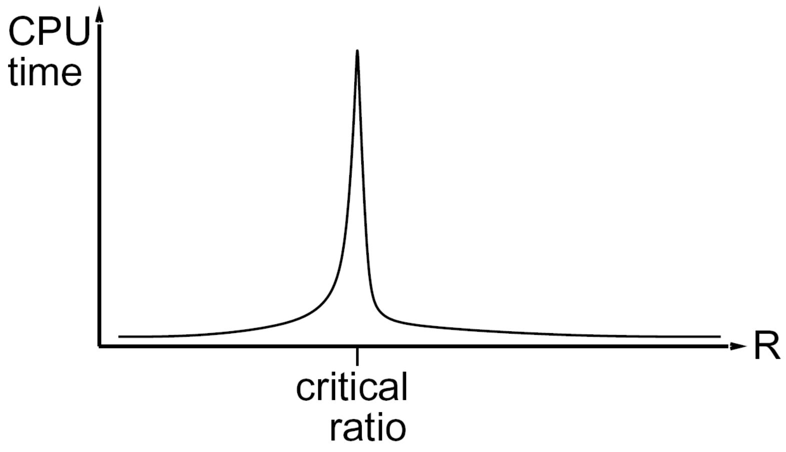

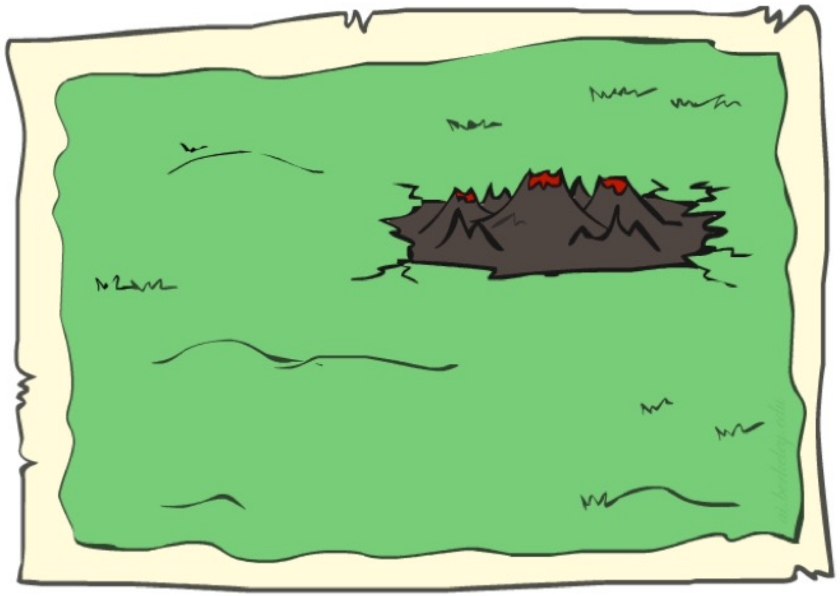

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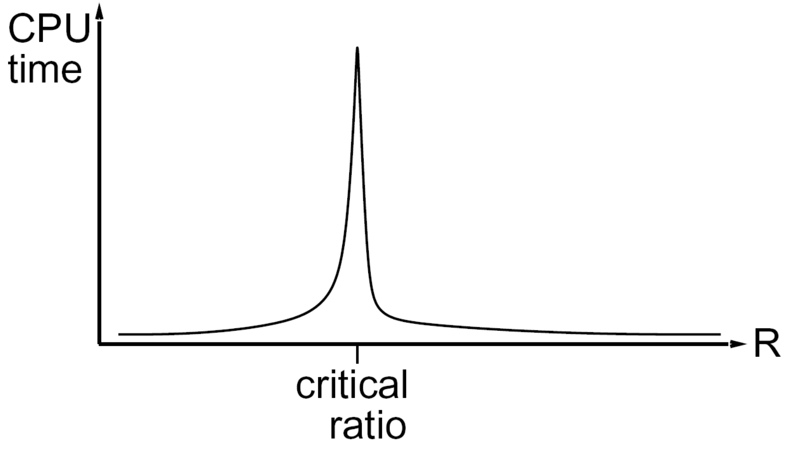

