CMSC 471
Artificial Intelligence

Search

KMA Solaiman – ksolaima@umbc.edu

Some material adopted from notes by Charles R. Dyer, University of Wisconsin-Madison

Many slides courtesy Tim Finin and Frank Ferraro

Classic uninformed search methods

- The four classic uninformed search methods
  - Breadth first search (BFS)
  - Depth first search (DFS)
  - Uniform cost search *(generalization of BFS)*
  - Iterative deepening *(blend of DFS and BFS)*
- To which we can add another technique
  - Bi-directional search *(hack on BFS)*

Informed (Heuristic) Search

- Heuristic search
- Best-first search
  - Greedy search
  - Beam search
  - A* Search
- Memory-conserving variations of A*
- Heuristic functions

Heuristics, More Formally

$h(n)$ is a **heuristic function**, that maps a state n to an estimated cost from n-to-goal

$h(n)$ is **admissible** iff $h(n) \leq$ the lowest actual cost from n-to-goal

$h(n)$ is **consistent** iff

$$h(n) \leq \text{lowestcost}(n, n') + h(n')$$

Informed methods add domain-specific information

- Select best path along which to continue searching
- h(n): estimates *goodness* of node n
- h(n) = **estimated cost** (or distance) of minimal cost path from n **to a goal state**.
- Based on domain-specific information and computable from current state description that estimates how close we are to a goal

Heuristics

- **All domain knowledge** used in search is encoded in the **heuristic function, h(<node>)**
- Examples:
  - 8-puzzle: number of tiles out of place
  - 8-puzzle: sum of distances each tile is from its goal
  - Missionaries & Cannibals: # people on starting river bank
- In general
  - $h(n) \geq 0$ for all nodes n
  - $h(n)$ = 0 implies that n is a goal node
  - $h(n) = \infty$ implies n is a dead-end that can't lead to goal

Best-first search

- Search algorithm that improves **depth-first search** by expanding most promising node chosen according to heuristic rule

- Order nodes on nodes list by increasing value of an evaluation function, **f(n)**, incorporating domain-specific information

Best-first search

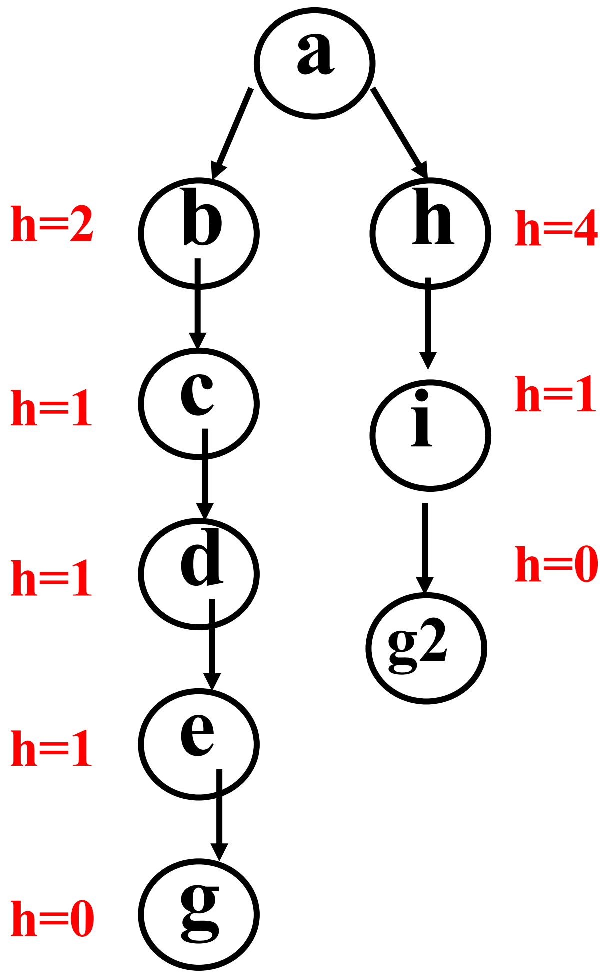
- Search algorithm that improves **depth-first search** by expanding most promising node chosen according to heuristic rule
- Order nodes on nodes list by increasing value of an evaluation function, **f(n)**, incorporating domain-specific information
- This is a generic way of referring to the class of informed methods

Greedy best first search

- A greedy algorithm makes locally optimal choices in hope of finding a global optimum
- Uses evaluation function $f(n) = h(n)$, sorting nodes by increasing values of f
- Selects node to expand appearing **closest** to goal (i.e., node with smallest f value)
- Not complete
- Not admissible, as in example
  - Assume arc costs = 1, greedy search finds goal g, with solution cost of 5
  - Optimal solution is path to goal with cost 3

Greedy best first search example

- Proof of non-admissibility
  - Assume arc costs = 1, greedy search finds goal g, with solution cost of 5
  - Optimal solution is path to goal with cost 3



Beam search

- Use evaluation function f(n), but maximum size of the nodes list is k, a fixed constant
- Only keep k best nodes as candidates for expansion, discard rest
- k is the *beam width*
- More space efficient than greedy search, but may discard nodes on a solution path
- As k increases, approaches best first search
- Complete?
- Admissible?

Beam search

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- As k increases, approaches best first search
- Not complete
- Not admissible

We’ve *got* to be able to do better, right?

Let’s think about car trips…

A* Search

Use an evaluation function

$$f(n) = g(n) + h(n)$$

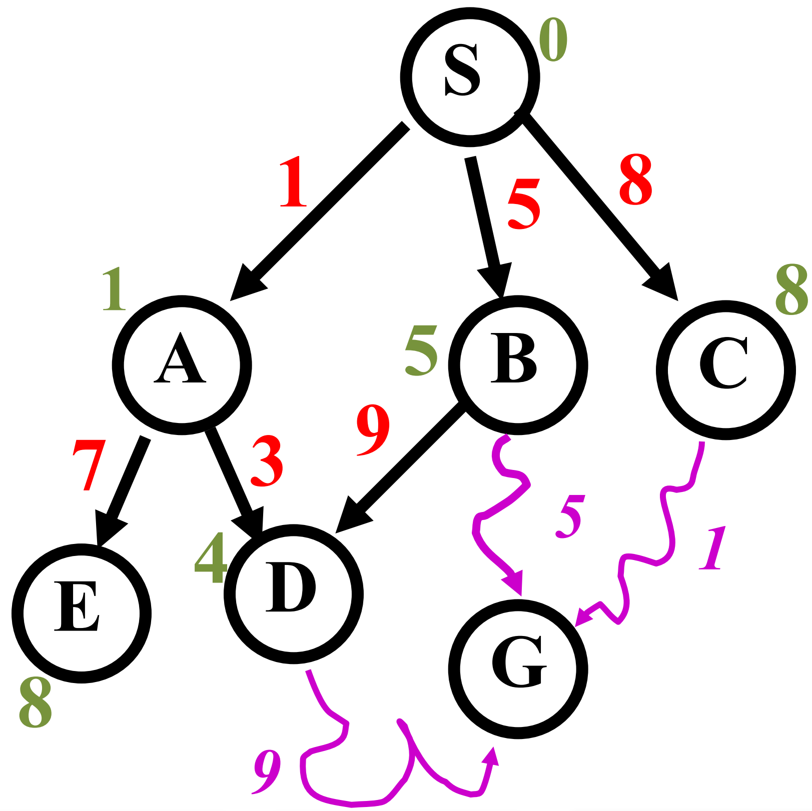
estimated total cost from start to goal via state n = minimal-cost path from the start state to state n + cost estimate from state n to the goal

A* Search

- Use as an evaluation function

  f(n) = g(n) + h(n)

- g(n) = minimal-cost path from the start state to state n
- g(n) adds "breadth-first" term to evaluation function
- Ranks nodes on search frontier by estimated cost of solution from start node ***via given node*** to goal



g(d)=4	g(b)=5	g(c)=8
h(d)=9	h(b)=5	h(c)=1
f(d)=13	f(d)=10	f(c)=9

A* Search

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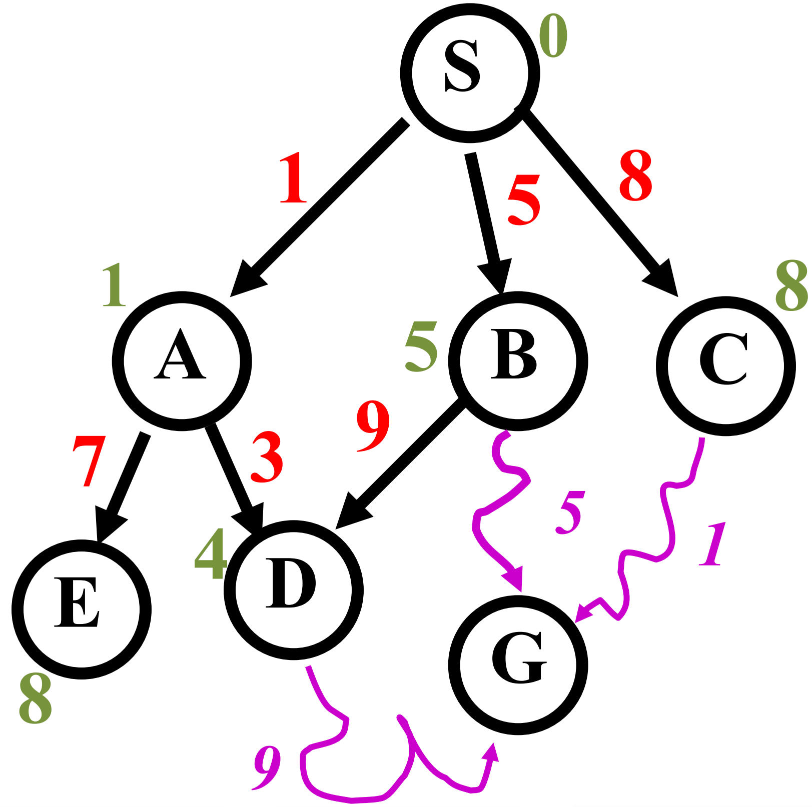


g(d)=4
h(d)=9
f(d)=13

g(b)=5
h(b)=5
f(d)=10

g(c)=8
h(c)=1
f(c)=9

C is chosen next to expand

A* Search

- Use an evaluation function

$$\mathbf{f(n) = g(n) + h(n)}$$

estimated total cost from start to goal via state n = minimal-cost path from the start state to state n + cost estimate from state n to the goal

- g(n) term adds "breadth-first" component to evaluation function
- Ranks nodes on search frontier by estimated cost of solution from start node *via given node* to goal
- Not complete if h(n) can = ∞
- Is it admissible?

A*

- Pronounced *"a star"*
- h is **admissible** when h(n) <= h*(n) holds
  - **h*(n)** = *true cost* of *minimal cost path* from n to a goal
- Using an admissible heuristic guarantees that 1st solution found will be an **optimal** one
  - With an admissible heuristic, A* is cost-optimal
- A* is **complete** whenever branching factor is finite and every action has fixed, positive cost
- A* is **admissible**

Hart, P. E.; Nilsson, N. J.; Raphael, B. (1968). "A Formal Basis for the Heuristic Determination of Minimum Cost Paths". *IEEE Transactions on Systems Science and Cybernetics SSC4* **4** (2): 100–107.

Implementing A*

Q: Can this be an instance of our general search algorithm?

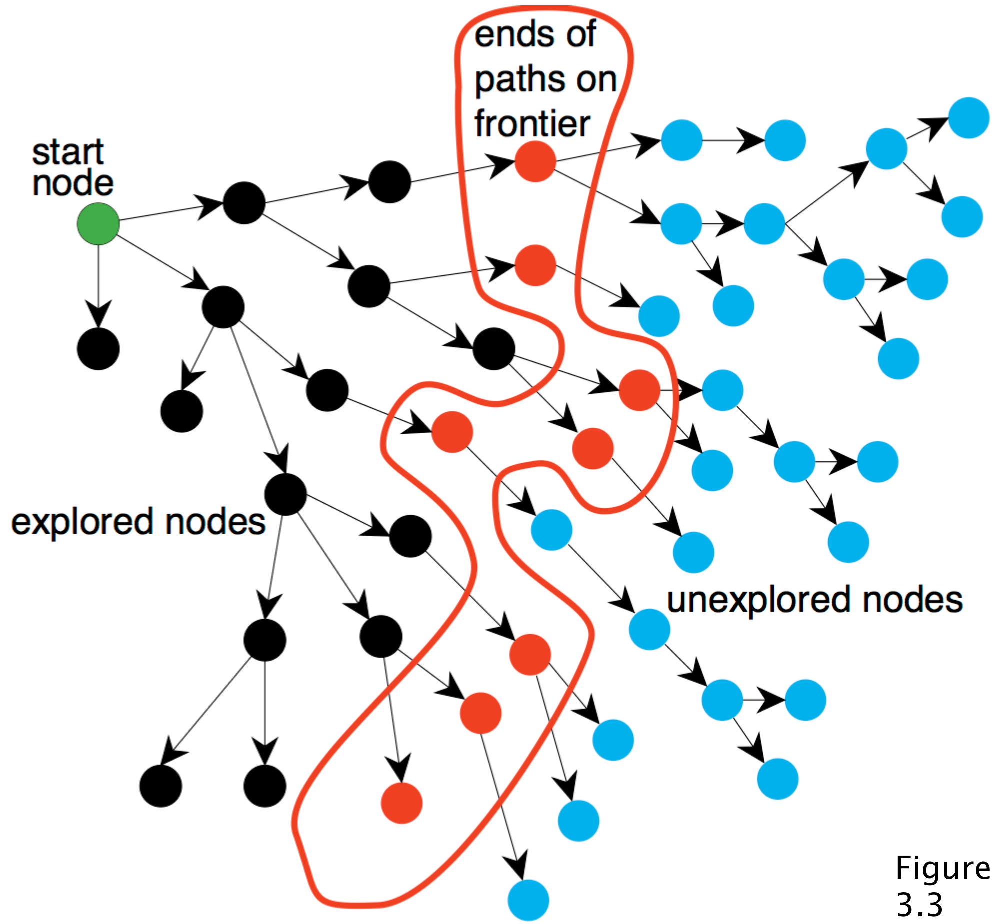


Figure 3.3

Implementing A*

Q: Can this be an instance of our general search algorithm?