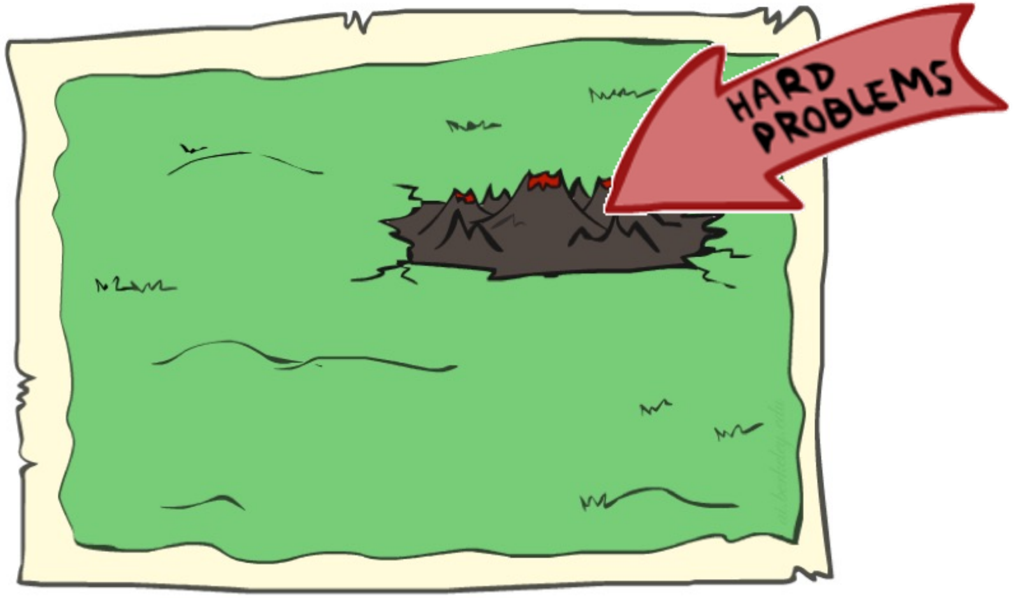

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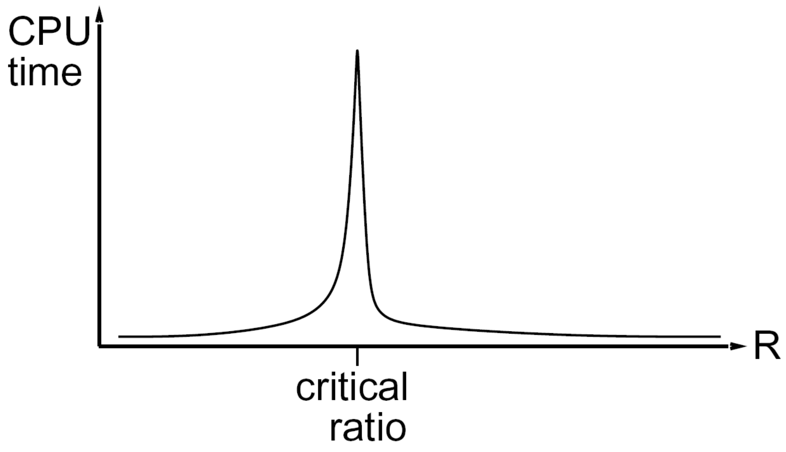
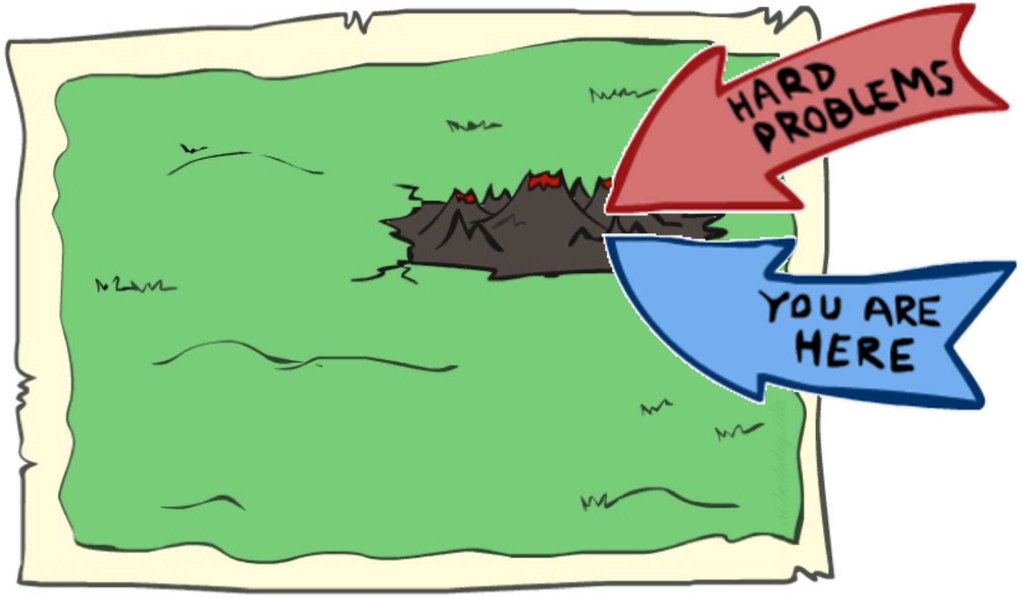
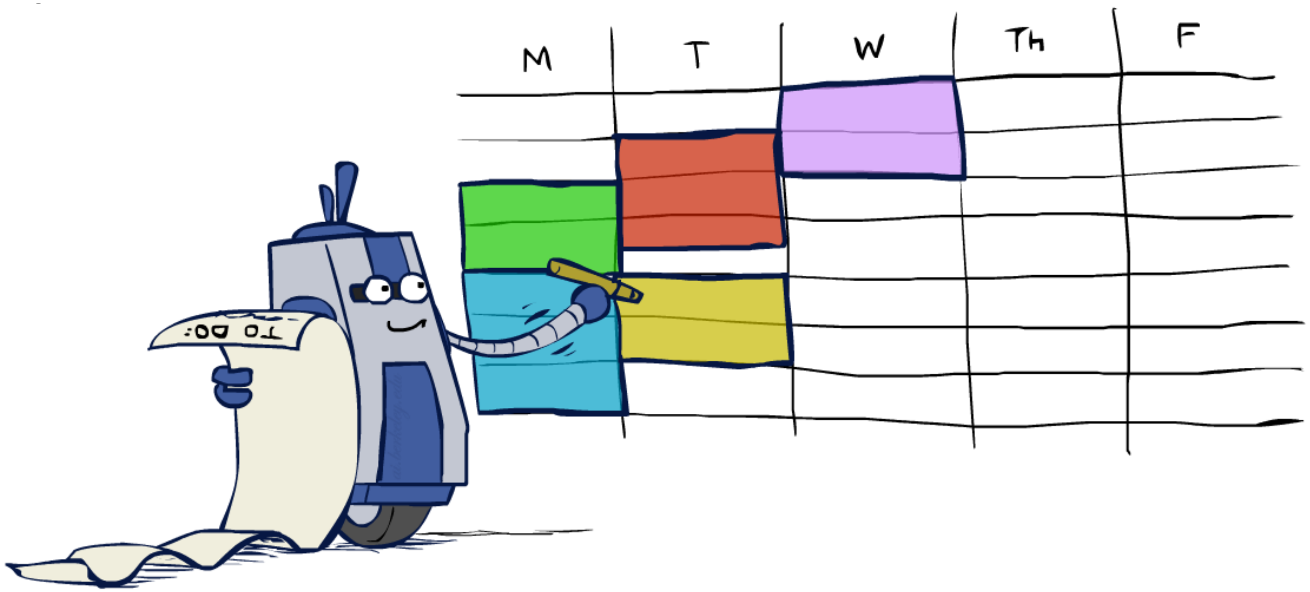




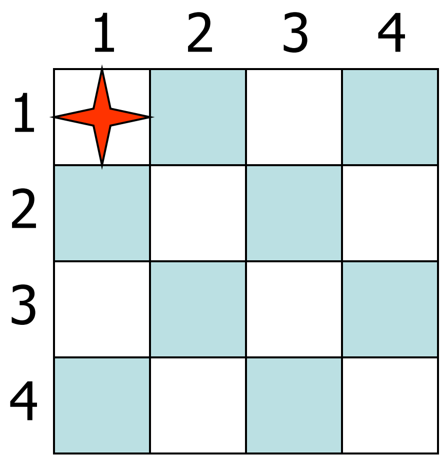
Summary: CSPs

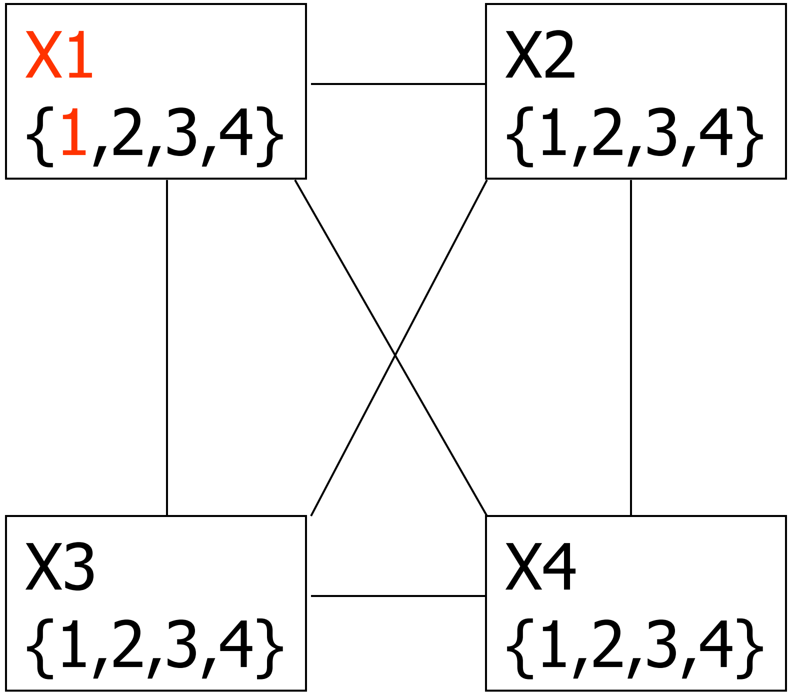
- CSPs are a special kind of search problem:
  - States are partial assignments
  - Goal test defined by constrai
- Basic solution: backtracking sea
- Speed-ups:
  - Ordering
  - Filtering
  - Structure
- Iterative min-conflicts is often effective in practice