A: Yup! Just make the fringe a priority queue ordered by $f(n)$

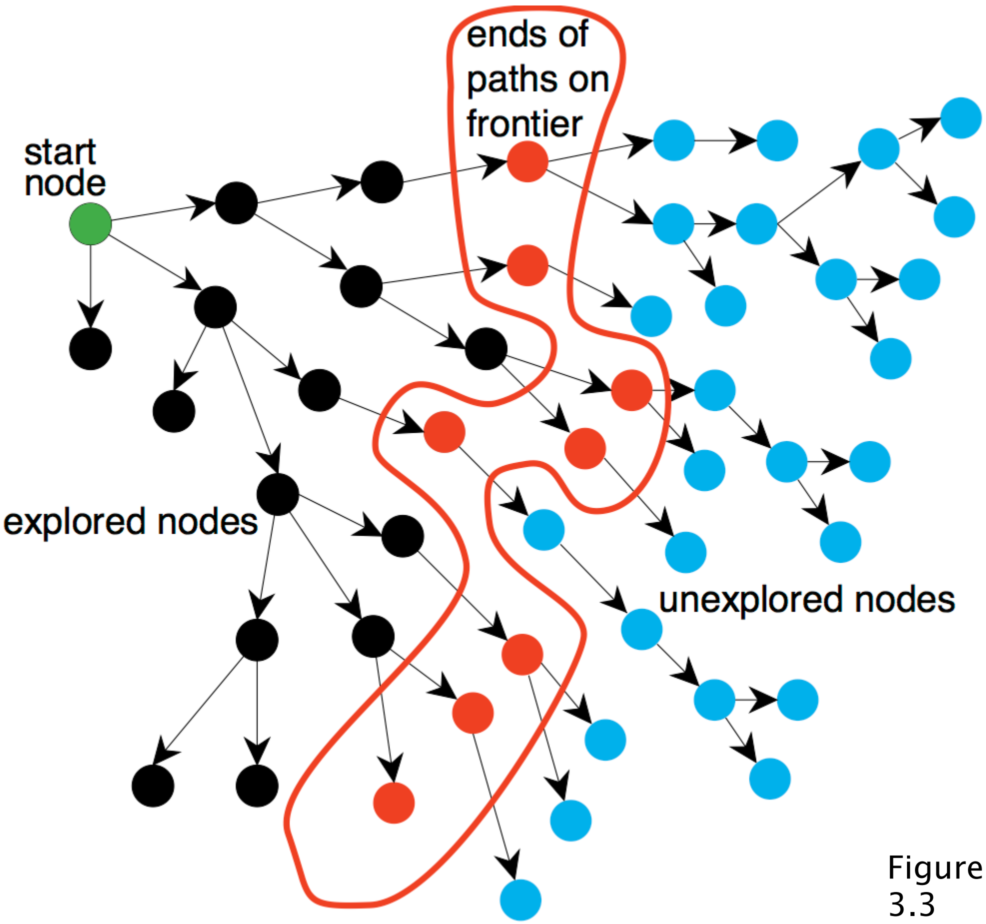


Figure 3.3

Alternative A* Pseudo-code

1 Put the start node S on the nodes list, called OPEN

2 If OPEN is empty, exit with failure

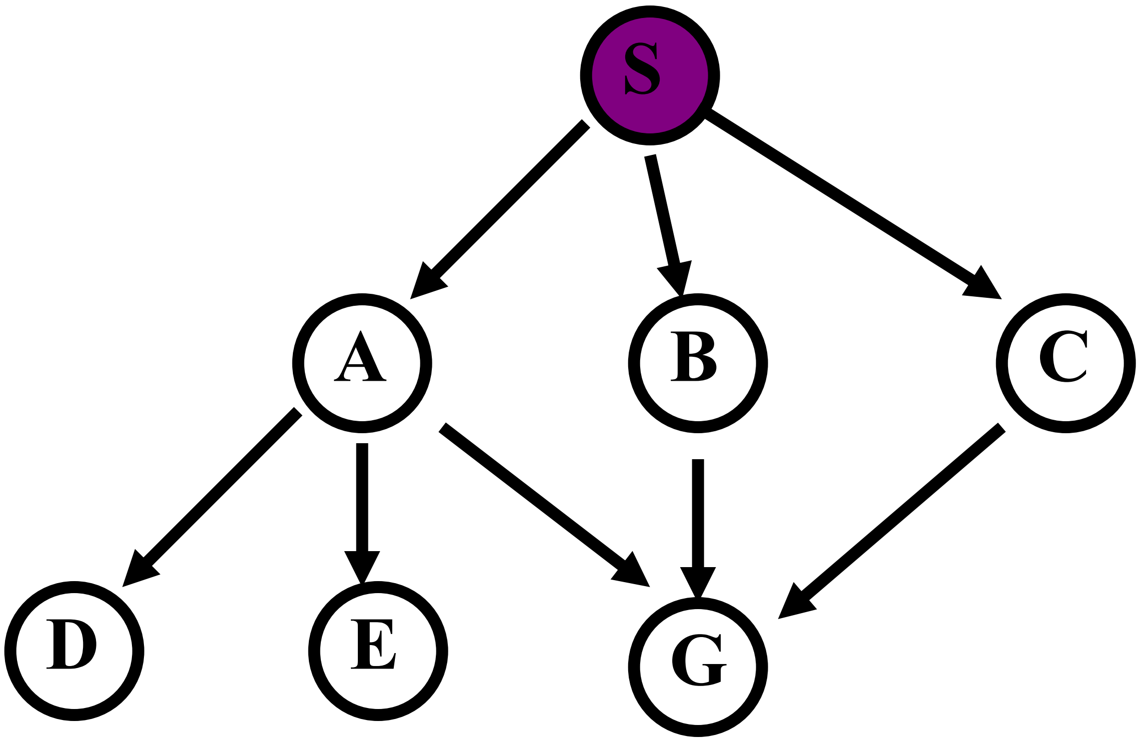
3 Select node in OPEN with minimal f(n) and place on CLOSED

4 If n is a goal node, collect path back to start and stop

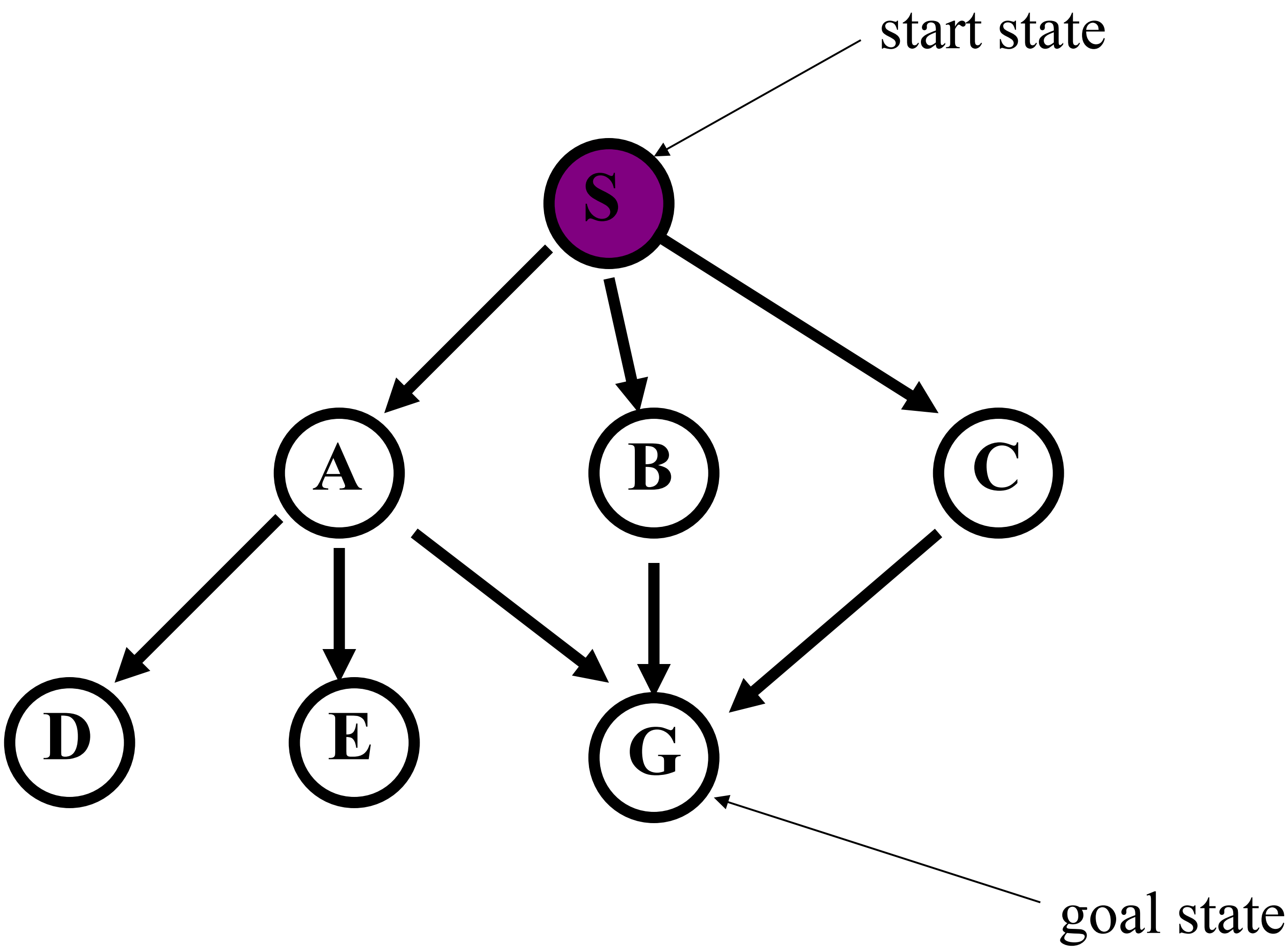
5 Expand n, generating all its successors and attach to them pointers back to n. For each successor n' of n

- **1** If n' not already on OPEN or CLOSED
  - put n' on OPEN
  - compute h(n'), g(n')=g(n)+ c(n,n'), f(n')=g(n')+h(n')
- **2** If n' already on OPEN or CLOSED and if g(n') is lower for new version of n', then:
  - Redirect pointers backward from n' on path with lower g(n')
  - Put n' on OPEN

Example search space



Example search space



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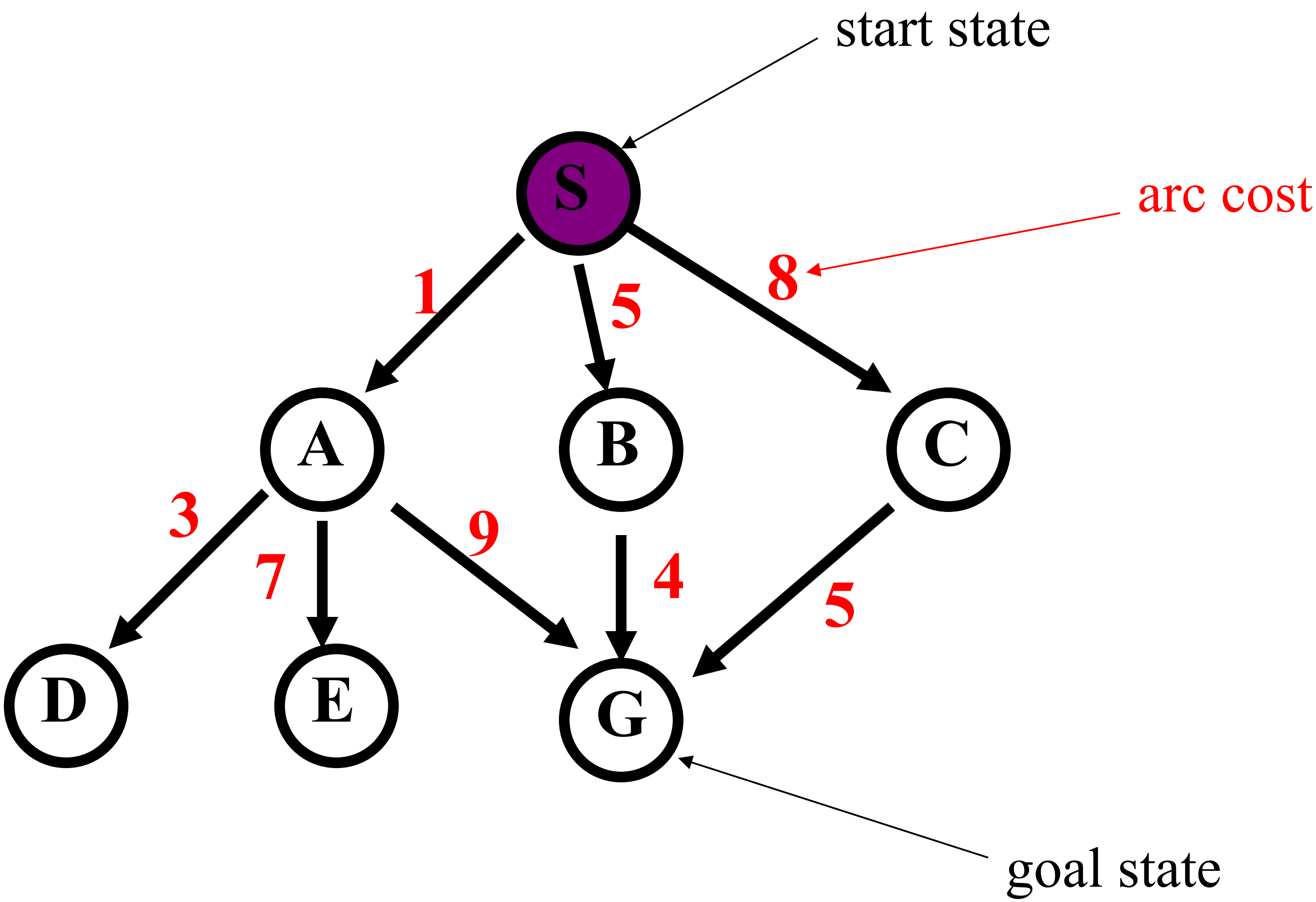