More Examples

4-Queens Problem





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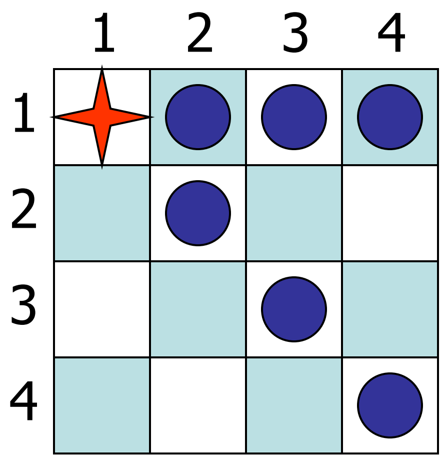

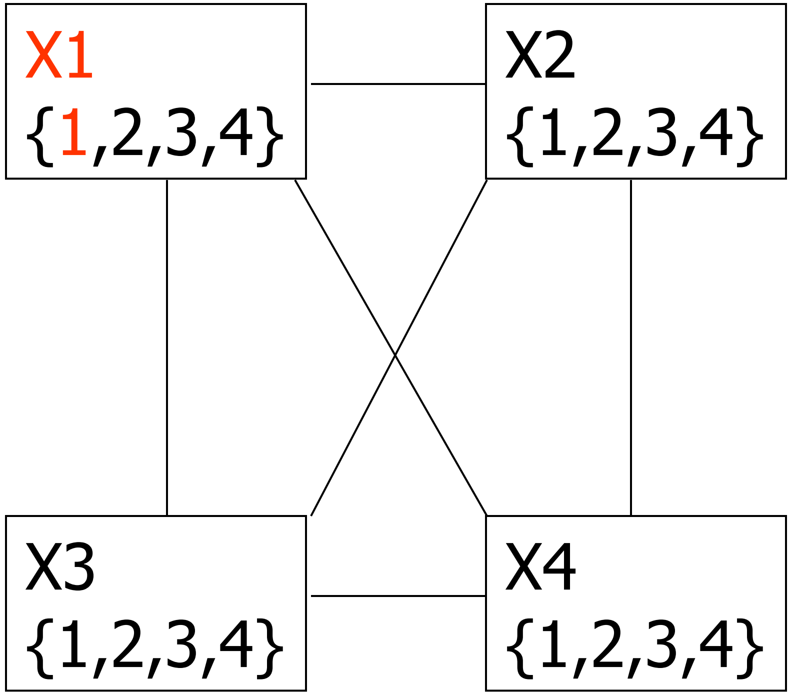

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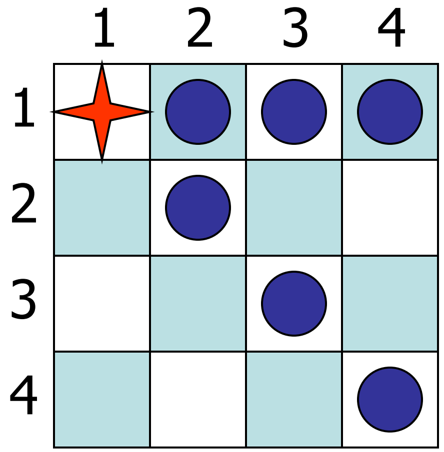

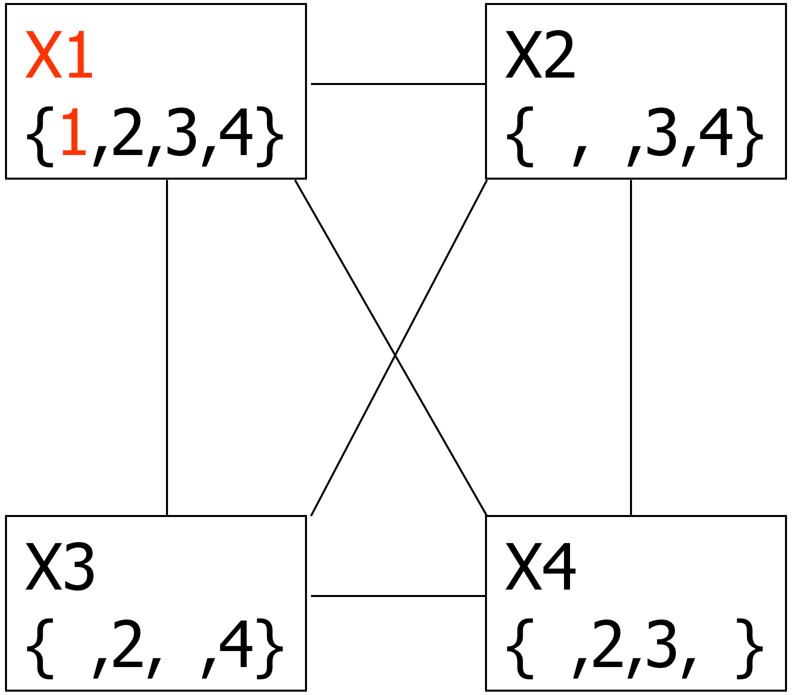

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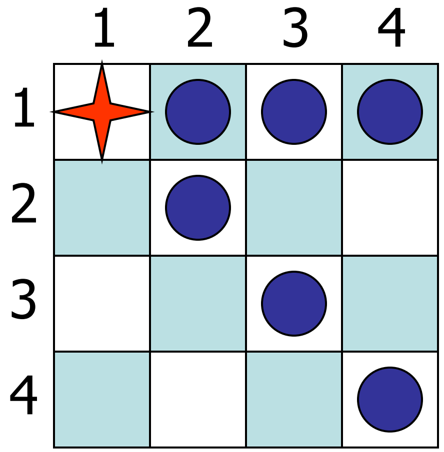

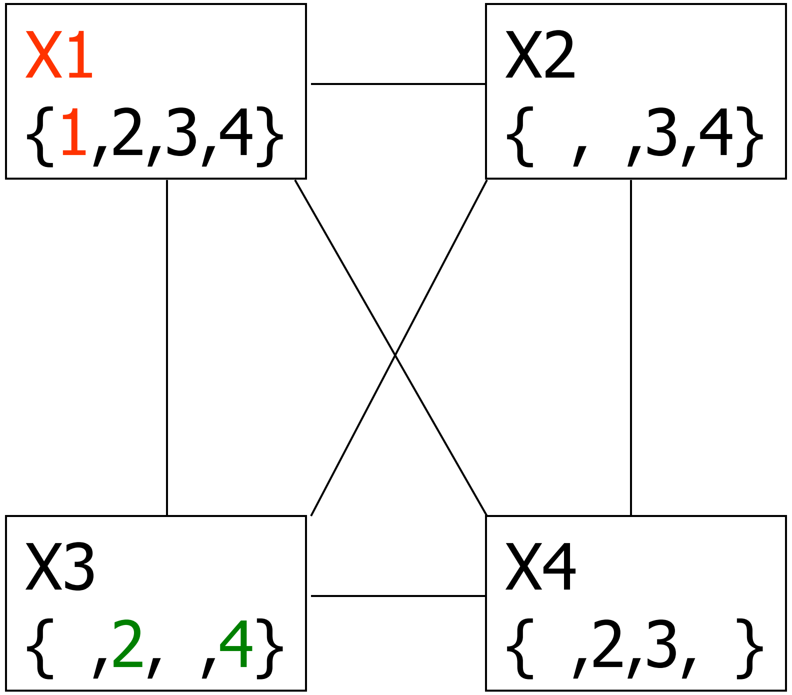

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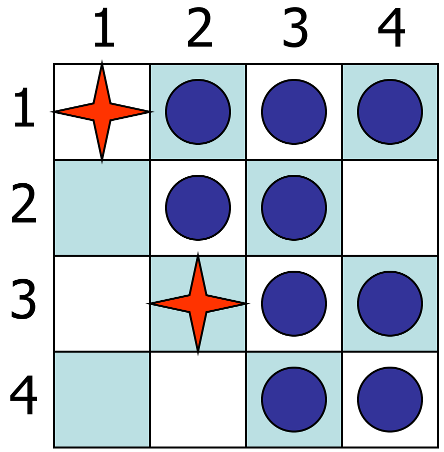

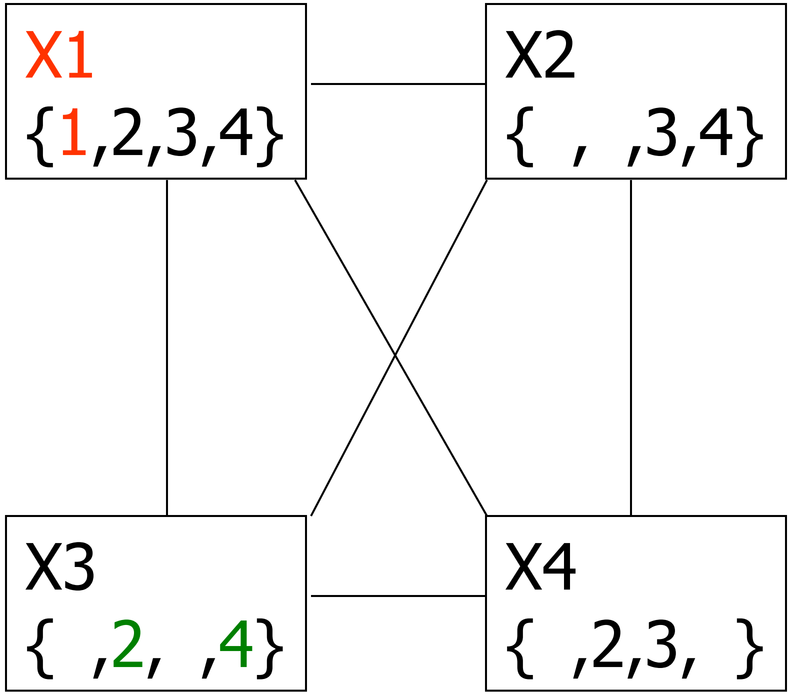

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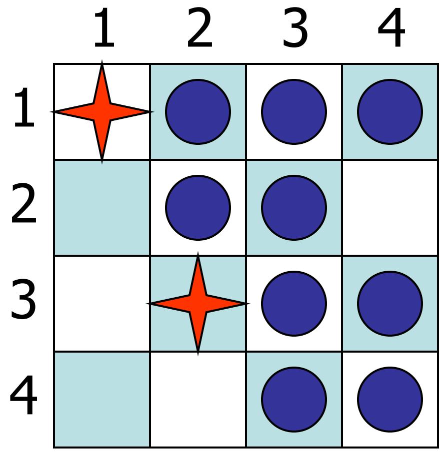

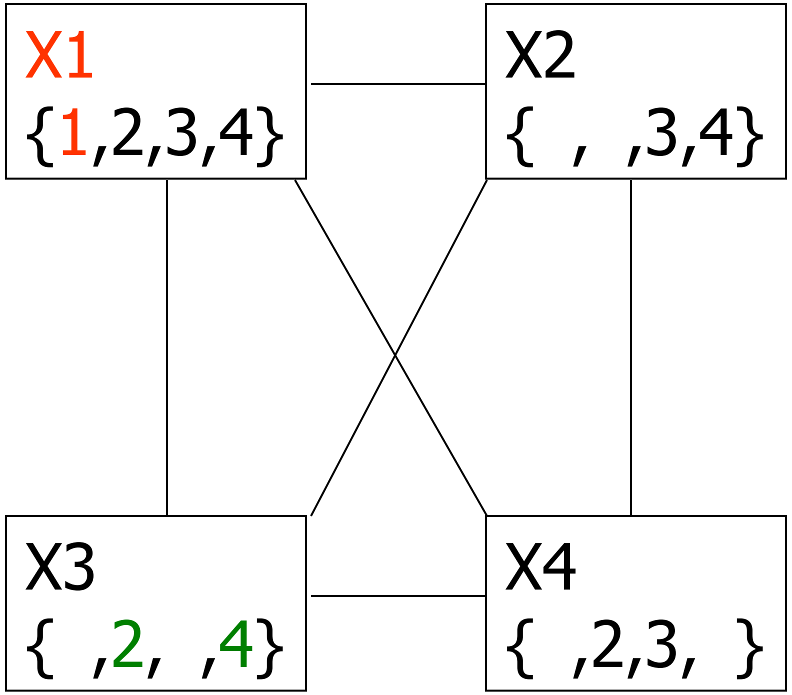

X2=3 eliminates { X3=2, X3=3, X3=4 }
⇒ inconsistent!

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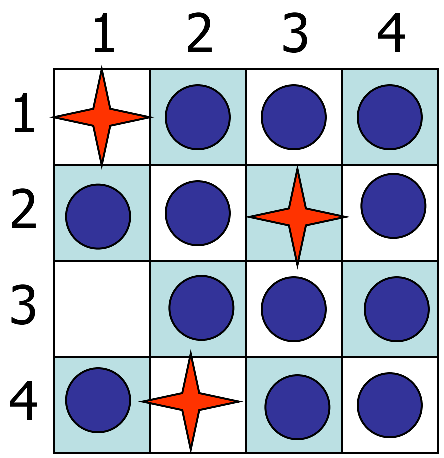

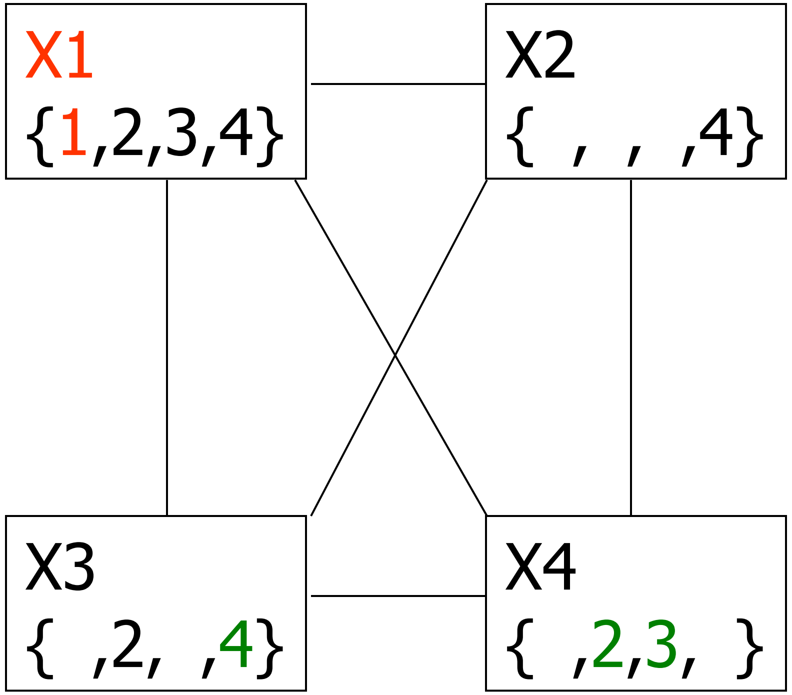

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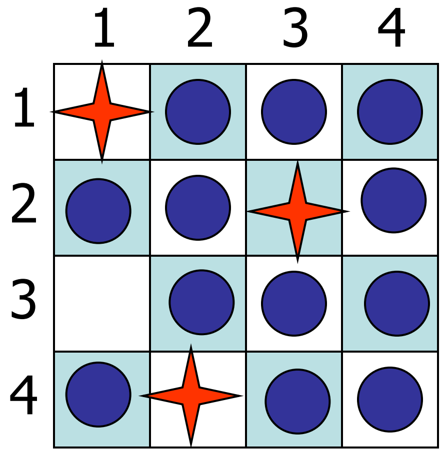