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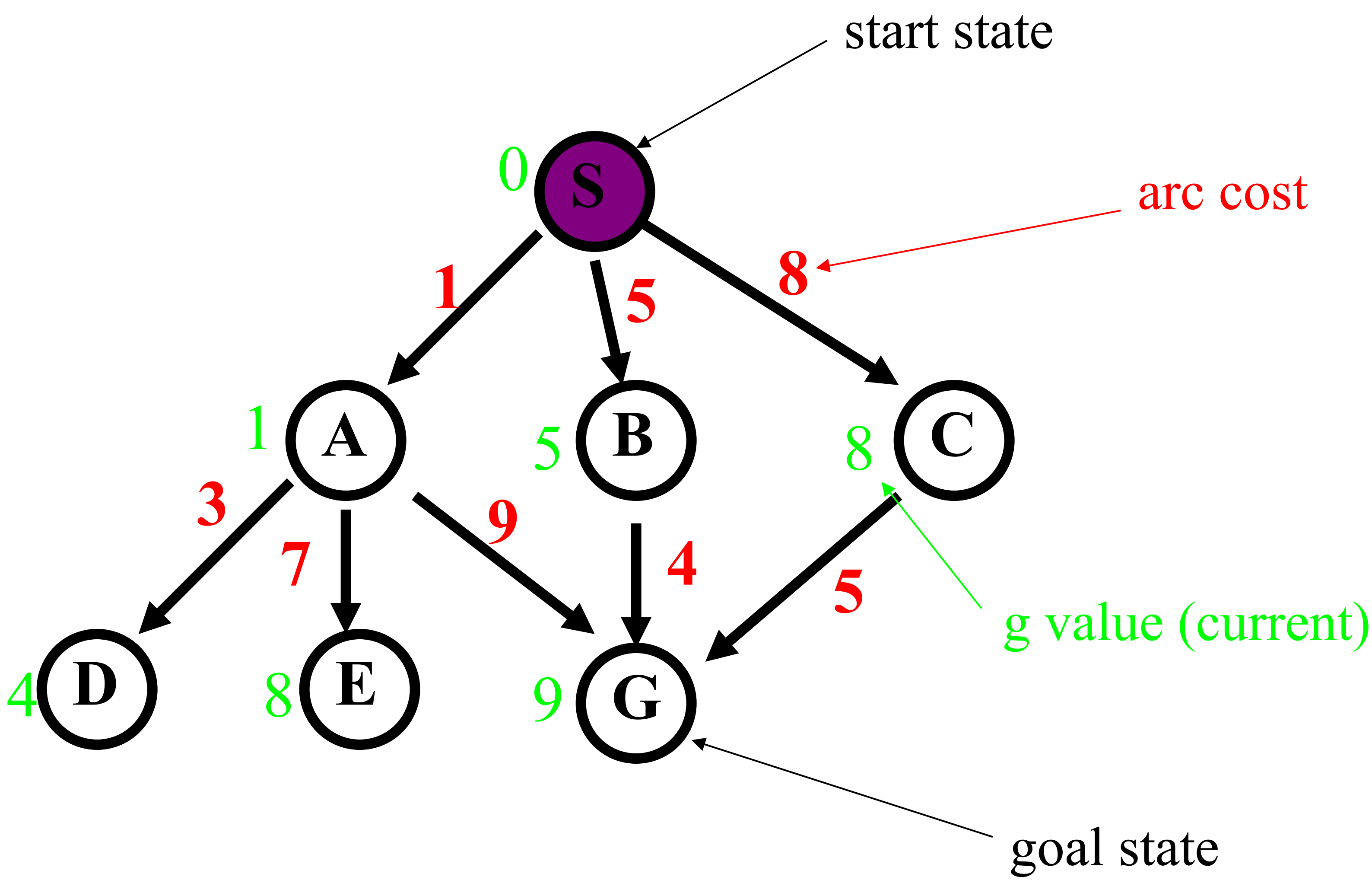