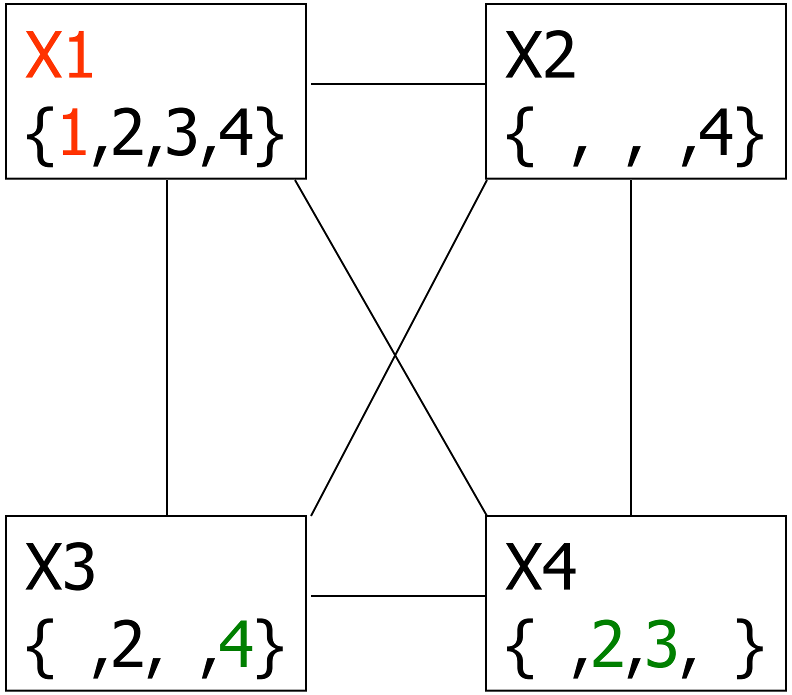

X2=4 ⇒ X3=2, which eliminates { X4=2, X4=3}
⇒ inconsistent!

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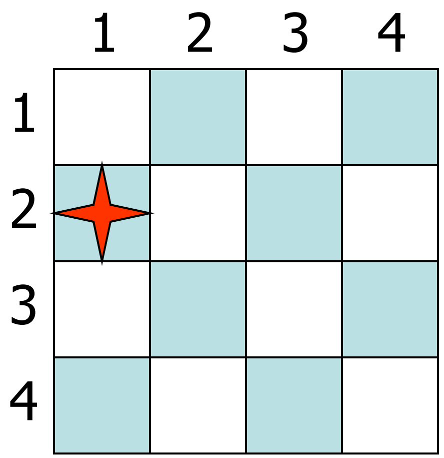

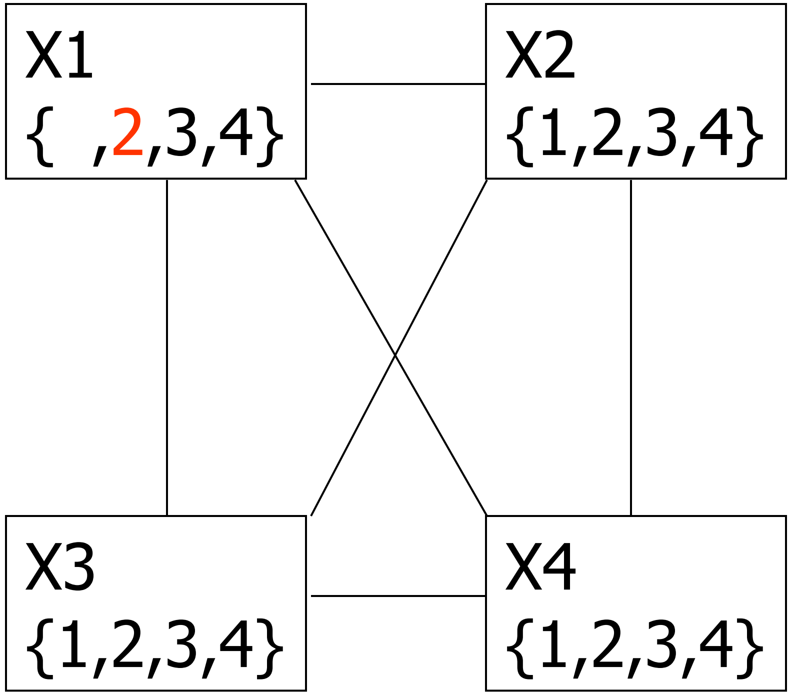

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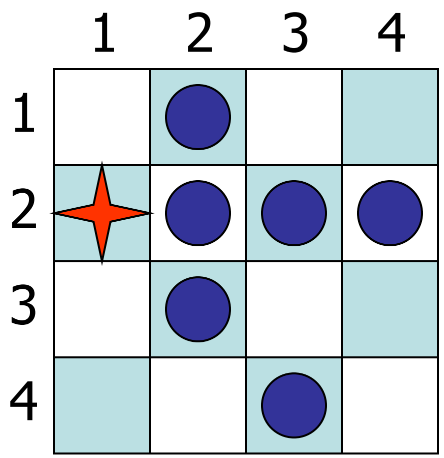