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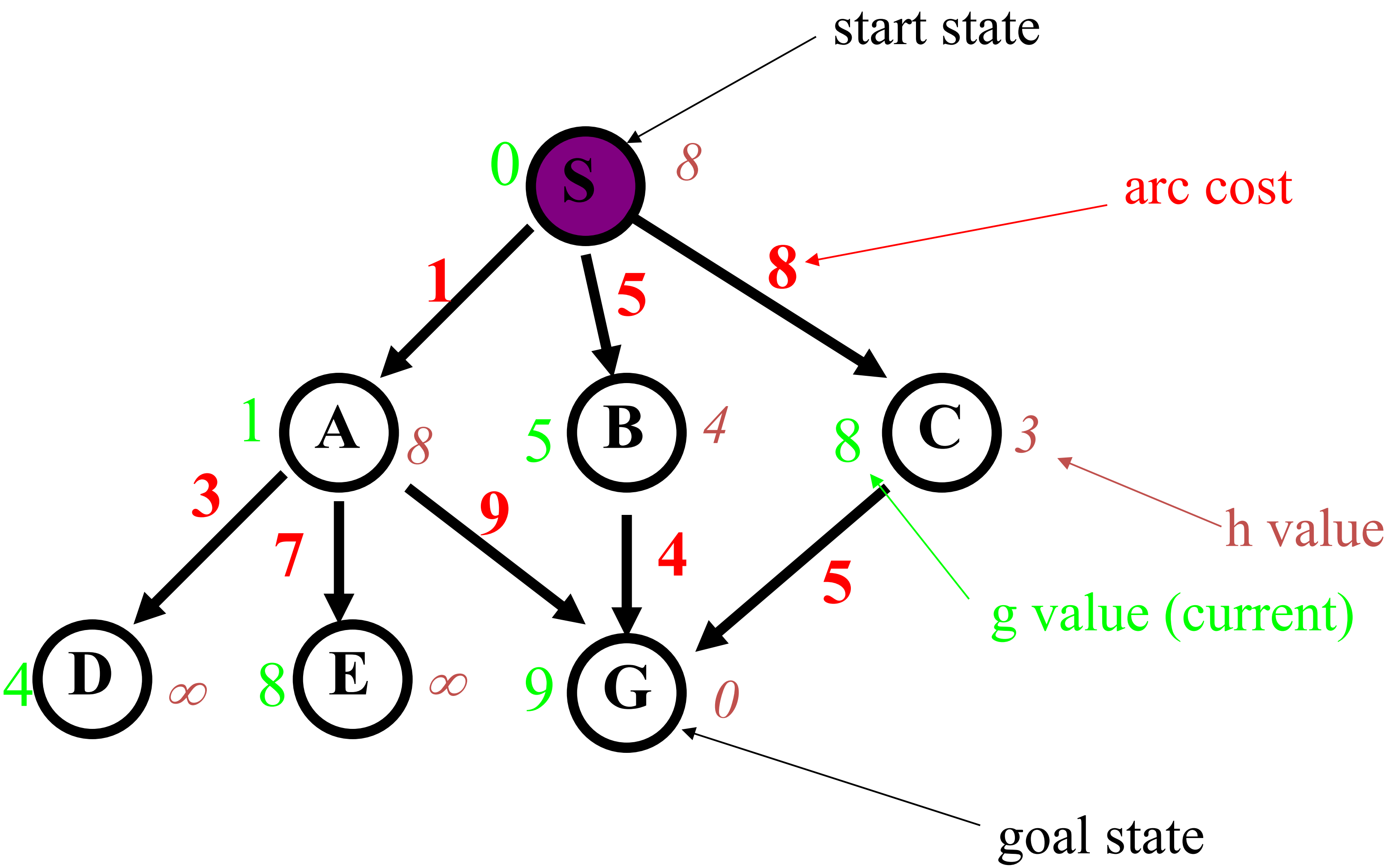

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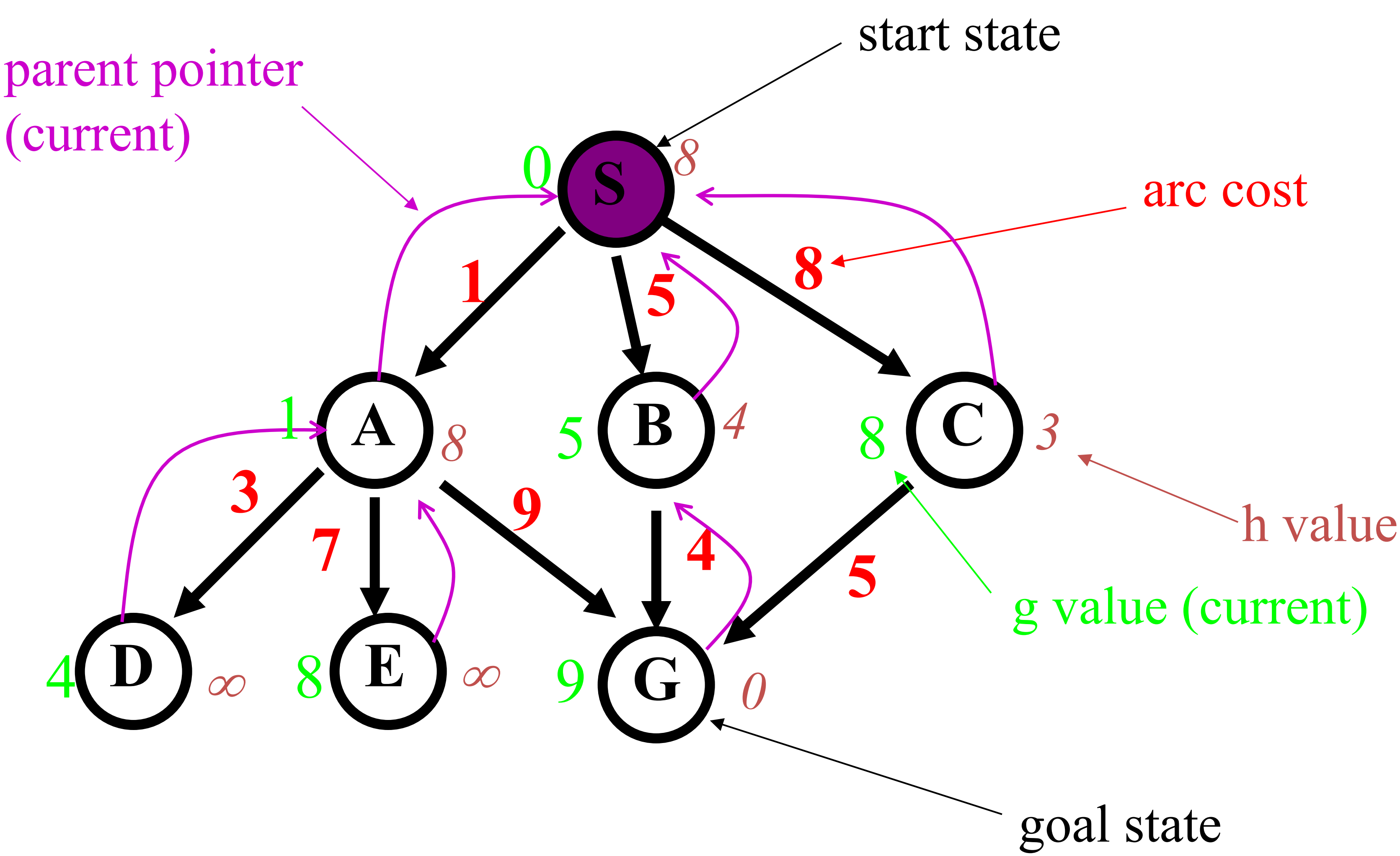
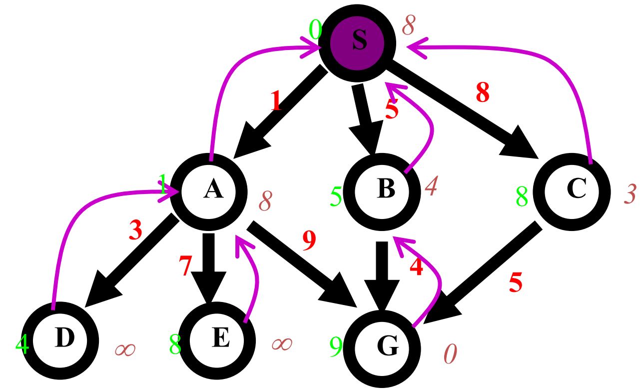