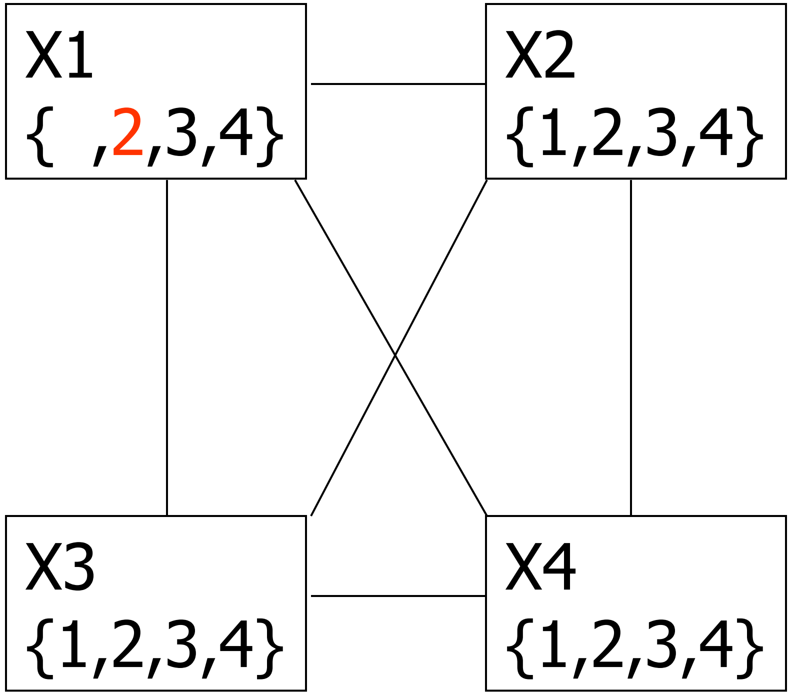

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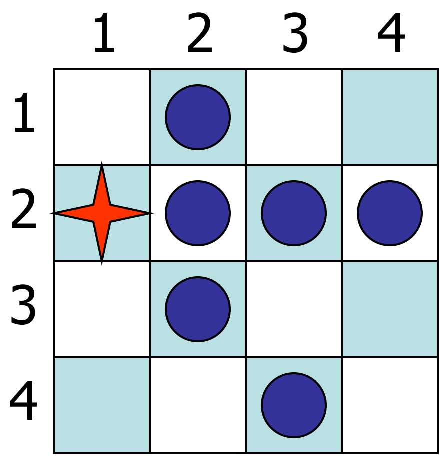
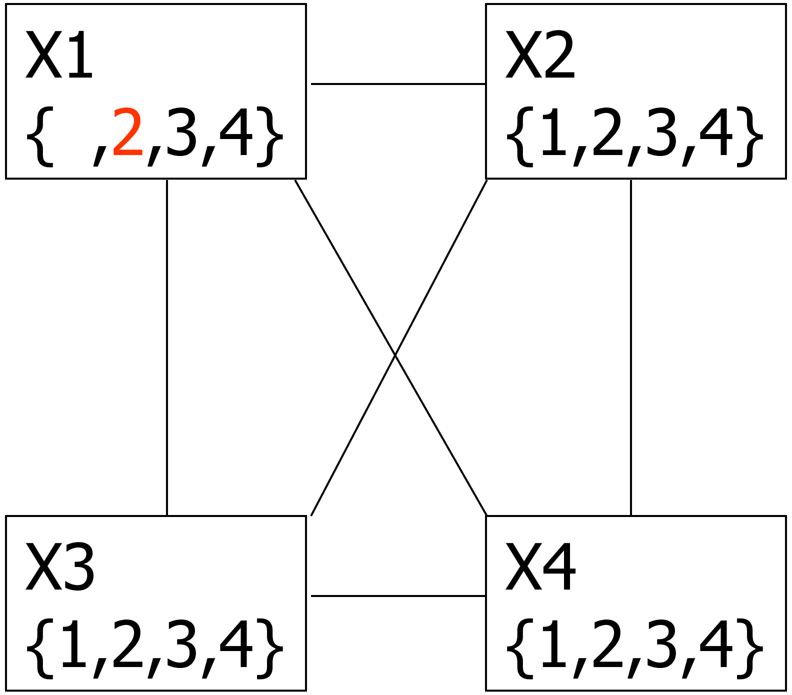
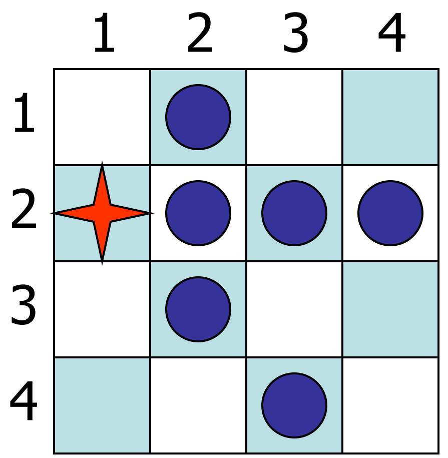
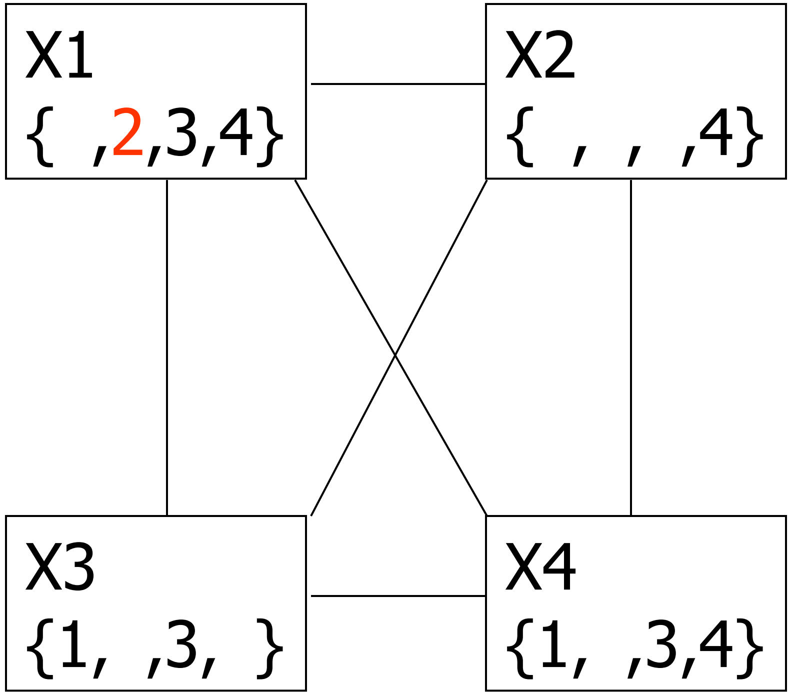




X1 can't be 1, let's try 2

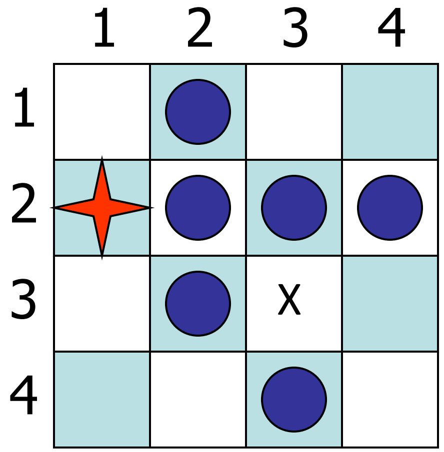
4-Queens Problem

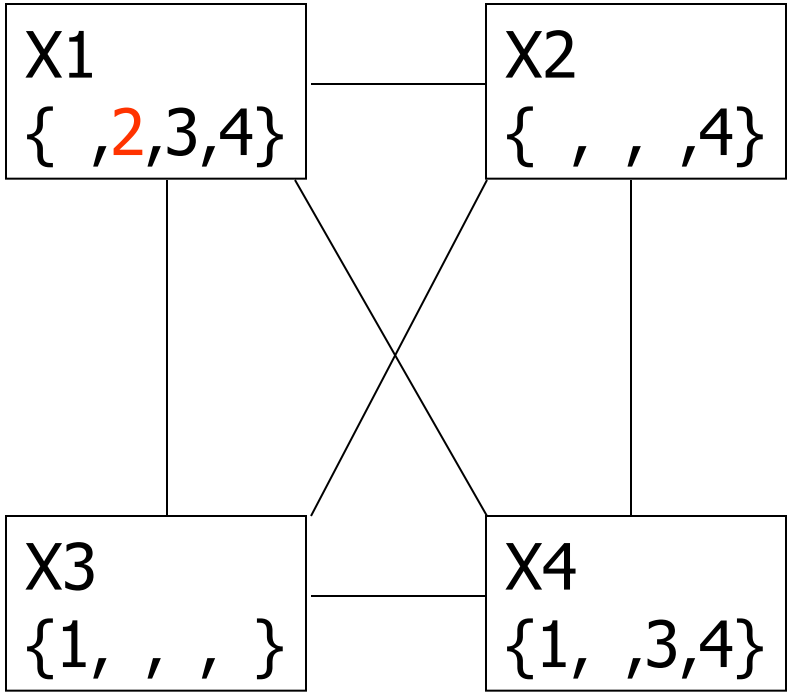




Can we eliminate any other values?

4-Queens Problem





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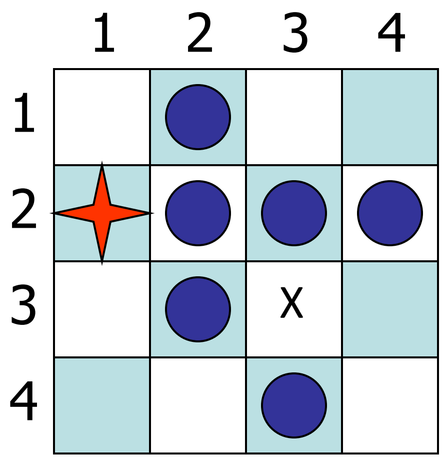

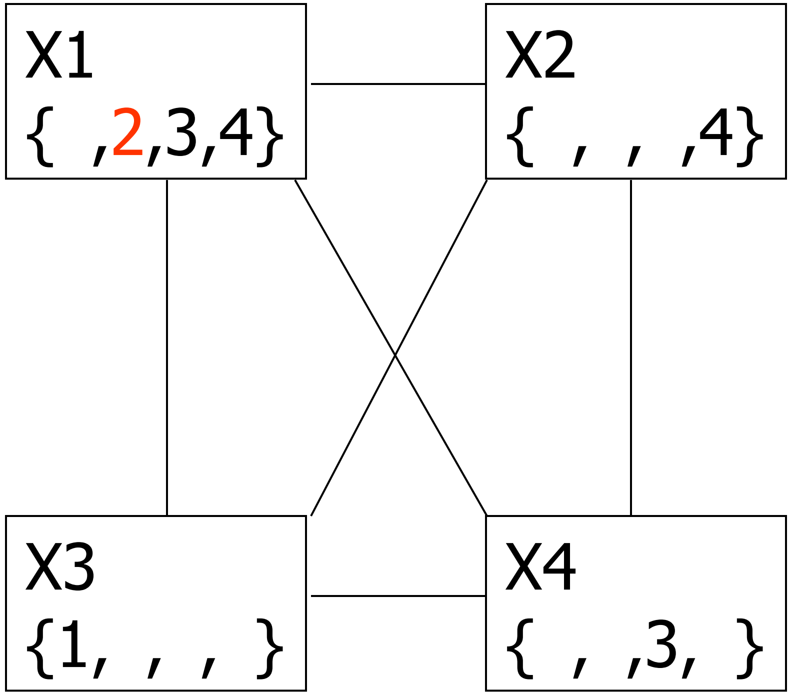

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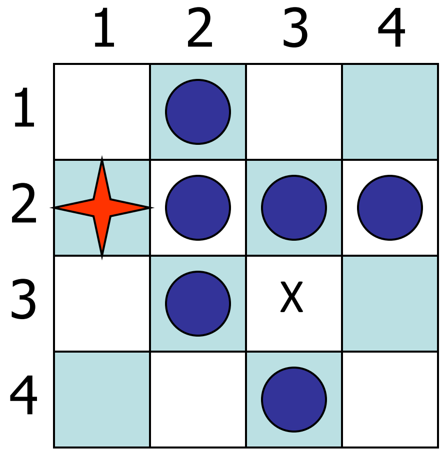
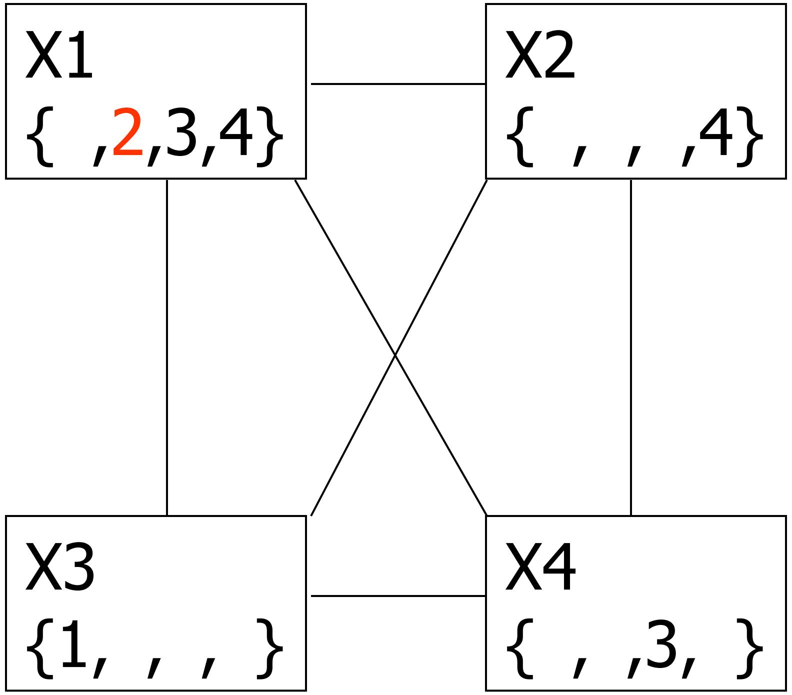
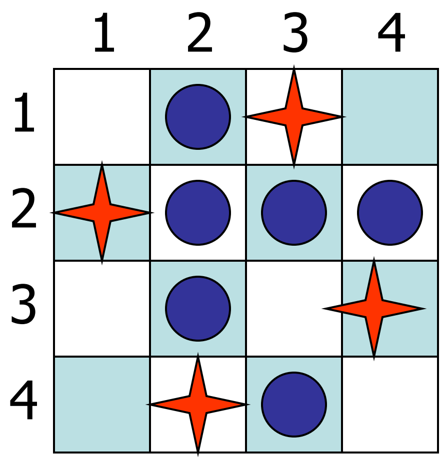
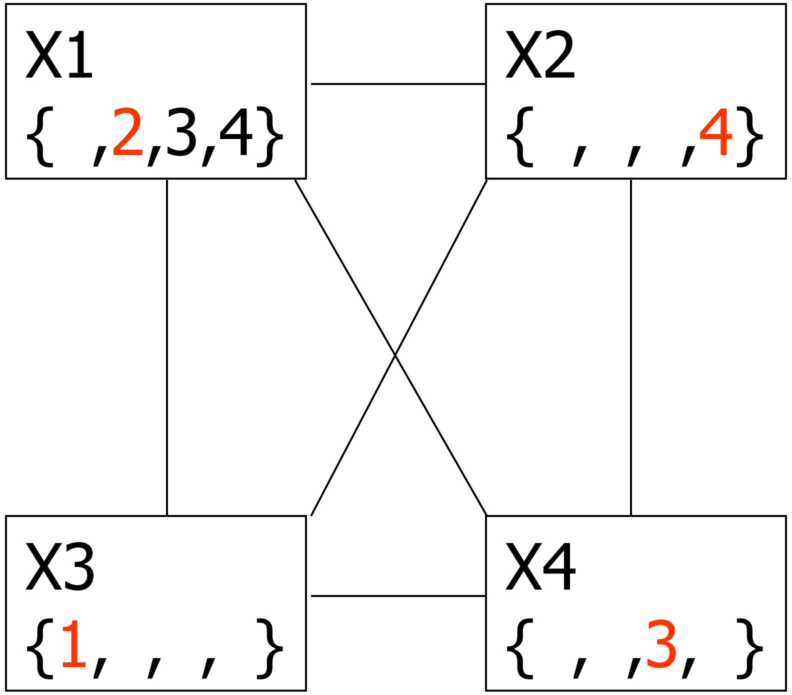