Example

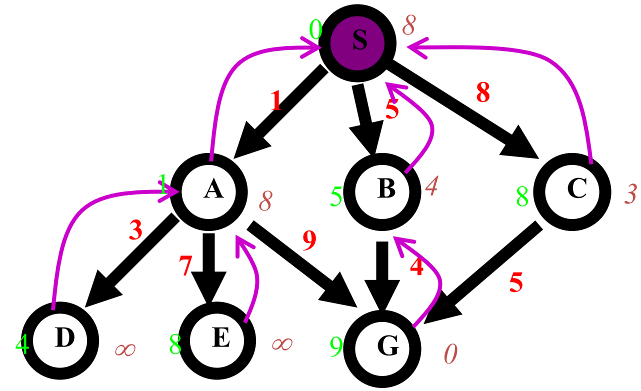


n	g(n)	h(n)	f(n)	h*(n)
S	0	8	8	9

- h*(n) is (hypothetical) perfect heuristic (an oracle)
- Since h(n) <= h*(n) for all n, h is admissible (optimal)
- Optimal path = *S B G* with cost 9

The table and graph show values for the entire space, but we must discover or compute them during the search

Example



n	g(n)	h(n)	f(n)	h*(n)
S	0	8	8	9
A	1	8	9	9
B	5	4	9	4
C	8	3	11	5
D	4	inf	inf	inf
E	8	inf	inf	inf
G	9	0	9	0

- h*(n) is (hypothetical) perfect heuristic (an oracle)
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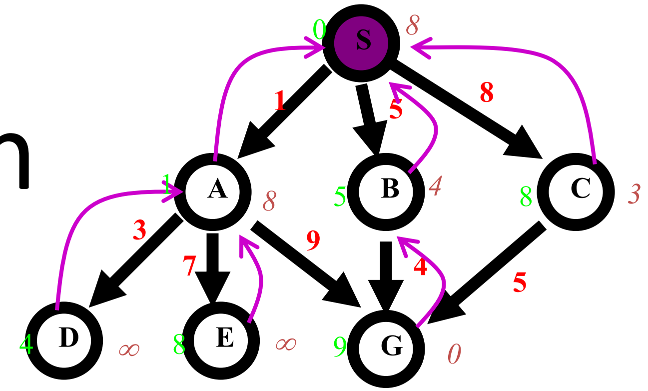
Greedy search



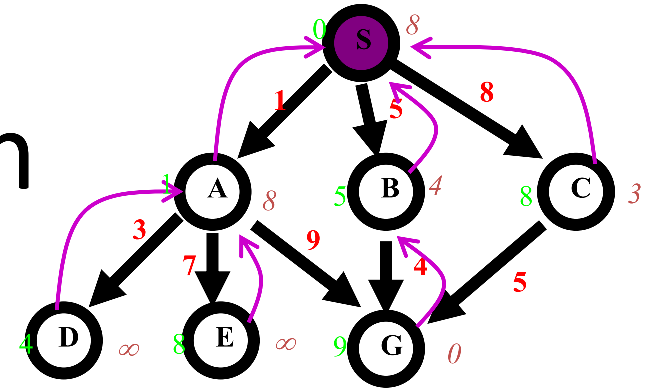
f(n) = h(n)

node expanded **nodes list**

{ S(8) }

what's next???

Greedy search

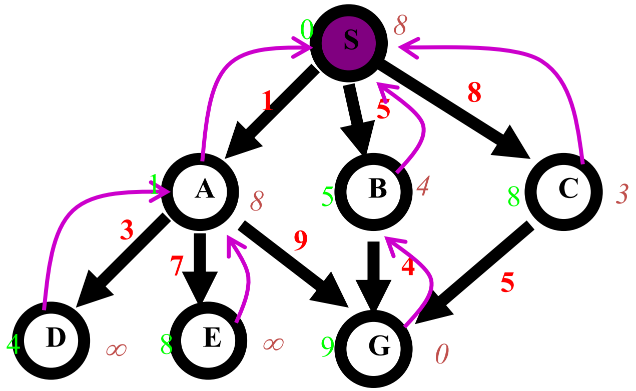


f(n) = h(n)

node expanded	**nodes list**
	{ S(8) }
S	{ C(3) B(4) A(8) }
C	{ G(0) B(4) A(8) }
G	{ B(4) A(8) }

- Solution path found is S C G, 3 nodes expanded.
- See how fast the search is!! But it is NOT optimal.

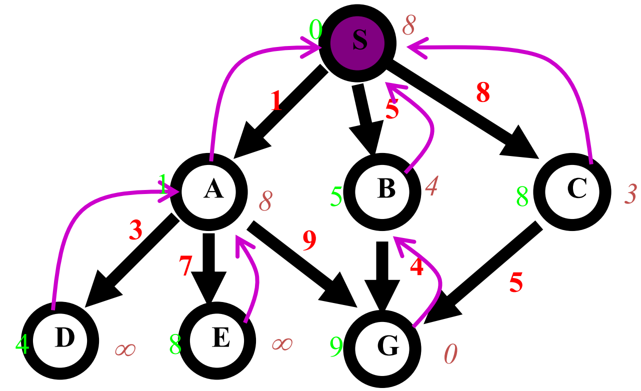
A* search



f(n) = g(n) + h(n)

```
node exp.      nodes list
            { S(8) }
                    What's next?
```

A* search



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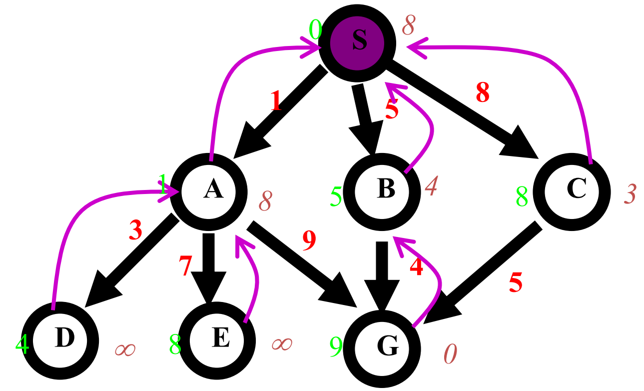
```
node exp.        nodes list
              { S(8) }
 S            { A(9) B(9) C(11) }
                   What’s next?
```

h(n)

h(S)=8
h(A)=8
h(B)=4
h(C)=3
h(D)=inf
h(E)=inf
h(G)=0

A* search

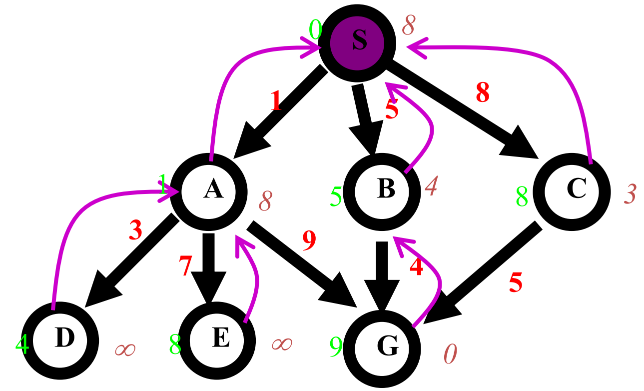


f(n) = g(n) + h(n)

node exp.	nodes list
	{ S(8) }
S	{ A(9) B(9) C(11) }
A	{ B(9) G(10) C(11) D(inf) E(inf) }

What’s next?