Arc constancy eliminates x3=3 because it's not consistent with X2's remaining values

4-Queens Problem





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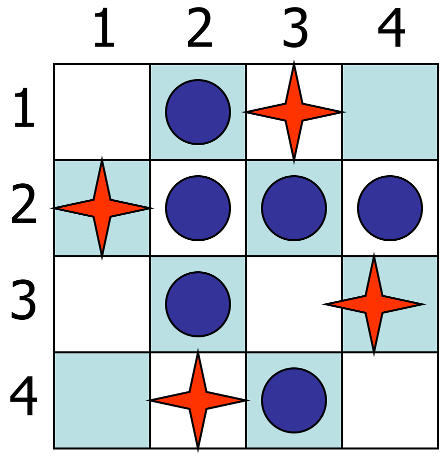

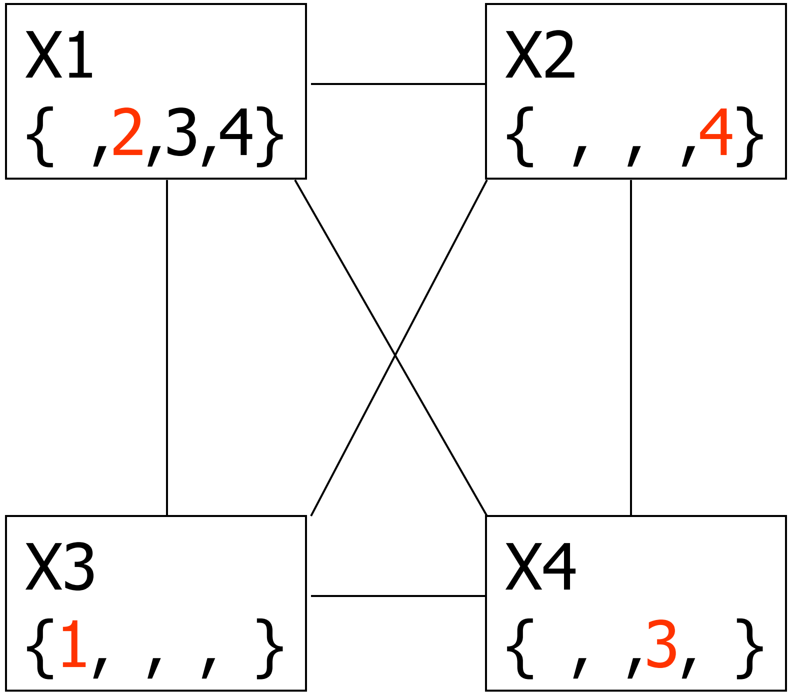

There is only one solution with X1=2

Sudoku

- Digit placement puzzle on 9x9 grid with unique answer
- Given an initial partially filled grid, fill remaining squares with a digit between 1 and 9
- Each column, row, and nine 3 × 3 sub-grids must contain all nine digits

	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

	1	2	3	4	5	6	7	8	9
A	4	8	3	9	2	1	6	5	7
B	9	6	7	3	4	5	8	2	1
C	2	5	1	8	7	6	4	9	3
D	5	4	8	1	3	2	9	7	6
E	7	2	9	5	6	4	1	3	8
F	1	3	6	7	9	8	2	4	5
G	3	7	2	6	8	9	5	1	4
H	8	1	4	2	5	3	7	6	9
I	6	9	5	4	1	7	3	8	2

- Some initial configurations are easy to solve and others very difficult

Sudoku Example

	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

initial problem

	1	2	3	4	5	6	7	8	9
A	4	8	3	9	2	1	6	5	7
B	9	6	7	3	4	5	8	2	1
C	2	5	1	8	7	6	4	9	3
D	5	4	8	1	3	2	9	7	6
E	7	2	9	5	6	4	1	3	8
F	1	3	6	7	9	8	2	4	5
G	3	7	2	6	8	9	5	1	4
H	8	1	4	2	5	3	7	6	9
I	6	9	5	4	1	7	3	8	2

a solution

How can we set this up as a CSP?

```
def sudoku(initValue):
  p = Problem()
  # Define a variable for each cell: 11,12,13...21,22,23...98,99
  for i in range(1, 10) :
    p.addVariables(range(i*10+1, i*10+10), range(1, 10))
  # Each row has different values
  for i in range(1, 10) :
    p.addConstraint(AllDifferentConstraint(), range(i*10+1, i*10+10))
  # Each column has different values
  for i in range(1, 10) :
    p.addConstraint(AllDifferentConstraint(), range(10+i, 100+i, 10))
  # Each 3x3 box has different values
  p.addConstraint(AllDifferentConstraint(), [11,12,13,21,22,23,31,32,33])
  p.addConstraint(AllDifferentConstraint(), [41,42,43,51,52,53,61,62,63])
  p.addConstraint(AllDifferentConstraint(), [71,72,73,81,82,83,91,92,93])

  p.addConstraint(AllDifferentConstraint(), [14,15,16,24,25,26,34,35,36])
  p.addConstraint(AllDifferentConstraint(), [44,45,46,54,55,56,64,65,66])
  p.addConstraint(AllDifferentConstraint(), [74,75,76,84,85,86,94,95,96])

  p.addConstraint(AllDifferentConstraint(), [17,18,19,27,28,29,37,38,39])
  p.addConstraint(AllDifferentConstraint(), [47,48,49,57,58,59,67,68,69])
  p.addConstraint(AllDifferentConstraint(), [77,78,79,87,88,89,97,98,99])

  # add unary constraints for cells with initial non-zero values
  for i in range(1, 10) :
    for j in range(1, 10):
      value = initValue[i-1][j-1]
      if value:
        p.addConstraint(lambda var, val=value: var == val, (i*10+j,))
  return p.getSolution()
```

```
# Sample problems
easy = [
 [0,9,0,7,0,0,8,6,0],
 [0,3,1,0,0,5,0,2,0],
 [8,0,6,0,0,0,0,0,0],
 [0,0,7,0,5,0,0,0,6],
 [0,0,0,3,0,7,0,0,0],
 [5,0,0,0,1,0,7,0,0],
 [0,0,0,0,0,0,1,0,9],
 [0,2,0,6,0,0,0,5,0],
 [0,5,4,0,0,8,0,7,0]]

hard = [
 [0,0,3,0,0,0,4,0,0],
 [0,0,0,0,7,0,0,0,0],
 [5,0,0,4,0,6,0,0,2],
 [0,0,4,0,0,0,8,0,0],
 [0,9,0,0,3,0,0,2,0],
 [0,0,7,0,0,0,5,0,0],
 [6,0,0,5,0,2,0,0,1],
 [0,0,0,0,9,0,0,0,0],
 [0,0,9,0,0,0,3,0,0]]

very_hard = [
 [0,0,0,0,0,0,0,0,0],
 [0,0,9,0,6,0,3,0,0],
 [0,7,0,3,0,4,0,9,0],
 [0,0,7,2,0,8,6,0,0],
 [0,4,0,0,0,0,0,7,0],
 [0,0,2,1,0,6,5,0,0],
 [0,1,0,9,0,5,0,4,0],
 [0,0,8,0,2,0,7,0,0],
 [0,0,0,0,0,0,0,0,0]]
```

Slide