A* search



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                    What's next?
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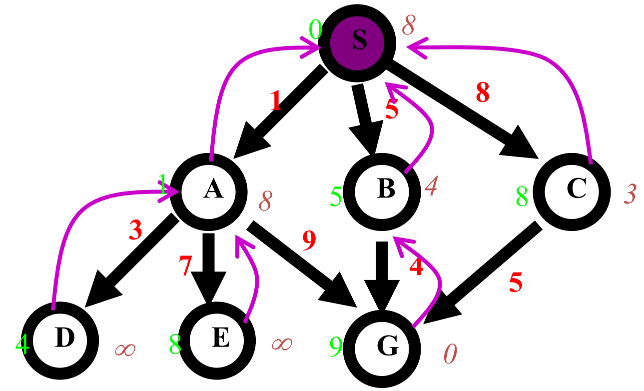
A* search



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 G          { C(11) D(inf) E(inf) }
```

- Solution path found is S B G, 4 nodes expanded..
- Still pretty fast. And optimal, too.

Observations on A*

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  - If A1* uses h1, and A2* uses h2, then every node expanded by A2* is also expanded by A1*

    i.e., A1 expands at least as many nodes as A2*
  - We say that A2* is *better informed* than A1*

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    i.e., A1 expands at least as many nodes as A2*
  - We say that A2* is *better informed* than A1*
- ***The closer h to h*, the fewer extra nodes expanded***

Proof of the optimality of A*

- Assume that A* has selected G2, a goal state with a suboptimal solution, i.e., $g(G2) > f^*$
- Proof by contradiction shows it's impossible

Proof of the optimality of A*

- Assume that A* has selected G2, a goal state with a suboptimal solution, i.e., g(G2) > f*
- Proof by contradiction shows it's impossible
  - Choose a node n on an optimal path to G
  - Because h(n) is admissible, f* >= f(n)
  - If we choose G2 instead of n for expansion, then f(n) >= f(G2)
  - This implies f* >= f(G2)
  - G2 is a goal state: h(G2) = 0, f(G2) = g(G2).
  - Therefore f* >= g(G2)   => g(G2) <= f*
  - Contradiction

Dealing with hard problems

- For large problems, A* may require too much space
- Variations conserve memory: IDA* and SMA*
- IDA*, iterative deepening A*, uses successive iteration with growing limits on f, e.g.
  - A* but don't consider a node n where f(n) >10
  - A* but don't consider a node n where f(n) >20
  - A* but don't consider a node n where f(n) >30, ...
- SMA* -- Simplified Memory-Bounded A*
  - Uses queue of restricted size to limit memory use

IDA*: iterative deepening A*

Use successive iteration with growing limits on f, e.g.

- A* but don't consider a node n where f(n) >10
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SMA*: Simplified Memory-Bounded A*

Uses queue of restricted size to limit memory use

How to find good heuristics

Some options (mix-and-match):

- If h1(n) < h2(n) <= h*(n) for all n, h2 is better than h1
  - h2 **dominates** h1
- **Relaxing problem:** remove constraints for easier problem; use its solution cost as heuristic function
- Max of two admissible heuristics is a **Combining heuristics**: admissible heuristic, and it’s better!
- Use statistical estimates to compute h; may lose admissibility
- Identify good features, then use **machine learning** to find heuristic function; also may lose admissibility

Pruning: Dealing with Large Search Spaces

Cycle pruning

Don’t add a node to the fringe if you’ve already expanded it (it’s already on a path you’ve considered/are considering)

Q: What type of search-space would this be approach be applicable for?

Multiple-path pruning

Pruning: Dealing with Large Search Spaces

Cycle pruning

Don’t add a node to the fringe if you’ve already expanded it (it’s already on a path you’ve considered/are considering)

Q: What type of search-space would this be approach be applicable for?

Multiple-path pruning

Core idea: there may be multiple possible solutions, but you only need one

Maintain an “explored” (sometimes called “closed”) set of nodes at the ends of paths; discard a path if a path node appears in this set

Q: Does this return an optimal solution?

Optimality with Multiple-Path Pruning

Some options to find the optimal solution (pulled from Ch 3.7.2)

- Make sure that the first path found to any node is a lowest-cost path to that node, then prune all subsequent paths found to that node. OR

Optimality with Multiple-Path Pruning

Some options to find the optimal solution (pulled from Ch 3.7.2)

- Make sure that the first path found to any node is a lowest-cost path to that node, then prune all subsequent paths found to that node. OR

- If the search algorithm finds a lower-cost path to a node than one already found, it could remove all paths that used the higher-cost path to the node. OR

Optimality with Multiple-Path Pruning

Some options to find the optimal solution (pulled from Ch 3.7.2)

- Make sure that the first path found to any node is a lowest-cost path to that node, then prune all subsequent paths found to that node. OR
- If the search algorithm finds a lower-cost path to a node than one already found, it could remove all paths that used the higher-cost path to the node. OR
- Whenever the search finds a lower-cost path to a node than a path to that node already found, it could incorporate a new initial section on the paths that have extended the initial path.

A* and Multiple-Path Pruning

If $h(n)$ is consistent, A* with multiple-path pruning will find an optimal solution

Core Idea: Why?

A* and Multiple-Path Pruning

If $h(n)$ is consistent, A* with multiple-path pruning will find an optimal solution

Core Idea: Why? (proof by contradiction: see Proposition 3.2 in Ch 3.7.2)

Summary: Informed search

- **Best-first search** is general search where minimum-cost nodes (w.r.t. some measure) are expanded first
- **Greedy search** uses minimal estimated cost h(n) to goal state as measure; reduces search time, but is neither complete nor optimal
- **A* search** combines uniform-cost search & greedy search: $f(n) = g(n) + h(n)$. Handles state repetitions & h(n) never overestimates
  - A* is complete & optimal, but space complexity high
  - Time complexity depends on quality of heuristic function
  - IDA* and SMA* reduce the memory requirements of A*

Summary (Fig 3.11)

Strategy	Selection from Frontier	Path found	Space
Breadth-first	First node added	Fewest arcs	Exponential
Depth-first	Last node added	No	Linear
Iterative deepening	—	Fewest arcs	Linear
Greedy best-first	Minimal $h(p)$	No	Exponential
Lowest-cost-first	Minimal cost (p)	Least cost	Exponential
A^*	Minimal cost $(p) + h(p)$	Least cost	Exponential
IDA^*	—	Least cost	Linear